Origin of Termination of Negative-Parity Bands

Contents

Work of

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2.Introduction

Strong reflection-asymmetric correlations or shapes near ground state

Probable possibilities:

- Octupole deformation
- Clustering : formation of di-cluster system

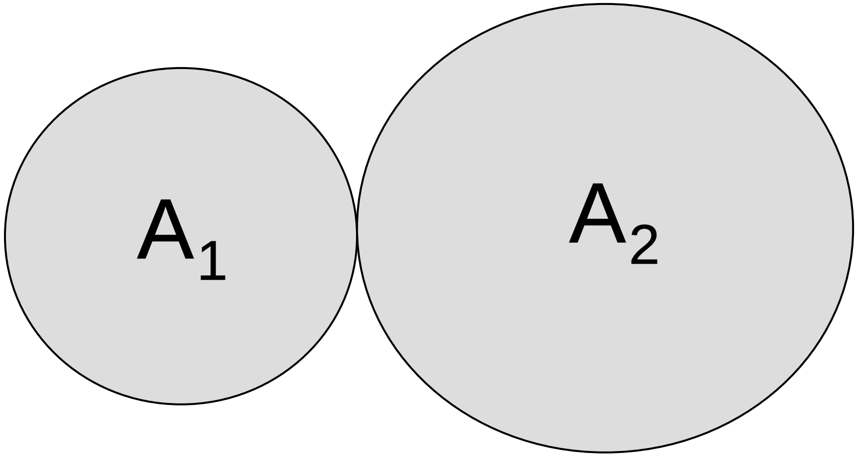
Di-cluster or Dinuclear or Molecular system

This system has two main degrees of freedom:

1. Relative motion ($\boldsymbol{R}$) of nuclei,
   molecular resonances,
   decay of the dinuclear system: fission, quasifission, emission of clusters
2. Transfer of nucleons between nuclei,
   change of mass and charge asymmetries

Mass asymmetry coordinate

$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$



$\eta = 0$ for $A_1 = A_2$, $\eta = \pm 1$ for A_1 or $A_2 = 0$

2. Dinuclear System (DNS) Model

Applications:

- Nuclear structure phenomena: normal- super- and hyper-deformed bands
- Cluster- and alpha-decay
- Fusion to heavy & superheavy nuclei
- Multi-nucleon transfer reactions
- Deep-inelastic reactions
- Fission

Mass asymmetry motion

For nuclear structure studies we assume η as a continuous coordinate and solve a Schrödinger equation in mass asymmetry:

$$\left(-\frac{\hbar^2}{2}\frac{d}{d\eta}\frac{1}{B(\eta)}\frac{d}{d\eta}+U(\eta,I)\right)\psi_I(\eta)=E_{n,I}\psi_I(\eta)$$

Wave function $\psi_I(\eta)$ contains different cluster and clusterless (mononucleus)configurations.

$$U(R,\eta,I) = B_1+B_2-B_0+V(R,\eta,I)$$

$$V(R,\eta,I) = V_{Coul}(R,\eta)+V_N(R,\eta)+\hbar^2 I(I+1)/\left(2\Im(R,\eta)\right)$$

Potential and moments of inertia

Clusterisation is most stable in minima of potential U as a function of η. Minima by shell effects, e.g. magic clusters.

Potential energy of dinuclear system:

$$U(R,\eta,I) = B_1 + B_2 - B_0 + V(R,\eta,I)$$

$$V(R,\eta,I) = V_{Coul}(R,\eta) + V_N(R,\eta) + \hbar^2 I(I+1)/(2\Im(R,\eta))$$

B_1, B_2, B_0 are negative binding energies of clusters and mononucleus ($|\eta|=1$).

$V(R,\eta,I)$ is the nucleus-nucleus potential.

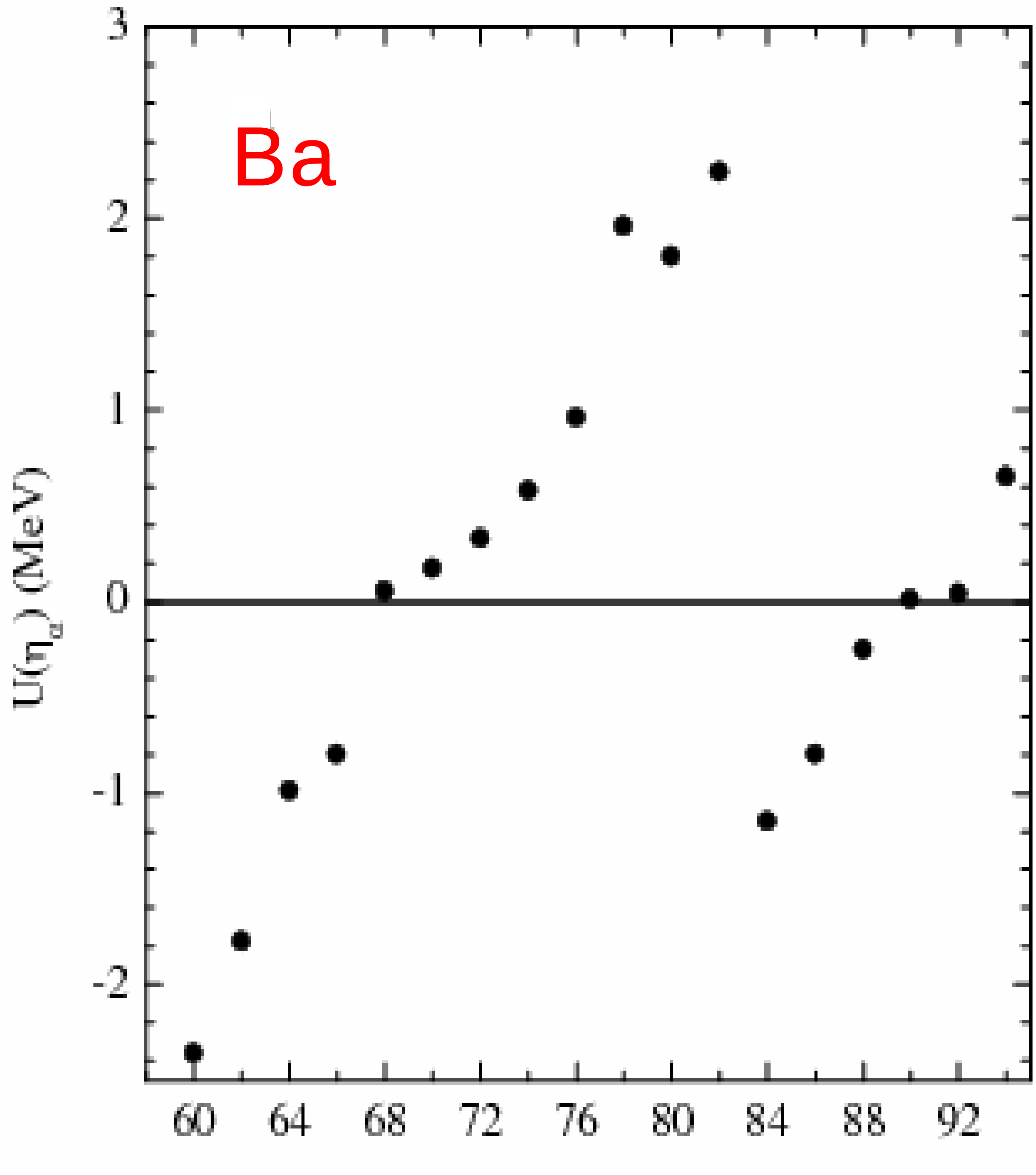
Moment of inertia of DNS:

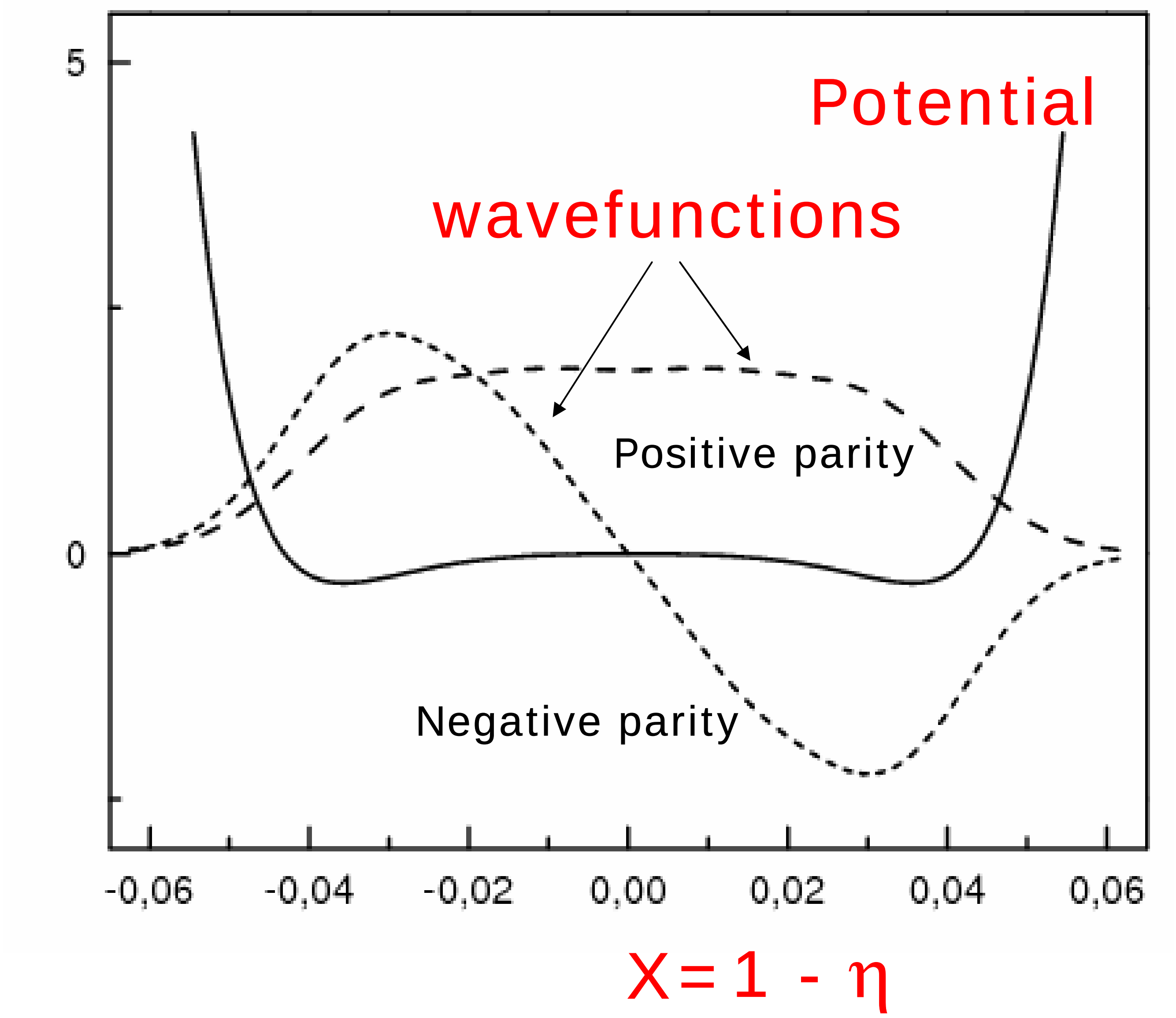
$$\Im = \Im_1 + \Im_2 + M\frac{A_1 A_2}{A}R_m^2$$

$\Im_1, \Im_2$: moments of inertia of DNS clusters

$$\Im_i = \frac{1}{5} M A_i (a_i^2 + b_i^2)$$

Exp.: moments of inertia of strongly deformed states about 85% of rigid body limit.


Ba
3
2
1
0
-1
-2
U(η_α) (MeV)
60
64
68
72
76
80
84
88
92
N



5
0
Potential
wavefunctions
Positive parity
Negative parity
-0,06
-0,04
-0,02
0,00
0,02
0,04
0,06
X=1 - η

Potential energy is invariant under inversion

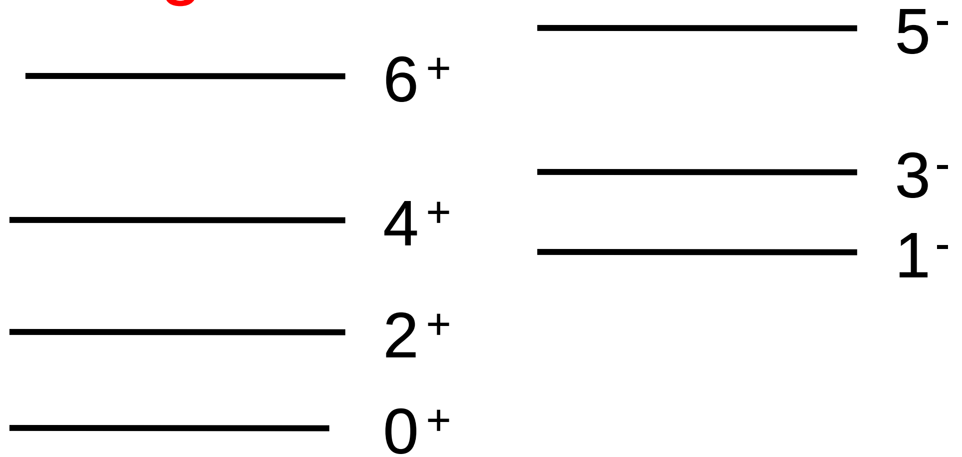
$\eta \to -\eta,$ every eigenfunction has definite parity.

- Rotation states are built on vibrational states in η.
- In even-even nuclei, set of states $J^\pi = 0^+, 1^-, 2^+, 3^-, 4^+, ...$
- Negative (Positive) parity states with given J are built on lowest odd (even) states in mass asymmetry at this J

Examples for mass asymmetry motion

Parity splitting

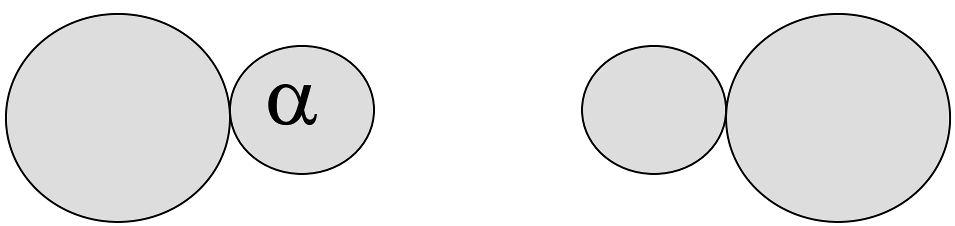
Ra, Th and U have positive and negative parity states which do not form an undisturbed rotational band. Negative parity states are shifted up. This is named parity splitting.

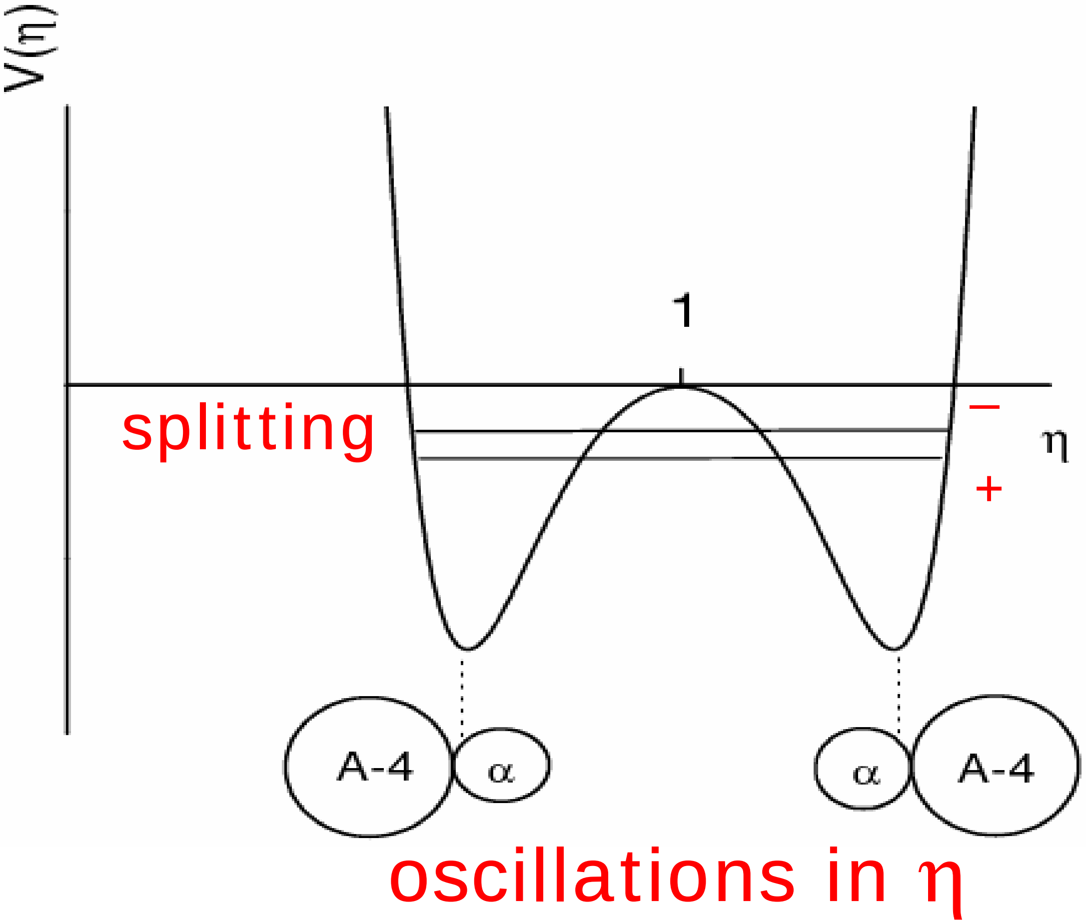


Parity splitting is explained by reflection-asymmetric shapes and is describable with an asymmetric mass clusterization.

The configuration with alpha-clustering has the largest binding energy.

$$^{A}Z \longrightarrow {}^{(A-4)}(Z-2) + \alpha \text{ - particle}$$





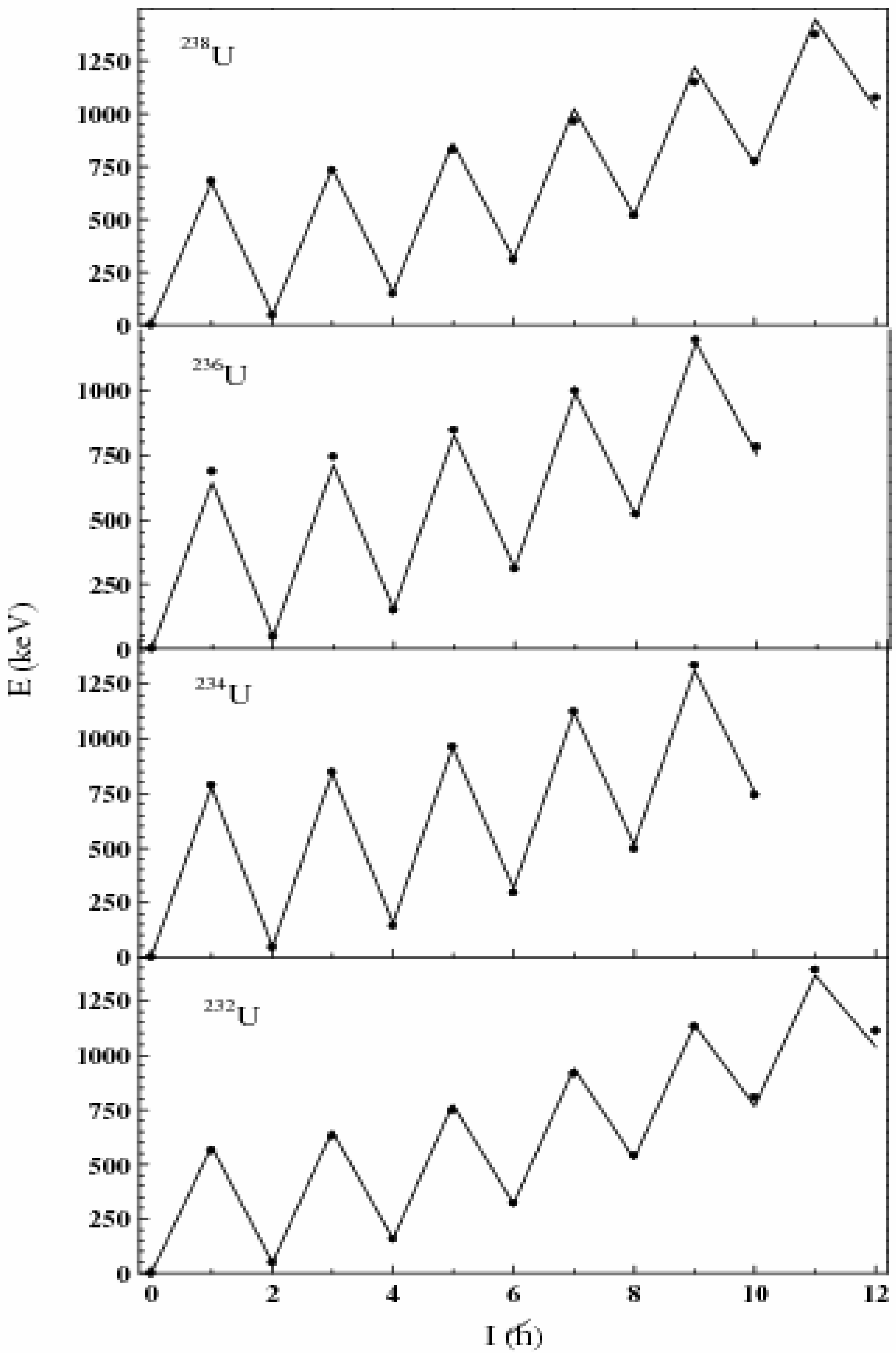
Lower state has positive parity, higher state negative parity. Energy difference depending on spin is parity splitting.

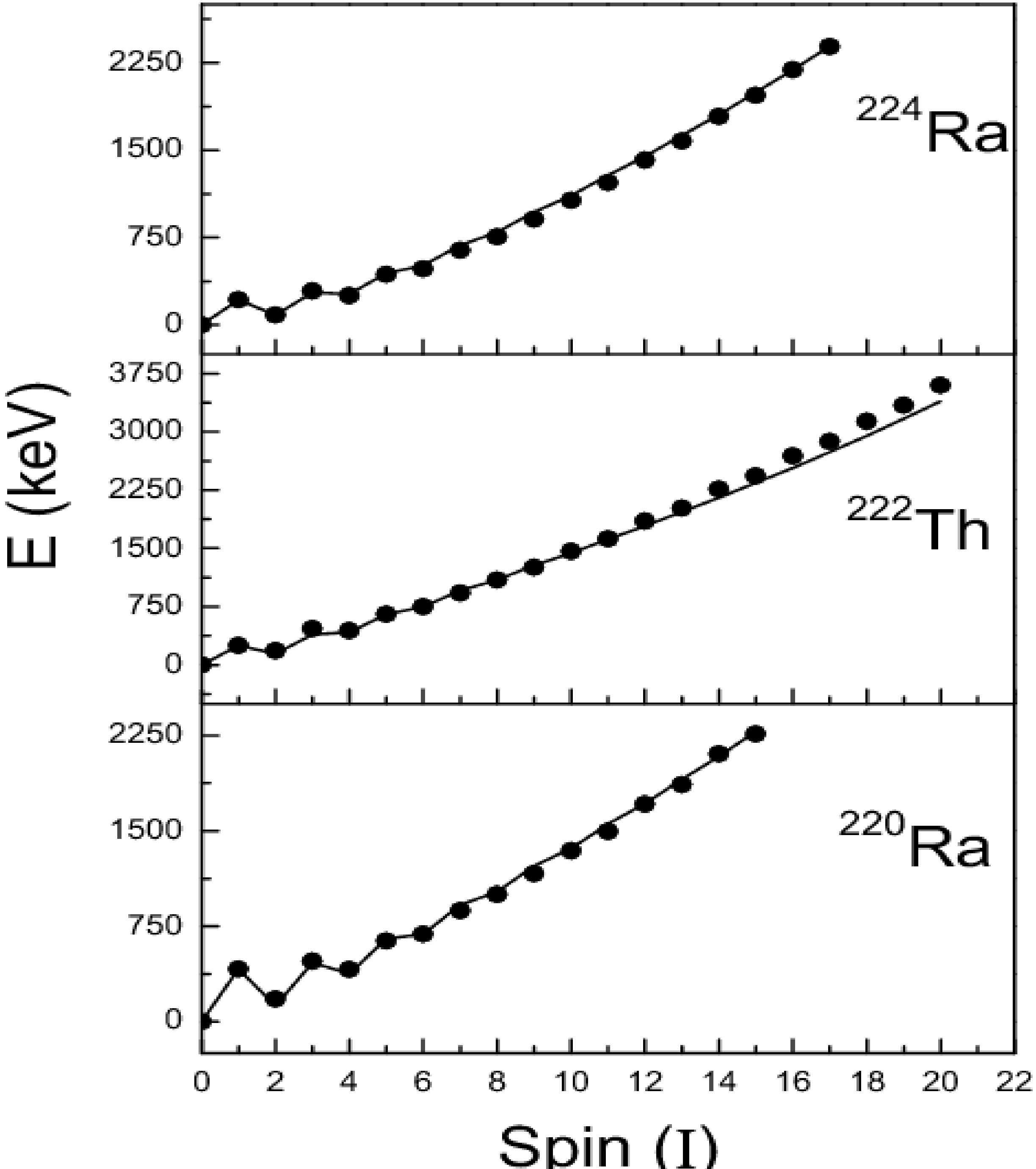
^{238}U

^{236}U

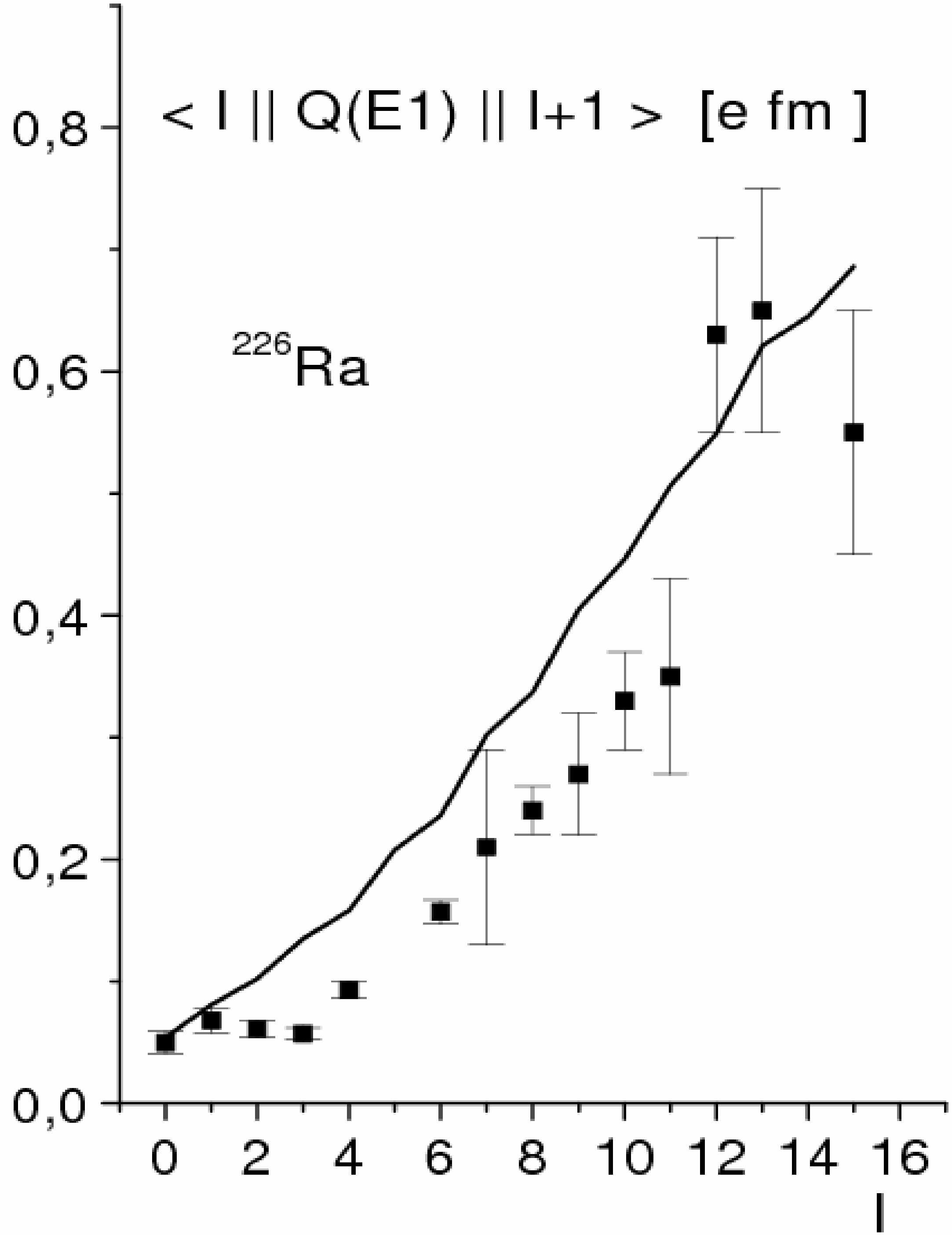
^{234}U

^{232}U

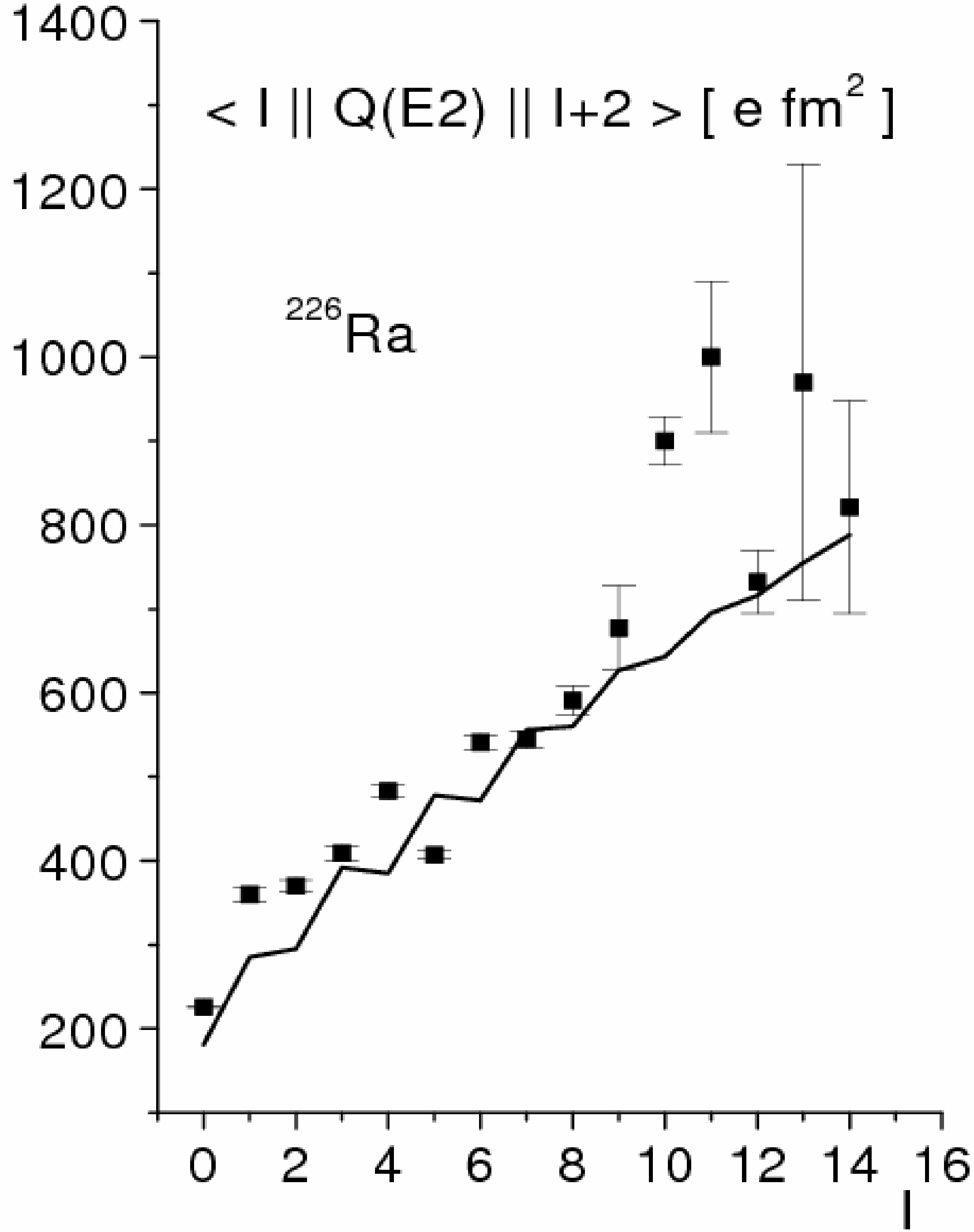




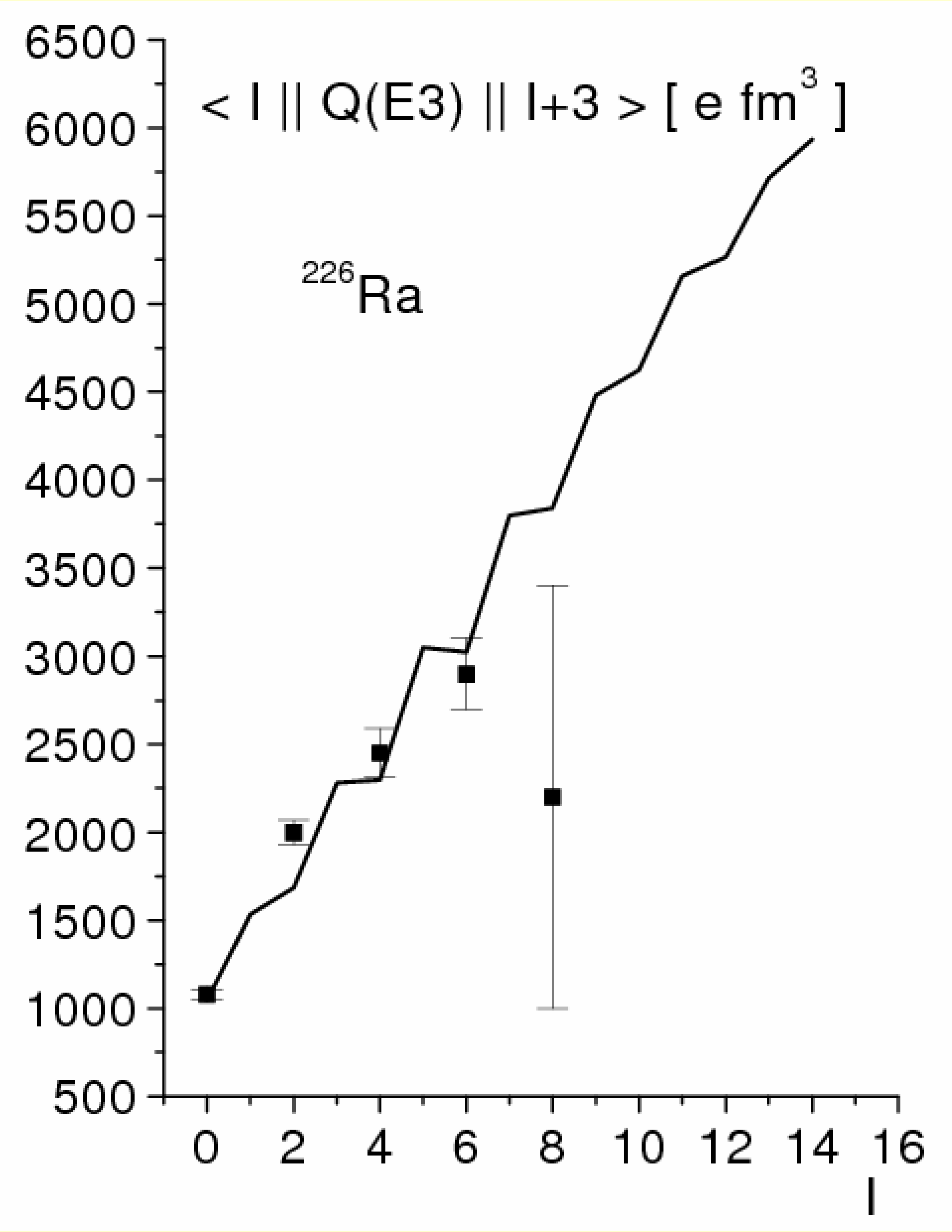
2250
1500
750
0
224Ra
3750
3000
2250
1500
750
0
222Th
2250
1500
750
0
220Ra
E (keV)
0
2
4
6
8
10
12
14
16
18
20
22
Spin (I)



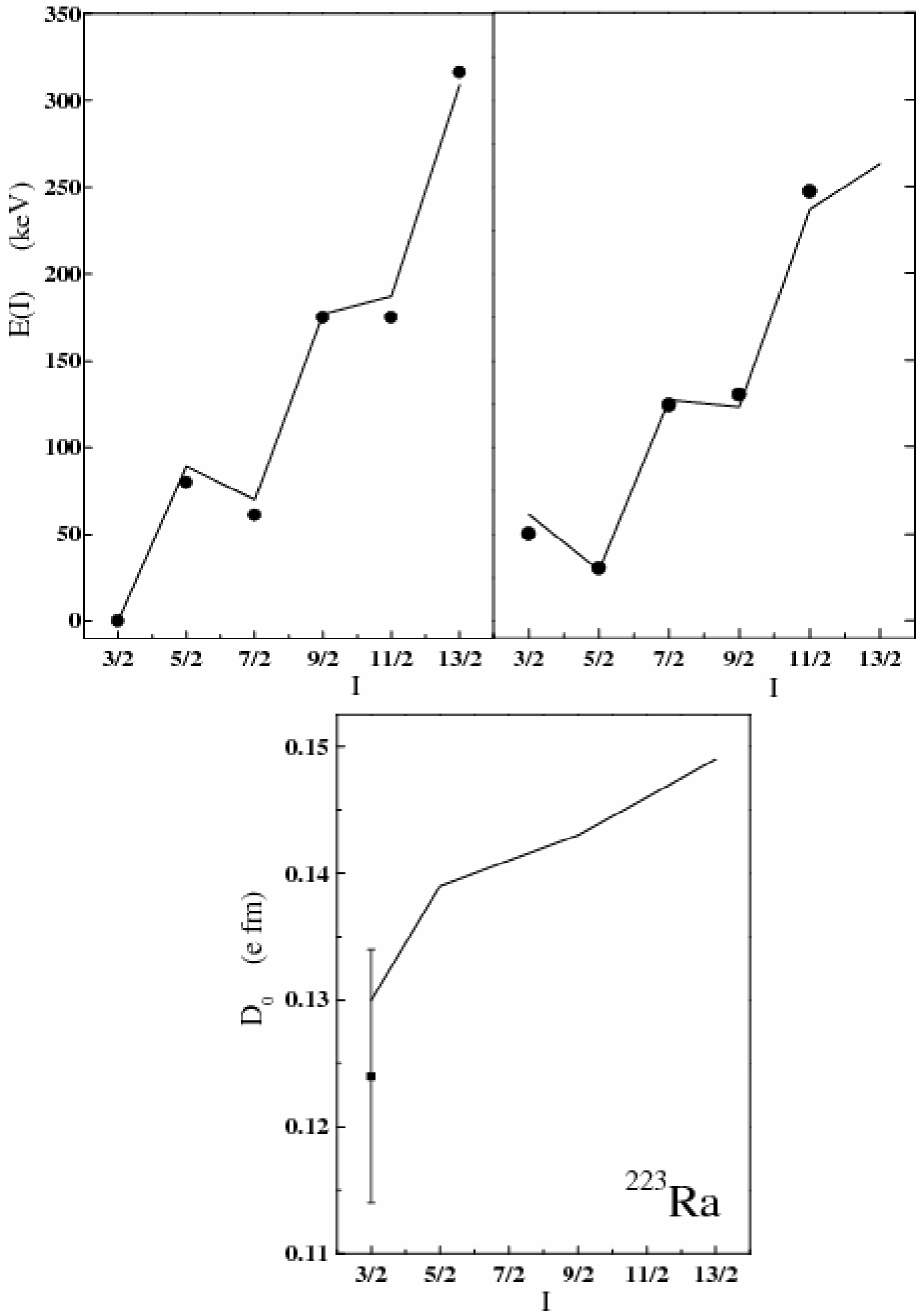
< I || Q(E1) || I+1 > [e fm]
226Ra
0,0
0,2
0,4
0,6
0,8
0
2
4
6
8
10
12
14
16
I



< I || Q(E2) || I+2 > [e fm^2]
^{226}Ra
1400
1200
1000
800
600
400
200
0
2
4
6
8
10
12
14
16
I


< I || Q(E3) || I+3 > [e fm^3]
^{226}Ra
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
2
4
6
8
10
12
14
16
I

^{223}Ra

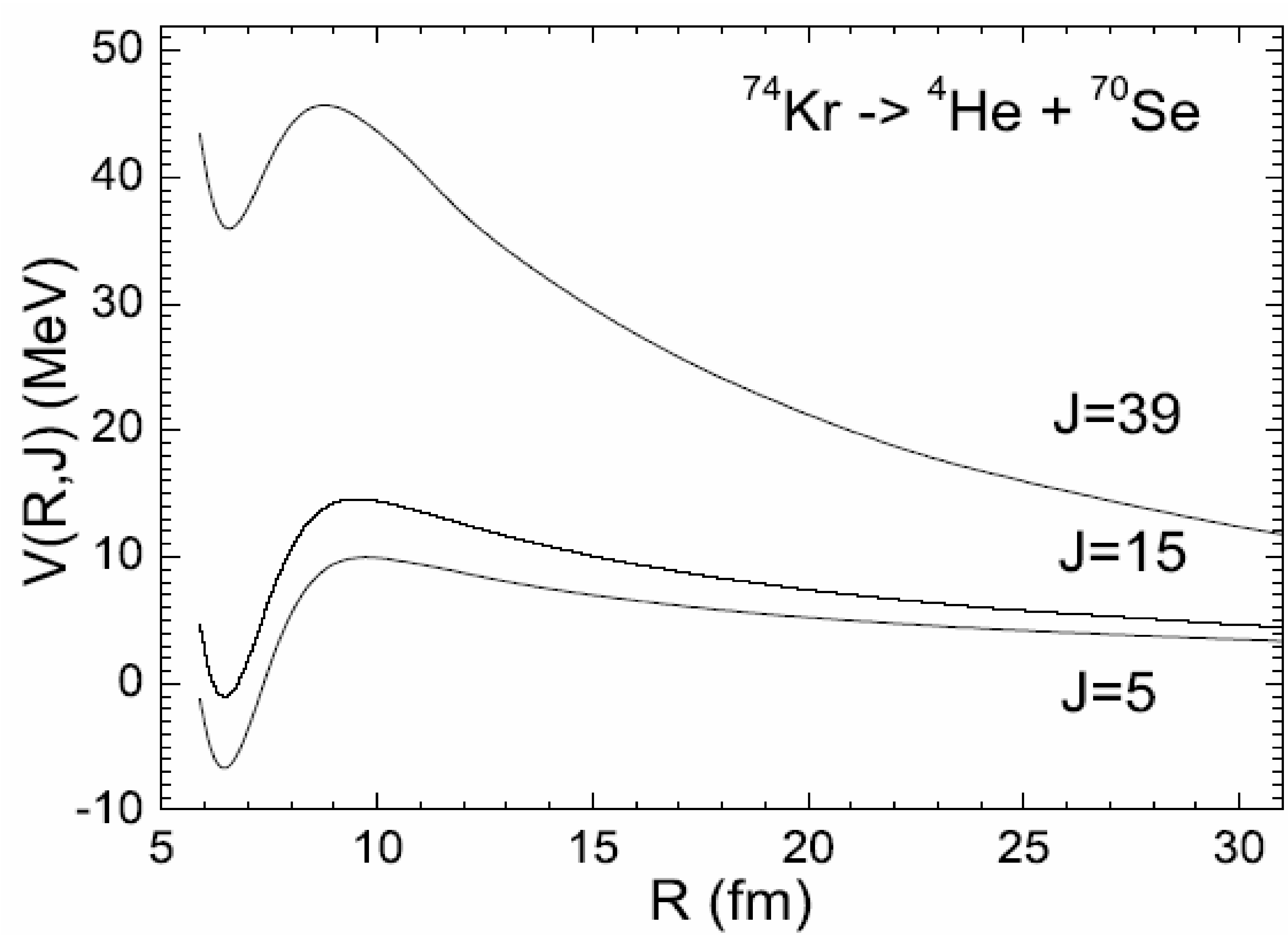


E(I) (keV)
0
50
100
150
200
250
300
350
3/2
5/2
7/2
9/2
11/2
13/2
I
D0 (e fm)
0.11
0.12
0.13
0.14
0.15
223Ra

3. Termination of lowest negative-parity bands

- Alpha-cluster gives the main contribution in w.f.
- In negative parity states with high spins, alpha-particle spectroscopic factor become close to unity
- It could be argued that reflection-asymmetric shape, especially at high spins, is a consequence of alpha-clustering in nucleus

Cluster-cluster interaction potential



Rotating alpha-cluster system has possibility to decay into two fragments by tunneling through potential barrier.

Alpha-Decay means Band Termination

at condition that with increasing J alpha-decay time $T_\alpha(J)$ becomes comparable and then smaller than γ-transition time $T_\gamma(J)$ in band.

Terminating spin J_{term} follows from condition:

$$T_\alpha(J_{term}) \ll T_\gamma(J_{term}).$$

Using the values of $\Im$ and the electric quadrupole moment of the DNS ($Q_2^{(c)}(i)$ ($i=1,2$) are the quadrupole moments of the DNS nuclei) [25]

$$Q_2^{(c)} = 2e\frac{A_2^2 Z_1 + A_1^2 Z_2}{A^2}R^2 + Q_2^{(c)}(1) + Q_2^{(c)}(2),$$

we obtain the energy $E_\gamma(J \to J-2) = J(J+1)/(2\Im) - (J-2)(J-1)/(2\Im) = (2J-1)/\Im$ and the time $T_\gamma(J)$ of the collective $E2$-transition between the rotational states with J and $J-2$ as in Ref. [23]:

$$T_\gamma(J) = \frac{408.1}{5/(16\pi)(Q_2^{(c)})^2(E_\gamma(J \to J-2))^5}, \tag{5}$$

where E_γ is in units of keV, $Q_2^{(c)}$ in $10^2(e\ \mathrm{fm}^2)$ and T_γ in s.

The process which competes with γ emission is tunneling through the barrier in R (α-decay). By employing the WKB-approach, the tunneling time through the barrier in R is estimated as

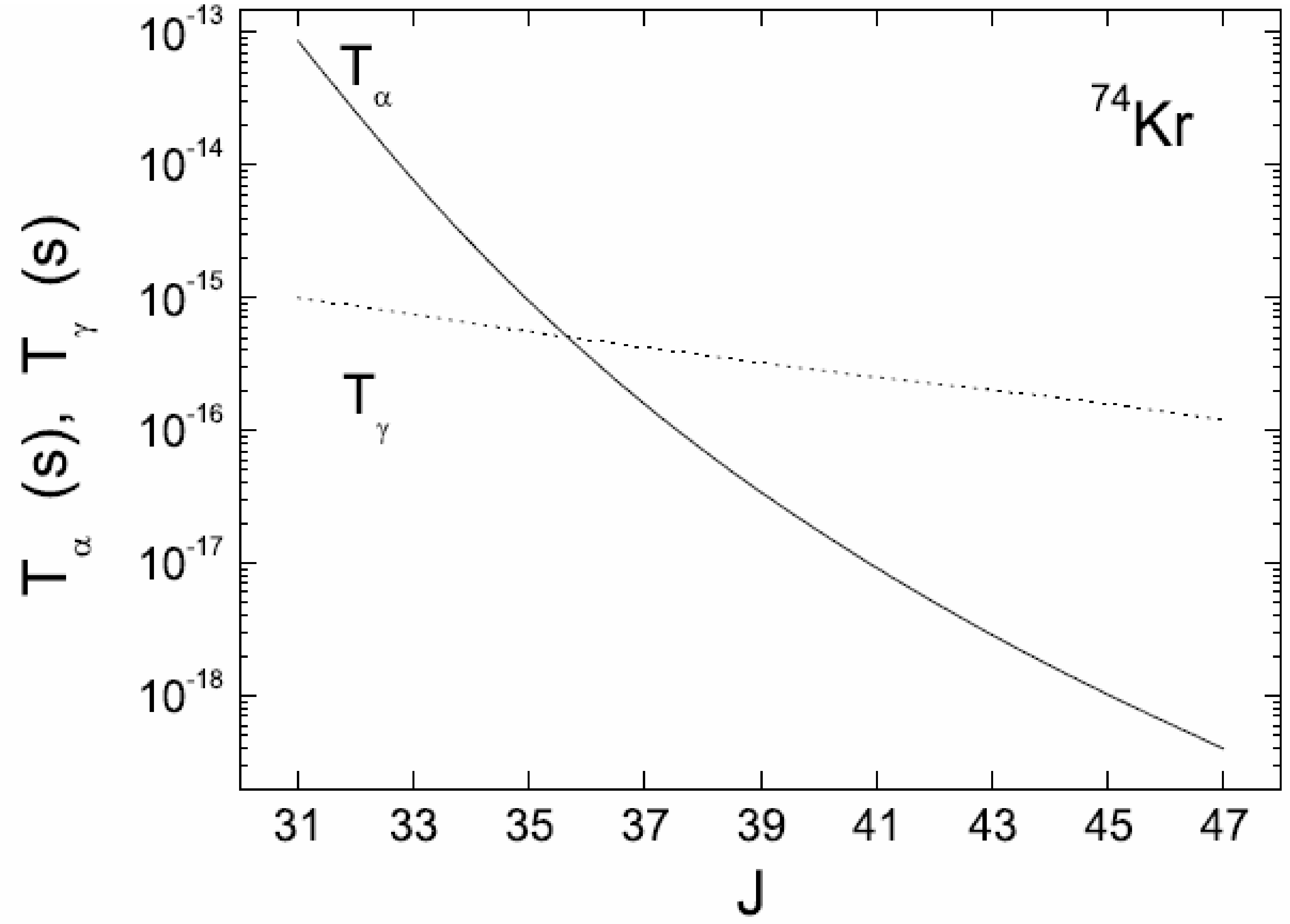
$$T_\alpha(J) = \frac{2\pi}{\omega_m(J)}(1 + \exp[2S_\alpha(J)/\hbar]), \tag{6}$$

where

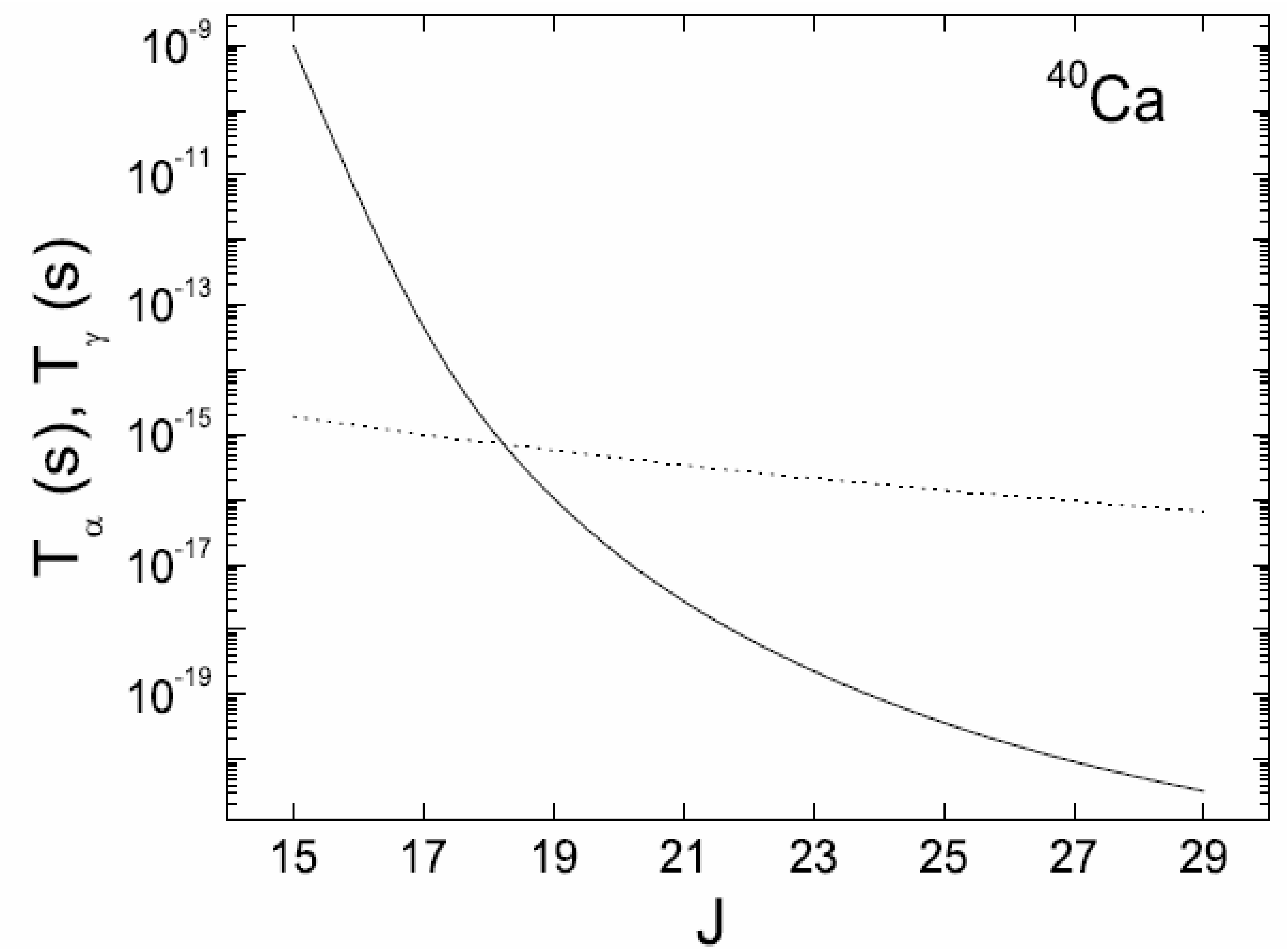
$$S_\alpha(J) = \int_{R_m}^{R_{ex}} dR[2\mu(V(R, J, \beta_2) - E_{\text{c.m.}})]^{1/2}$$

is the classical action in R, R_{ex} is the external turning point, and $\omega_m = \sqrt{\partial^2 V/\partial R^2|_{R=R_m}/\mu}$ is the assault frequency in the potential pocket at given value of J.

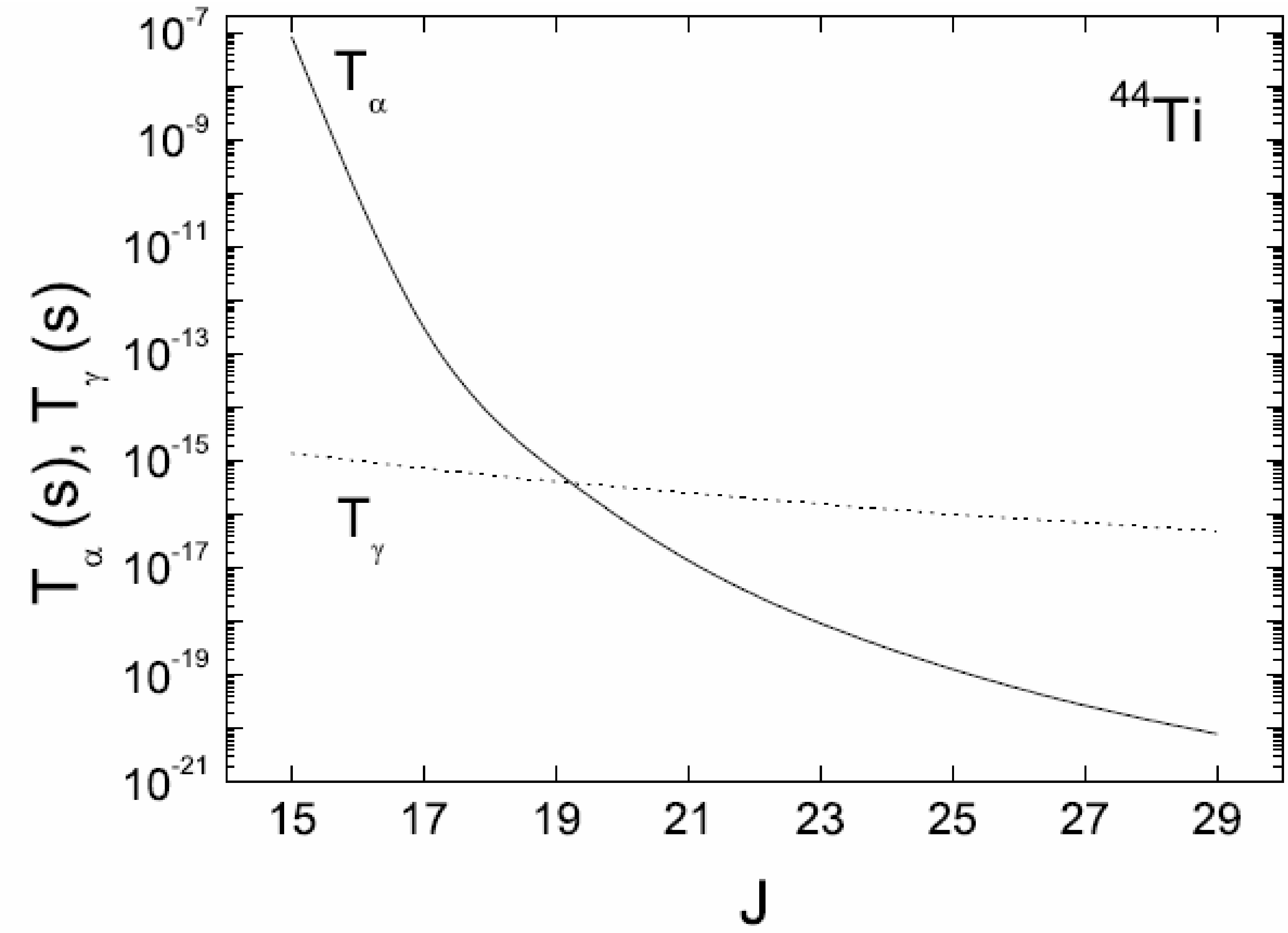
Times of E2-transition and alpha-decay vs spin J



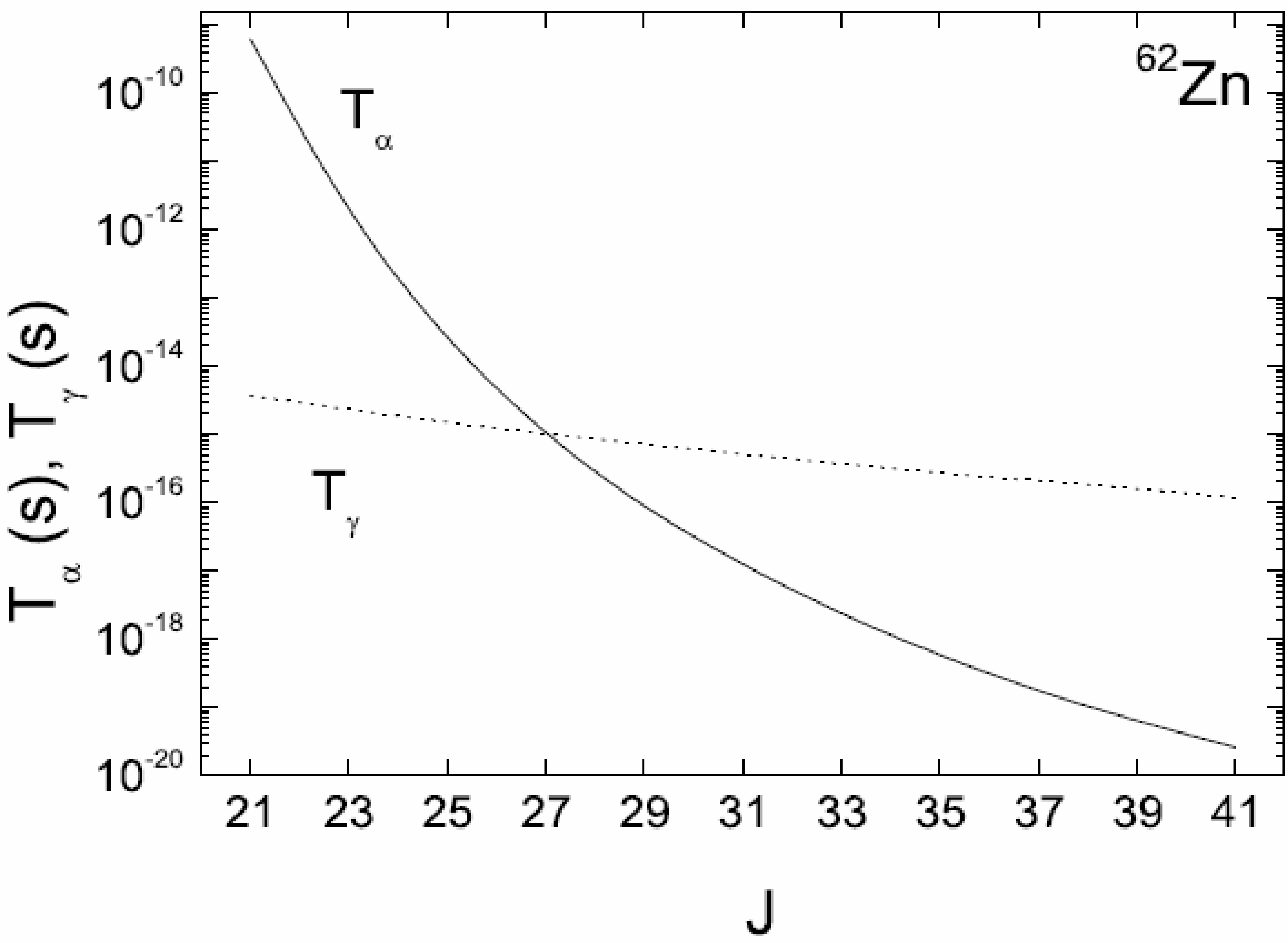
Times of E2-transition and alpha-decay vs spin J



Times of E2-transition and alpha-decay vs spin J

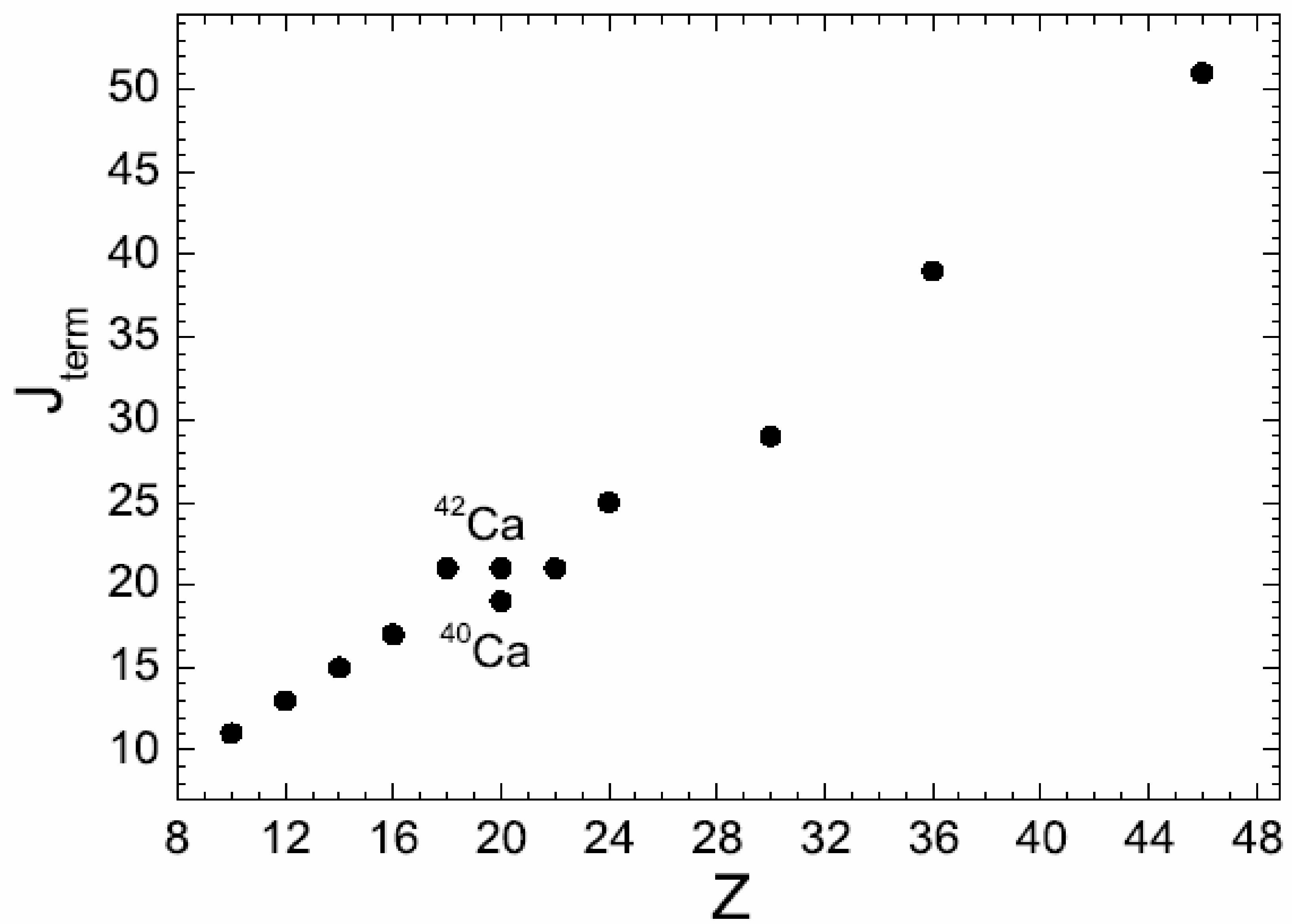


Times of E2-transition and alpha-decay vs spin J



- Thus at $J \geq J_{term}$ alpha-cluster system is unstable and related negative-parity band does not exist
- Negative parity band disappear upon reaching this terminating state with spin $J = J_{term}$
- Alpha-decay is origin of band termination

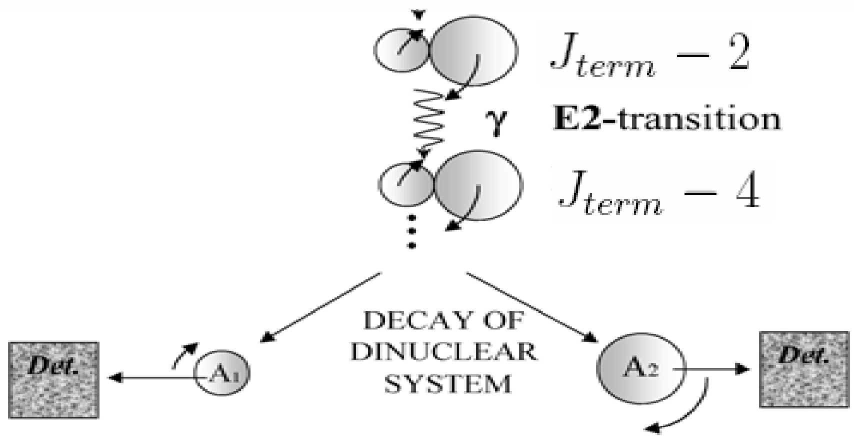
Termination spin vs atomic number Z



Experimental verification:

Decay of alpha-cluster system by γ-transitions from negative parity state with $J = J_{term} - 2$ to lower J-values in coincidence with decay fragments of alpha-cluster configuration.

Heavy ion experiments with coincidences of γ-rays and decay fragments could verify the cluster origin of low-lying negative parity states.



$J_{term} - 2$
γ
E2-transition
$J_{term} - 4$
DECAY OF
DINUCLEAR
SYSTEM
Det.
A1
A2
Det.

4. Summary

- Reflection-asymmetric shape is consequence of alpha clustering

- Physical origin of termination of negative-parity rotational band is alpha-decay

- To verify in experiments by measuring E2- or E1-transitions [in vicinity of terminating spin] in coincidence with decay fragments of alpha-cluster system

2.1 Deformation

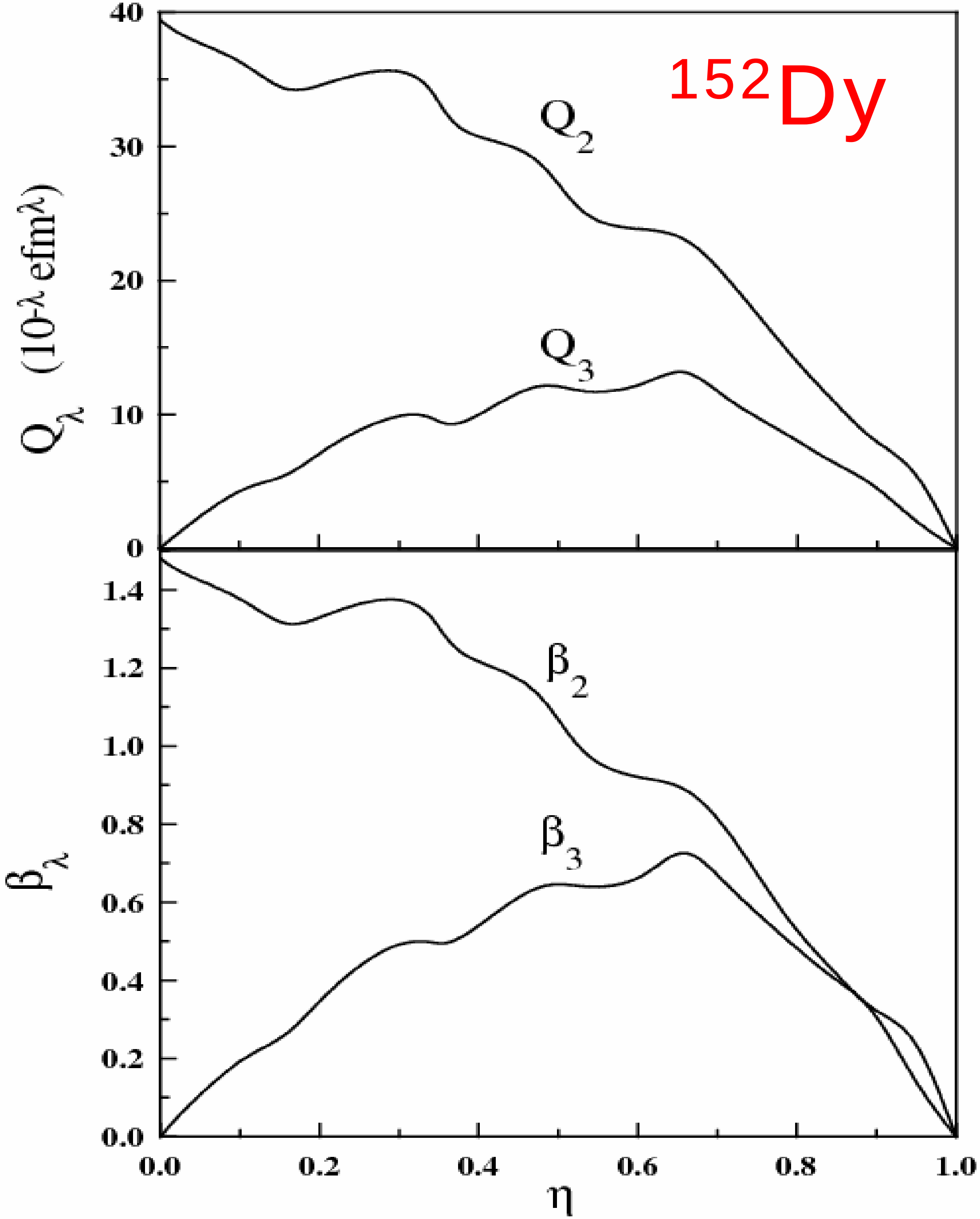
Dinuclear configuration describes quadrupole- and octupole-like deformations and extreme deformations as super- and hyperdeformations.

Multipole moments of dinuclear system:

$$Q_{\lambda,\mu}^{\text{(mass or charge)}} = \sqrt{\frac{16\pi}{2\lambda+1}} \int \rho_{\text{DNS}}^{\text{(m. or ch.)}}(\boldsymbol{r}) r^{\lambda} Y_{\lambda,\mu}(\Omega) d\tau$$

Comparison with deformation of axially deformed nucleus described by usual shape parameters:

$$R = R_0 \left(1 + \sum_{\lambda=0} \beta_\lambda Y_{\lambda 0}(\Omega)\right) \longrightarrow \underline{\beta_\lambda = \beta_\lambda(\eta \text{ or } \eta_Z)}$$



152Dy
Q2
Q3
Qλ (10-λ efmλ)
β2
β3
βλ
η
0
10
20
30
40
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.2
0.4
0.6
0.8
1.0

2.1 Deformation

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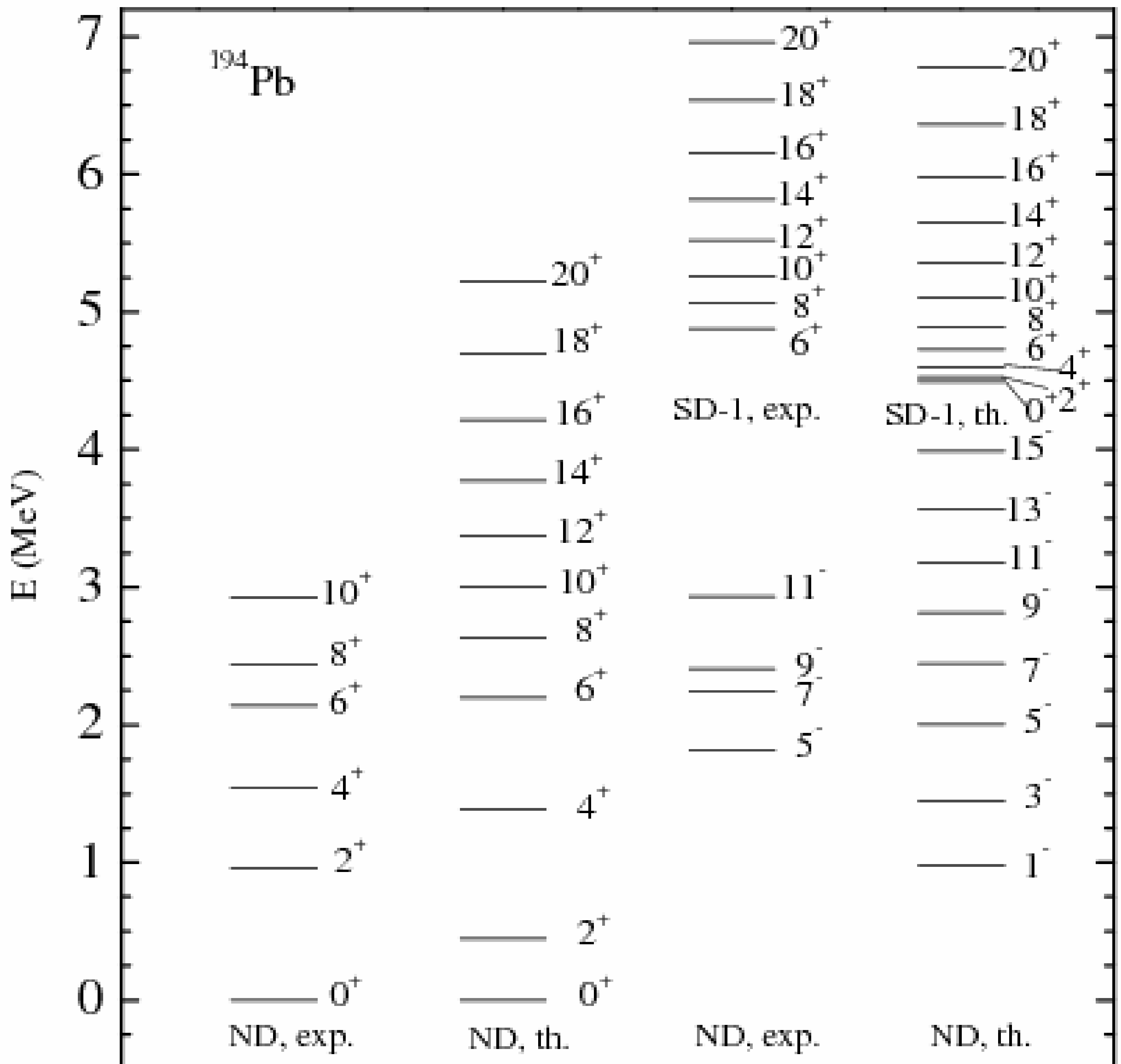


194Pb
E (MeV)
0
1
2
3
4
5
6
7
ND, exp.
ND, th.
ND, exp.
ND, th.
SD-1, exp.
SD-1, th.
0+
2+
4+
6+
8+
10+
12+
14+
16+
18+
20+
1-
3-
5-
7-
9-
11-
13-
15-

5. Summary and conclusions

- The concept of the dinuclear system describes nuclear structure phenomena connected with cluster structures, the fusion of heavy nuclei to superheavy nuclei, the quasifission and fission.
- The dynamics of the dinuclear system has two main degrees of freedom: the relative motion of the nuclei and the mass asymmetry degree of freedom.

Dinuclear system model is used in various ranges of η:

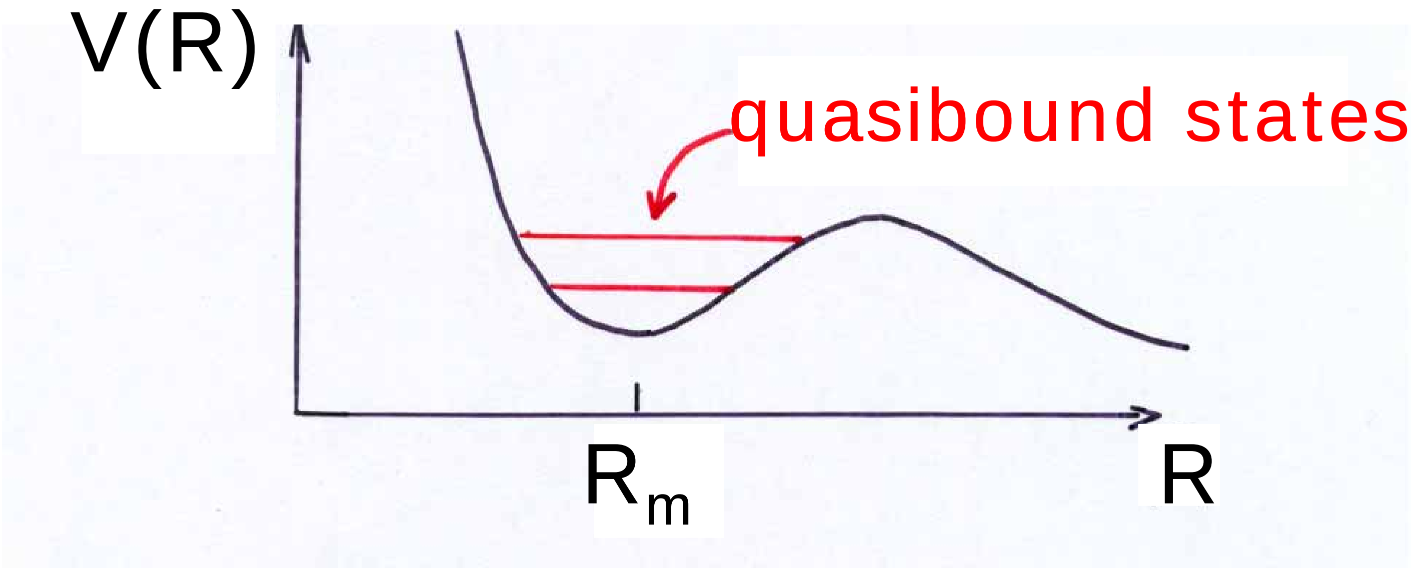
- $\eta=0$ - 0.3: large quadrupole deformation, hyperdeformed states
- $\eta=0.6$ - 0.8: quadrupole and octupole deformations are similar, superdeformed states
- $\eta \sim 1$: linear increase of deformations, parity splitting

Dinuclear System Model

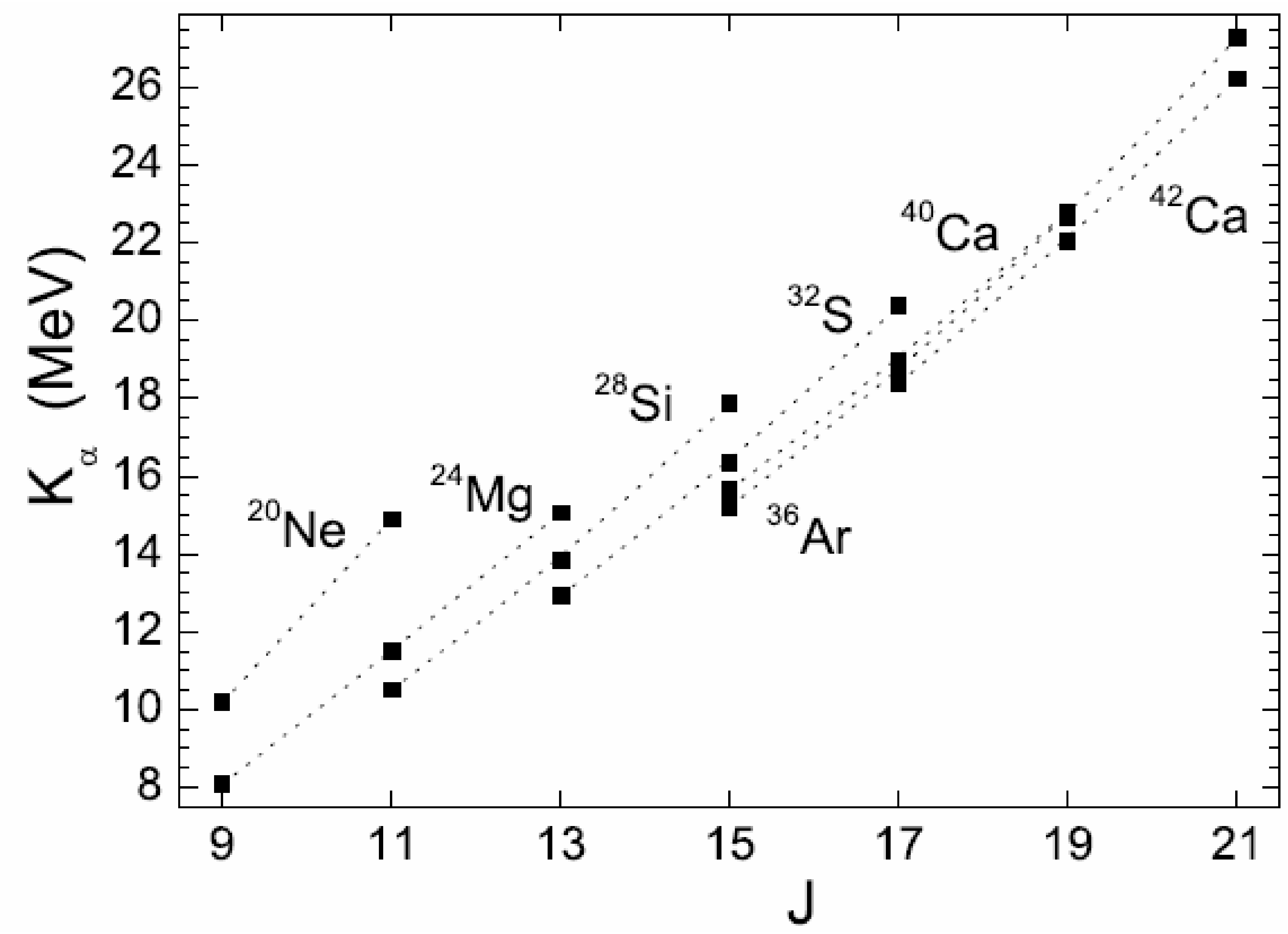
Applications:

- Nuclear structure phenomena: normal- super- and hyper-deformed bands
- Fusion to heavy & superheavy nuclei
- Quasifission in nuclear reactions
- Fission

Alpha-cluster states are quasibound states of the dinuclear system.



Kinetic energy of alpha vs angular momentum J



Kinetic energy of alpha vs spin J:

$$K_\alpha = \frac{A-4}{A}\left[V(R_b, J=0) + \frac{\hbar^2 J(J+1)}{2\mu R_b^2}\right]$$