Systematic Evaluation of the Nuclear Binding Energies in the Valence Shells

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Outline of the talk:

1. Origin and Definition of F-spin

2. Dynamical Symmetries as phases of the nuclear system
   - 2.1 Simplectic symmetries
   - 2.2 Classification of nuclei with Sp(4,R) –quantum numbers F_0 and N – physical interpretation

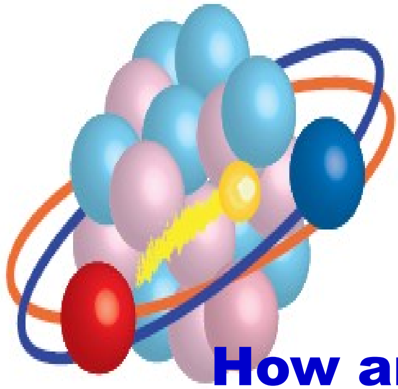
3. Systematic investigation of nuclear properties
   - 3.1 Empirical investigation of collective states
   - 3.2 Generalized description - introducing phases, phase transitions and control parameters

4. Application of the classification for Evaluation of the Nuclear Binding Energies in the Valence Shells

5. Predictions and possible Generalizations

ORIGIN AND DEFINITION OF F-SPIN

THE NUCLEAR STRUCTURE



- A two-fluid (neutrons and protons), finite N system (IBM2)
- interacting via strong, short and long - range forces.

How are complex systems built from a few, simple ingredients?

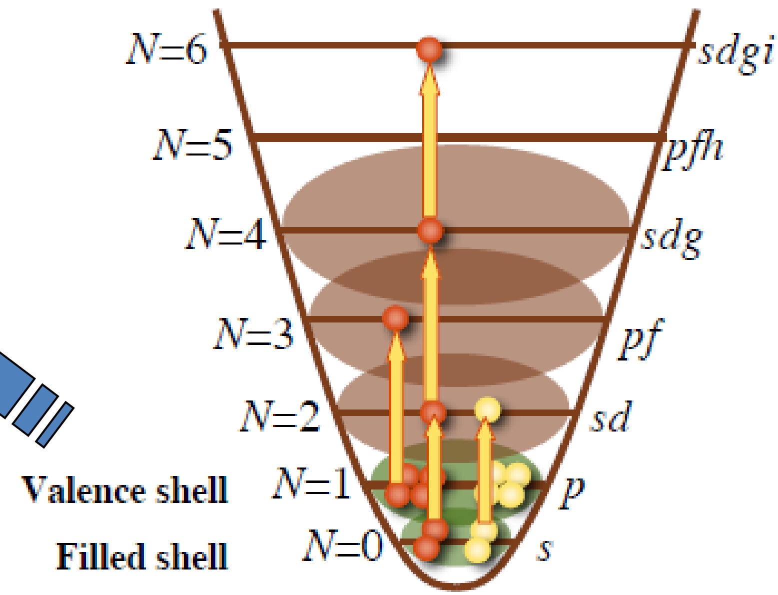
- Shell Structure – number of particles in each shell, magic numbers
- Pairing – pairs of particles
- Collective modes

Excitations of the nucleons in the valence shells

Vertical classification

Shell Structure

Horisontal classification



Symplicity in complexity

How to discover the simplicity in the nuclear spectral properties? **The manifestation of simplicity is**

Symmetry

GROUP THEORY — **the language of symmetries**

Dynamical Symmetry of the system -

- **chains of group-subgroup structures**

Classification of the basis states — **Hamiltonians and interactions**

leads to exact analytic solutions for the eigenvalue problems.

Dynamic symmetries apply to purely bosonic or purely fermionic systems. Supersymmetries

Symplectic Dynamical Symmetries

The symplectic symmetry includes all this!

$$\sum p_{si}p_{sj}, \sum x_{si}p_{sj} \pm x_{sj}p_{si}, \sum x_{si}x_{sj}$$

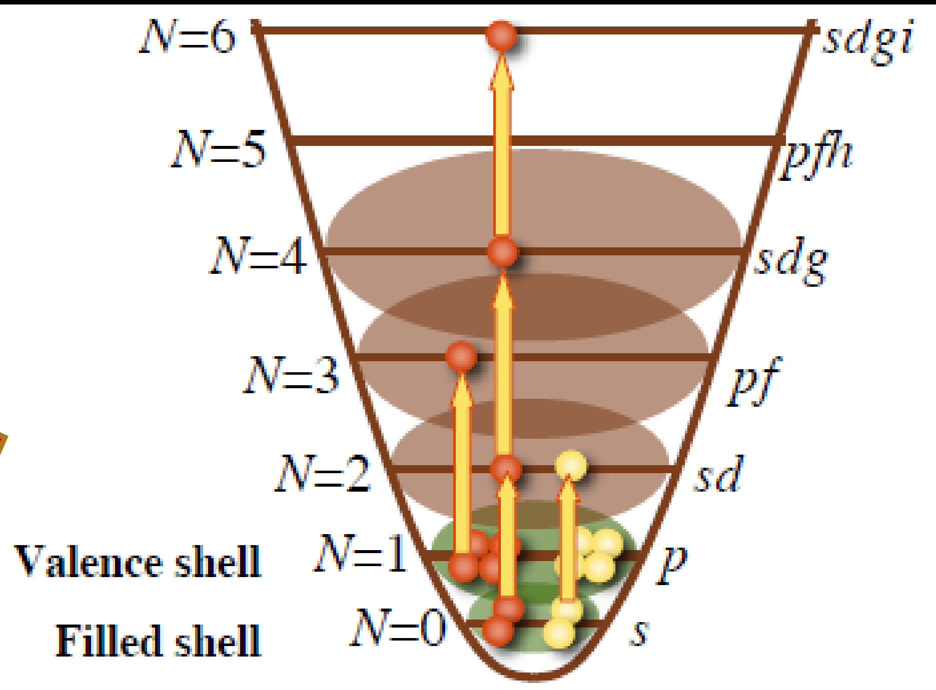
mass quadrupole moment

Angular momentum

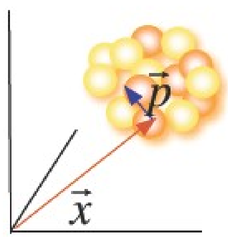
many-particle kinetic energy

monopole and quadrupole collective excitations

vorticity (from irrotational to rigid rotor flow)



i,j s



Nucleus with A nucleons

Change the number of particles, biger represantation spaces

Realisation of the Algebras

in terms of boson creation and annihilation operators

$$\sum_s p_{si}p_{sj},\ \sum_s x_{si}p_{sj} \pm x_{sj}p_{si},\ \sum_s x_{si}x_{sj}$$

$$u_m^+(\alpha) = (x_m(\alpha) - iq_m(\alpha))/\sqrt{2}$$
$$u^m(\alpha) = (x^m(\alpha) + iq^m(\alpha))/\sqrt{2}.$$

$$q_m(\alpha) = -i\ \partial/\partial x^m(\alpha)$$

$q_m \sim p_j$

$u^+_{n\beta}\, u_{m\alpha}$

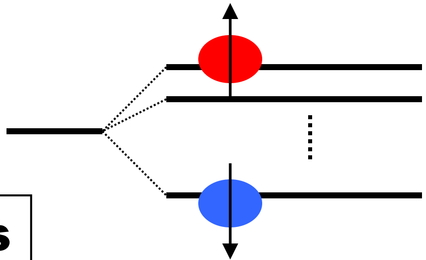
$\alpha = \pm 1/2$, n,m=l,l-1,…-l+1,-l

l=0, n,m= 0 scalars

l=1, n,m= -1,0,1 vectors **IVBM**

l=0, n,m= 0 S-, l=2, n,m= 0,±1,±2 D-boson **IBM 1**

IBM 2 → F=1/2 **spinors**



$(F_0 = \alpha = 1/2)$ *p* boson

$(F_0 = \alpha = -1/2)$ *n* boson

$U_\alpha(K)$

F-spin

$[u_{n\alpha}, u^+_{m\beta}] = \delta^n_m\, \delta_{\alpha\beta}$

$u^+_{n\,\alpha}\, u_{m\alpha}$

$\alpha = \pm 1/2$, k,m=l,l-1,…-l+1,-l

Example

Sp(4,R)

Bosons

π, π^{+} ν, ν^{+}

Algebra generators

$\pi^{+}\pi^{+}$, $\pi^{+}\nu^{+}$, $\nu^{+}\nu^{+}$

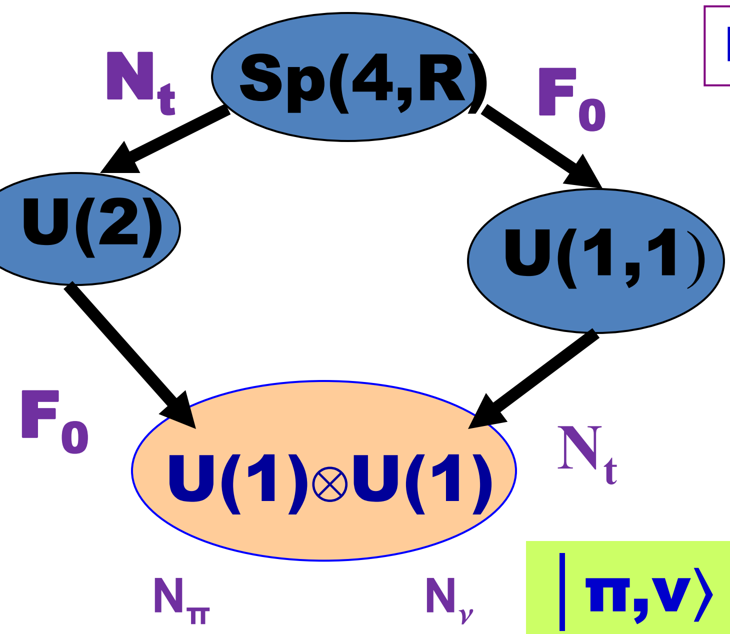
$\pi\pi$, $\pi\nu$, $\nu\nu$

$l=0, \alpha=\pm 1/2$

Exhatations of the nucleons in the valence shells

Reduction Operators

$\pi^{+}\pi$, $\pi^{+}\nu$, $\nu^{+}\pi$, $\nu^{+}\nu$



$\pi^{+}\nu^{+}$, $-\nu\pi$, $\pi^{+}\pi$, $-\nu\nu^{+}$

Basis states

$|\pi,\nu\rangle = (\pi^{+})^{N\pi}(\nu^{+})^{N\nu}|0\rangle$

Casimir invariants

$N_t = N_\pi + N_\nu$

$F_0 = 1/2(N_\pi - N_\nu)$

$N_\pi = \pi^{+}\pi$ $N_\nu = \nu^{+}\nu$

PHYSICAL INTERPRETATION OF THE ALGEBRA GENERATORS

$P_+ = A_0 = \pi^\dagger \nu^\dagger,$

$P_- = \nu \ \pi$

$N_t = N_\pi + N_\nu$

$N_\pi = \pi^\dagger \pi = 1/2(Z - Z_{min})$ $U_\pi(1)$

$N_\nu = \nu^\dagger \nu = 1/2(N - N_{min})$ $U_\nu(1)$

Number of proton and neutron valence pairs

U(1,1)

Z and **N** are the number of proton and neutron for a given nucleus.

Z_{min} and N_{min} the numbers of protons and neutrons of the double magic nucleus at the beginning of the shell

The F-spin group $U_F(2)$

$$F_+ = \sum_{a=0}^{5} \pi_a^+ \nu_a, \qquad F_- = \sum_{a=0}^{5} \nu_a^+ \pi_a, \qquad F_0 = \tfrac{1}{2}(N_\pi - N_\nu)$$

Sp(4,R) classification scheme

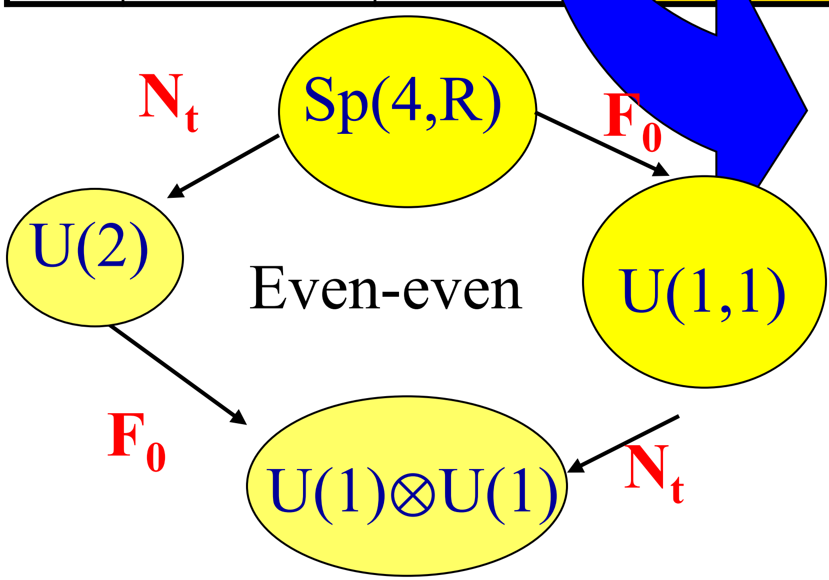
Rows – fixed $N_t = N_p + N_n$
Columns-fixed $F_0 = N_p - N_n$

F_0	-2	-1	0	1	2	3 …
0			$\vert 0,0 \rangle$			
2		$\vert 2,0 \rangle$	$\vert 1,1 \rangle$	$\vert 0,2 \rangle$		
4	$\vert 4,0 \rangle$	$\vert 3,1 \rangle$	$\vert 2,2 \rangle$	$\vert 1,3 \rangle$	$\vert 0,4 \rangle$	
N_t	…	…	…	…	…	…

K= 1

Vacuum $|0\rangle$

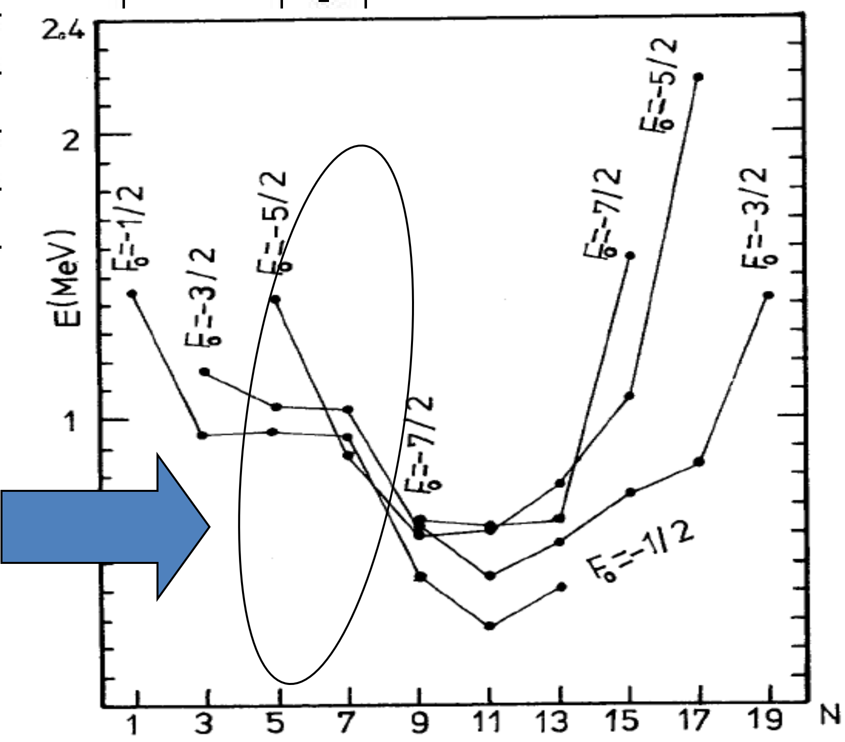
Mapping on the nuclei from a major shell



N/F_0	1	0	-1	-2	-3	-4	-5	
0		^{56}Ni 0						22
2	^{60}Ge	^{60}Zn 1	^{60}Ni 0					20
4	^{64}Se	^{64}Ge 2	^{64}Zn 1	^{64}Ni 0				18
6		^{68}Se 3	^{68}Ge 2	^{68}Zn 1	^{68}Ni 0			16
8		^{72}Kr 4	^{72}Se 3	^{72}Ge 2	^{72}Zn 1	^{72}Ni*0		14
10		^{76}Sr 5	^{76}Kr 4	^{76}Se 3	^{76}Ge 2	^{76}Zn*1	^{76}Ni 0	12
12		^{80}Zr 6	^{80}Sr 5	^{80}Kr 4	^{80}Se 3	^{80}Ge 2	^{80}Zn 1	10
14		^{84}Mo 7	^{84}Zr 6	^{84}Sr 5	^{84}Kr 4	^{84}Se 3		8
16		^{88}Ru 8	^{88}Mo 7	^{88}Zr 6	^{88}Sr 5			6
18		^{92}Pd 9	^{92}Ru 8	^{92}Mo 7				4
20		^{96}Cd 10	^{96}Pd 9	*				2
22		^{100}Sn 11						
		1	2	3	4	5		$\bar{F}_0$

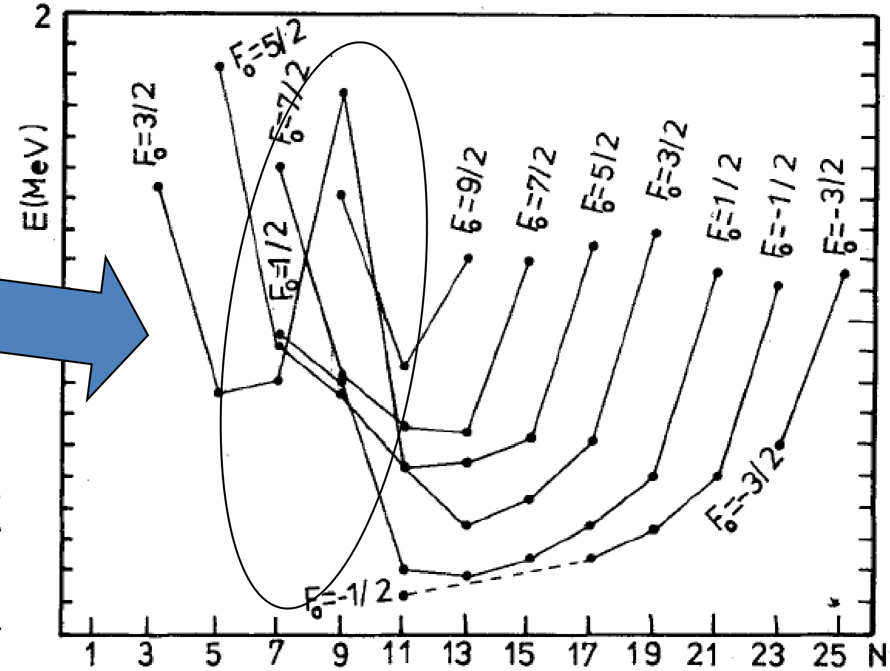
N/F_0	1	0	-1	-2	-3	-4	-5	
0		^{56}Ni 0						22
2	^{60}Ge	^{60}Zn 1	^{60}Ni 0					20
4	^{64}Se	^{64}Ge 2	^{64}Zn 1	^{64}Ni 0				18
6		^{68}Se 3	^{68}Ge 2	^{68}Zn 1	^{68}Ni 0			16
8		^{72}Kr 4	^{72}Se 3	^{72}Ge 2	^{72}Zn 1	^{72}Ni*0		14
10		^{76}Sr 5	^{76}Kr 4	^{76}Se 3	^{76}Ge 2	^{76}Zn*1	^{76}Ni 0	12
12		^{80}Zr 6	^{80}Sr 5	^{80}Kr 4	^{80}Se 3	^{80}Ge 2	^{80}Zn 1	10
14		^{84}Mo 7	^{84}Zr 6	^{84}Sr 5	^{84}Kr 4	^{84}Se 3		8
16		^{88}Ru 8	^{88}Mo 7	^{88}Zr 6	^{88}Sr 5			6
18		^{92}Pd 9	^{92}Ru 8	^{92}Mo 7				
20		^{96}Cd 10	^{96}Pd 9	*				
22		^{100}Sn 11						
		1	2	3	4			

Subshell



Multiplet (28, 28|50, 50)$_-$. Dependence of the 2^+ levels on N at fixed F_0 (see Table III).

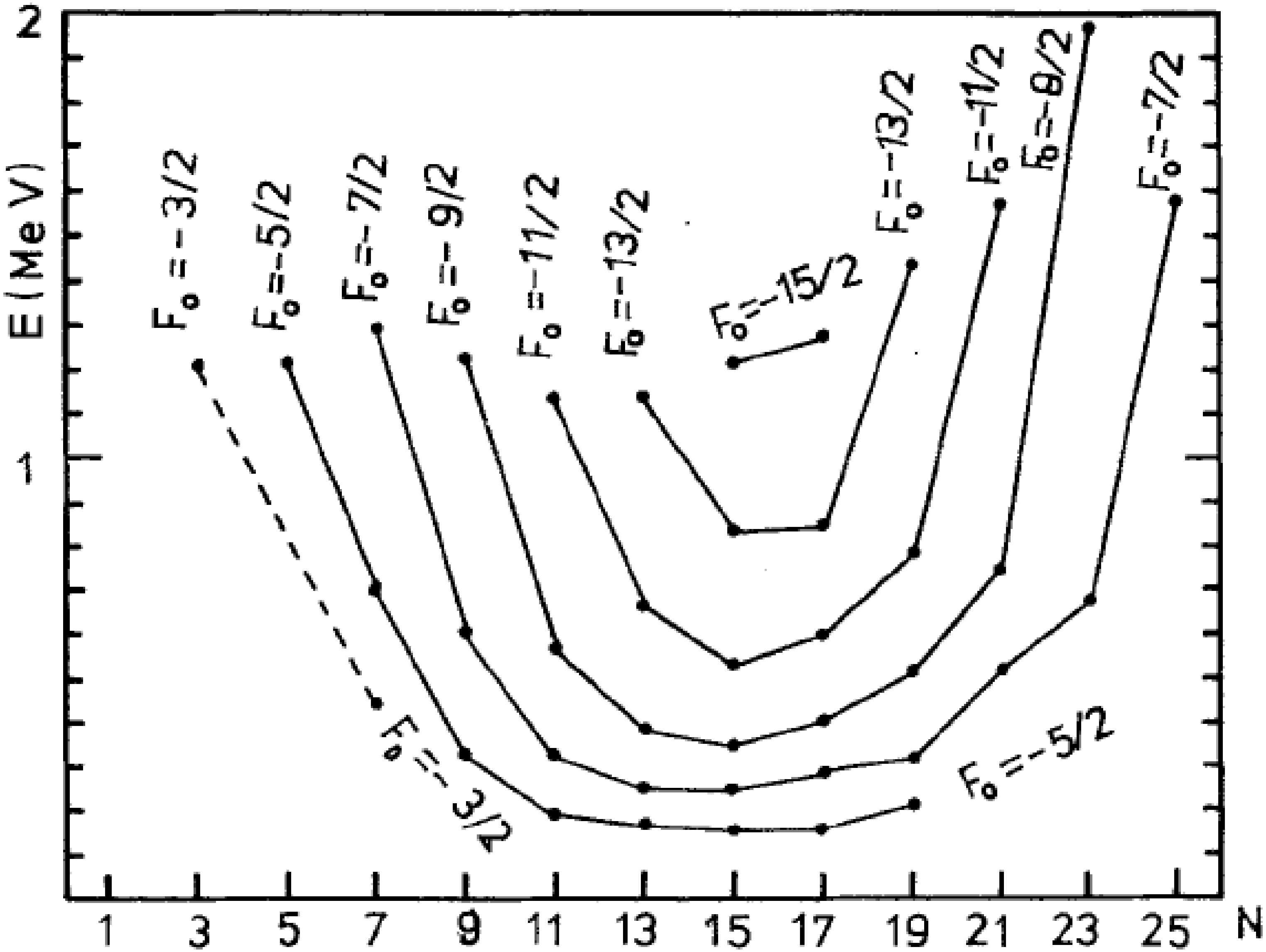
Subshell



Multiplet (28, 50|50, 82)_. Dependence of the 2^+ levels on N at fixed F_0 (see Table IV).

Table IV. Multiplet (28, 50|50, 82)

	F_0							
N	9/2	7/2	5/2	3/2	1/2			
3				^{84}Se	^{84}Ge			
5			^{88}Sr	^{88}Kr	^{88}Se			
7		^{92}Mo	^{92}Zr	^{92}Sr	^{92}Kr			
9	^{96}Pd	^{96}Ru	^{96}Mo	^{96}Zr	^{96}Sr			
11	^{100}Cd	^{100}Pd	^{100}Ru	^{100}Mo	^{100}Zr	^{100}Sr		
13	^{104}Sn	^{104}Cd	^{104}Pd	^{104}Ru	^{104}Mo			
15		^{108}Sn	^{108}Cd	^{108}Pd	^{108}Ru	^{108}Mo		
17			^{112}Sn	^{112}Cd	^{112}Pd	^{112}Ru		
19				^{116}Sn	^{116}Cd	^{116}Pd		
21					^{120}Sn	^{120}Cd		
23						^{124}Sn	^{124}Cd	
25							^{128}Sn	
27								^{132}Sn



E (Me V)
2
1
N
1
3
5
7
9
11
13
15
17
19
21
23
25
Fo = -3/2
Fo = -5/2
Fo = - 7/2
Fo = - 9/2
Fo = -11/2
Fo = -13/2
Fo = -15/2
Fo = -13/2
Fo = -11/2
Fo = -9/2
Fo = -7/2
Fo = - 3/2
Fo = -5/2

A. I. Georgieva, M. I. Ivanov, P. P. Raychev and R. P. Roussev
"Boson Representations of Symplectic Algebras"
Int. J. Theor. Phys. 25 (1986) 1181

A. I. Georgieva, M. I. Ivanov, P. P. Raychev and R. P. Roussev
"On The Boson Representations of Symplectic Algebras: General Scheme"
Compt. Rend. Bulg. Acad. Sci., 40, N 3, (1987), 29

A. Georgieva, M. Ivanov, P. Raychev, R. Roussev
"F-spin and Sp(24,R) Classification of the Even - Even Nuclei"
Compt. Rend. Bulg. Acad. Sci., 41, (1988), 55

A. Georgieva, M. Ivanov, P. Raychev, R. Roussev
"Classification of the Even - Even Nuclei in Symplectic Multiplets"
Int. J. Theor. Phys. 28 (1989) 769

GDG — Generalized Dynamical Group

Its representations contain the spectra of a set of nuclei

DG — Group of Dynamical Symmetry

CG — Classification Group

Its representations contain the spectra of collective states of a given nucleus

Phase transitions **Phases**

$$GDG \supset CG \otimes DG$$

Classification q[illegible]tum numbers

Control Parameters

$$H = \alpha(K_i)\, V(x_i, L_m)$$

Sp(4,R) classification scheme

Define the interactions

$E_L = \alpha(K_i) L(L+\omega)$

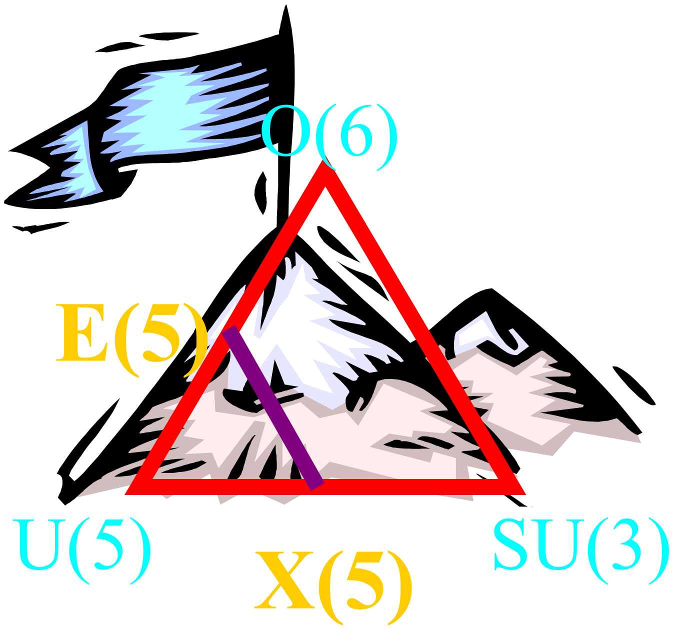
Function on the classification quantum numbers

$$A_p = N_p^{(1)} + N_p^{(2)}, \qquad A_n = N_n^{(1)} + N_n^{(2)},$$

$$N = N_\pi + N_\nu, \qquad F_0 = \tfrac{1}{2}(N_\pi - N_\upsilon),$$

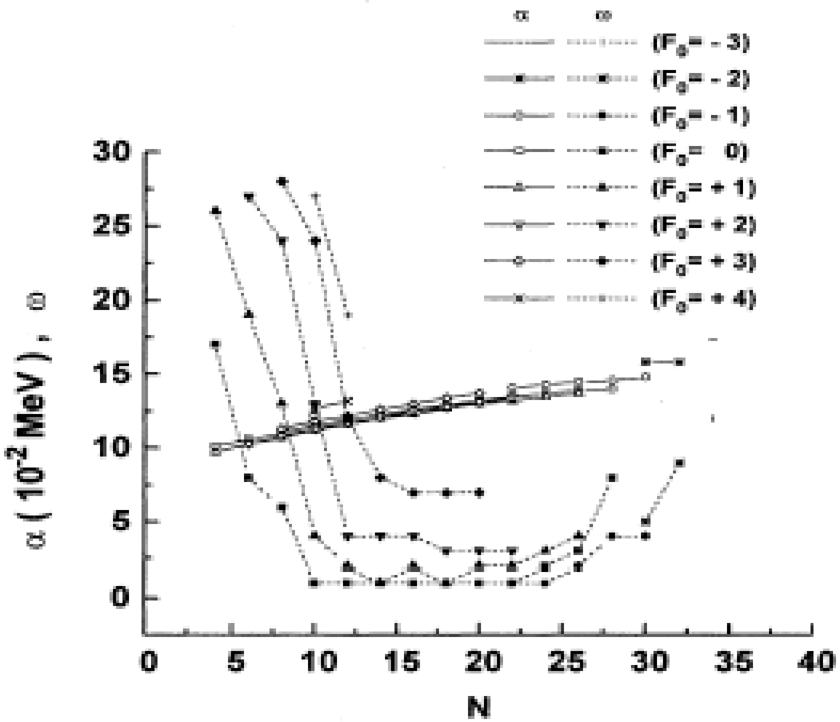
$$\bar{N} = \bar{N}_\pi + \bar{N}_\nu, \qquad \bar{F}_0 = \tfrac{1}{2}(\bar{N}_\pi - \bar{N}_\nu),$$

$$\begin{aligned}\alpha(A_p, A_n, N, F_0, \bar{N}, \bar{F}_0) = {} & D_1 + D_2 N + D_3 \bar{N} + D_4 F_0 + D_5 \bar{F}_0 \\ & + D_6 N^2 + D_7 \bar{N}^2 + D_8 F_0^2 + D_9 \bar{F}_0^2 \\ & + D_{10} N F_0 + D_{11} \overline{N F_0} + D_{12} A_p \\ & + D_{13} A_n . \end{aligned} \tag{19}$$



$$R_2(\omega) = 2 + \frac{4}{(2+\omega)}$$

Geometrical parameter



S. Drenska, A. Georgieva, V. Gueorguiev,

R. Roussev and P. Raychev,

"Unified description of the low lying states of the ground bands of even-even nuclei",
Phys. Rev. C 52, (1995) 1853.

927 low lying states of 271 nuclei from 5 major shells

σ = 45 keV

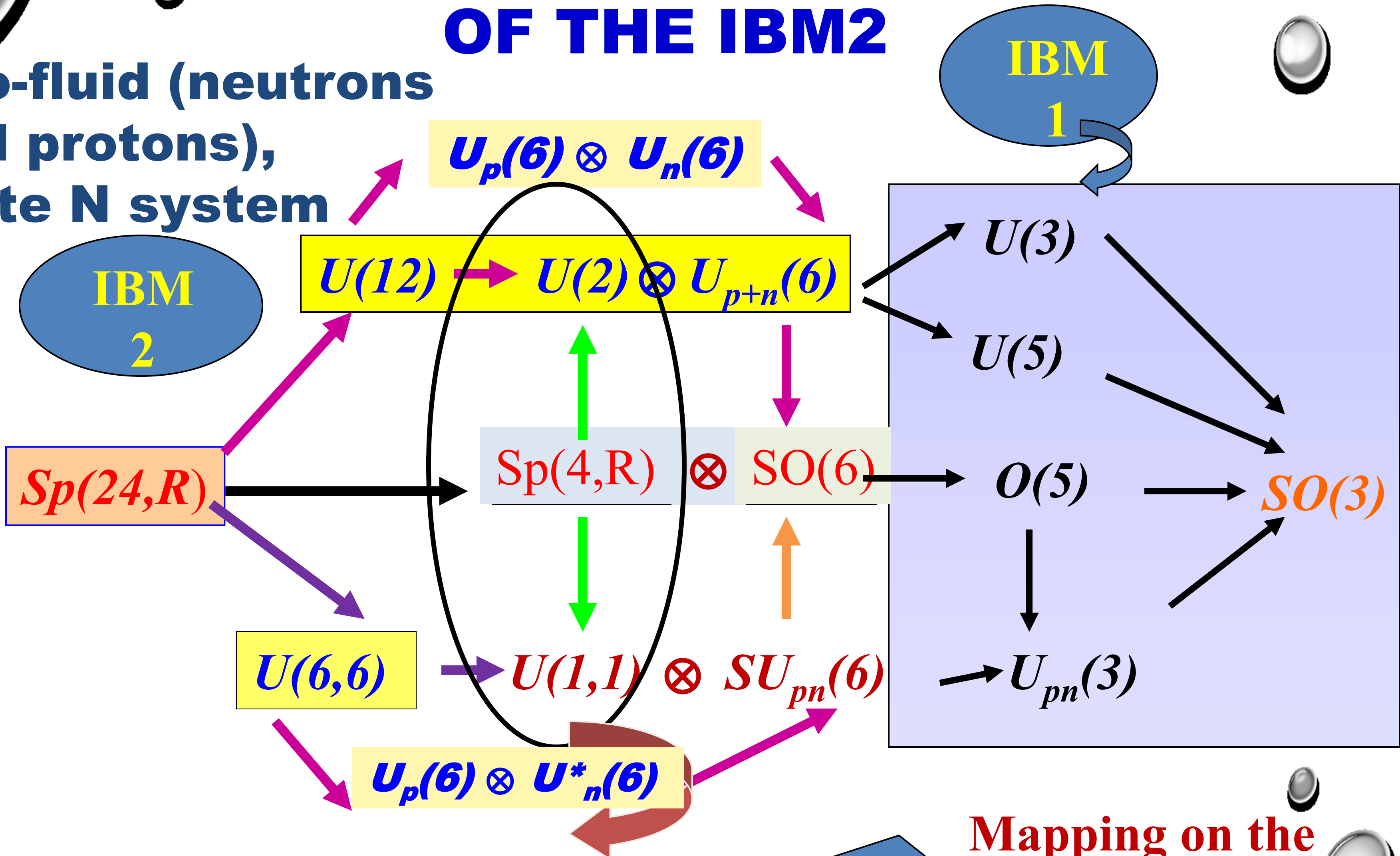
$\chi^2 = 1.0001$

TABLE XI. Parameters D_i .

i	D_i[MeV]	$\pm \Delta D_i$[MeV]
1	+0.0261526	0.0001740
2	−0.0009279	0.0000284
3	−0.0021273	0.0000232
4	−0.0323361	0.0001986
5	−0.0302709	0.0001770
6	−0.0000098	0.0000002
7	+0.0000059	0.0000001
8	+0.0009395	0.0000064
9	−0.000730	0.0000049
10	+0.0001599	0.0000013
11	−0.000153	0.0000011
12	+0.001724	0.0000107
13	−0.001318	0.0000064

GENERALIZED REDUCTION SCHEME OF THE IBM2

two-fluid (neutrons and protons), finite N system

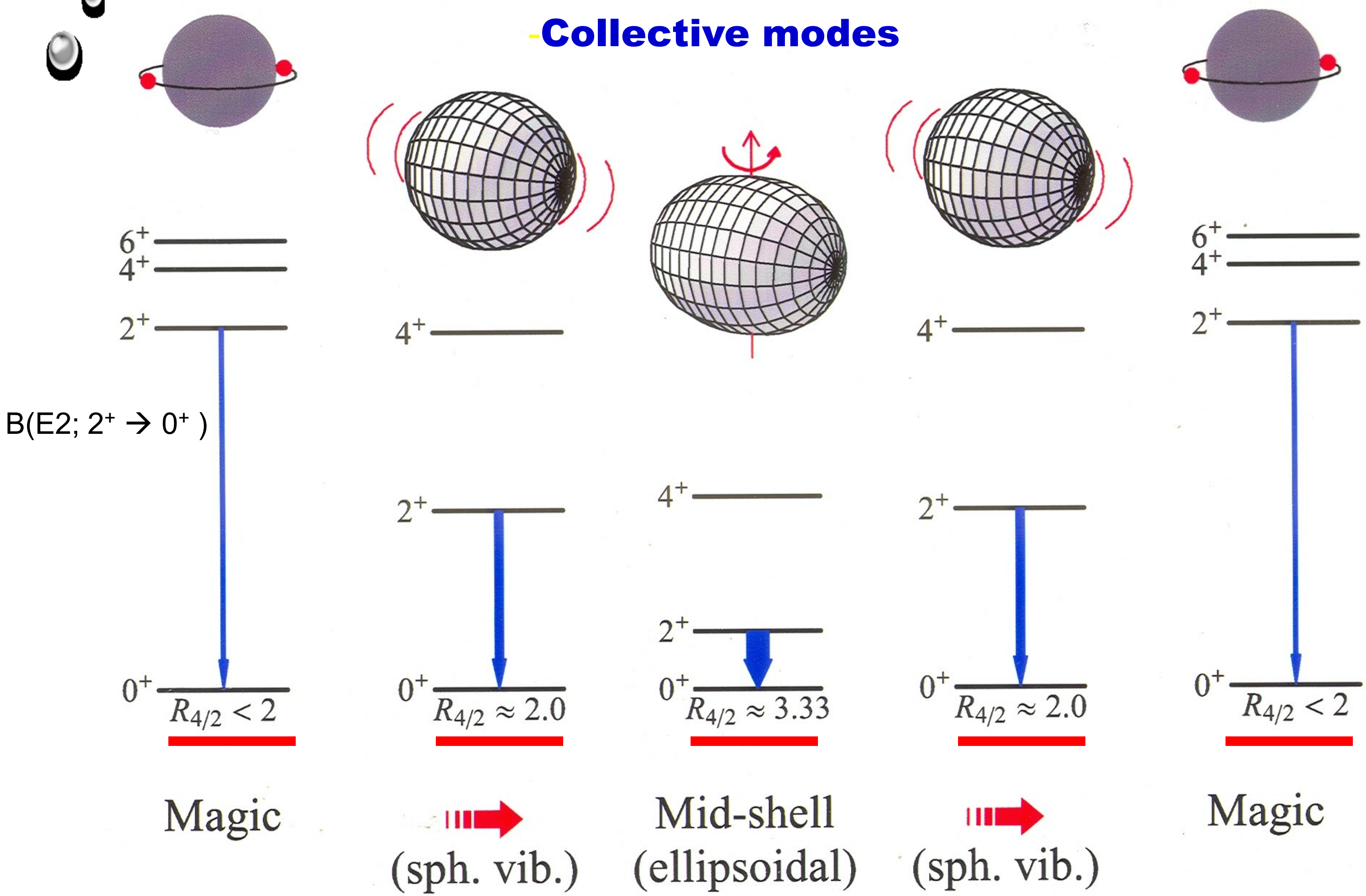


Sp(4,R) classification scheme

Mapping on the nuclei from a major shell

Evolution of nuclear structure

(as a function of nucleon number)



Phase/Shape transitions

$\omega = 1$ → **mid shell** → Deformed nuclei → SU(3)

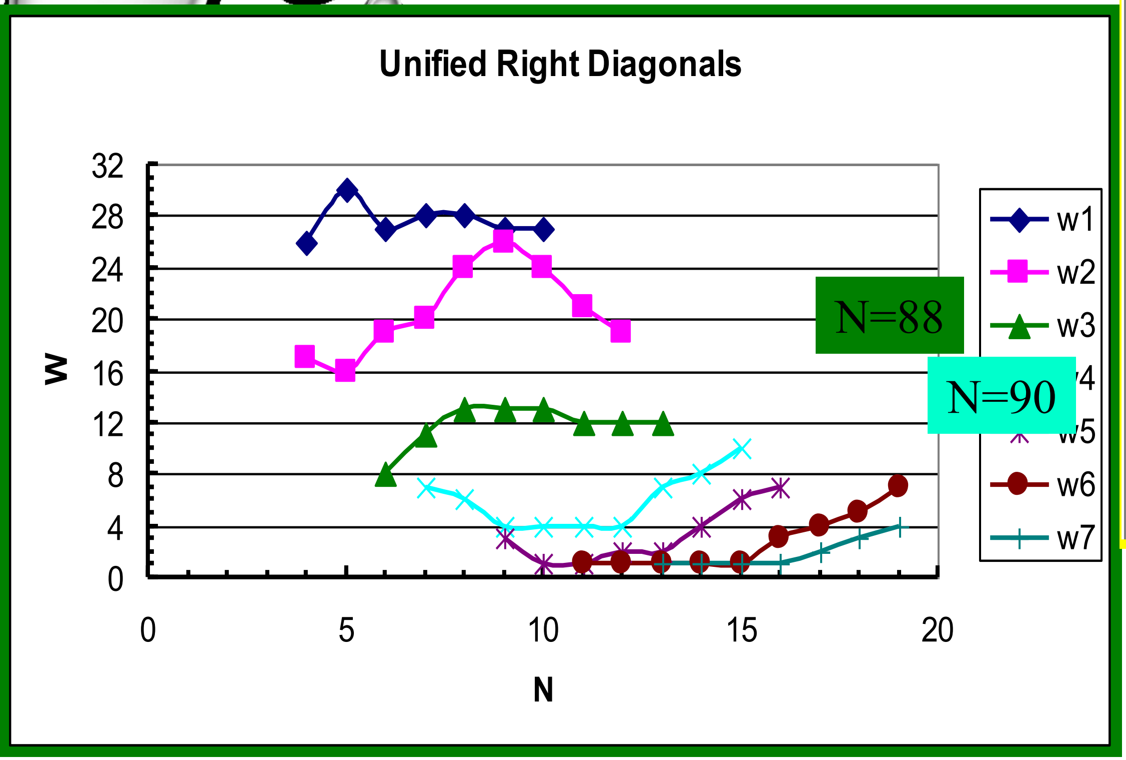
$1 < \omega < 20$ → **?Transitional nuclei?** → O(6)

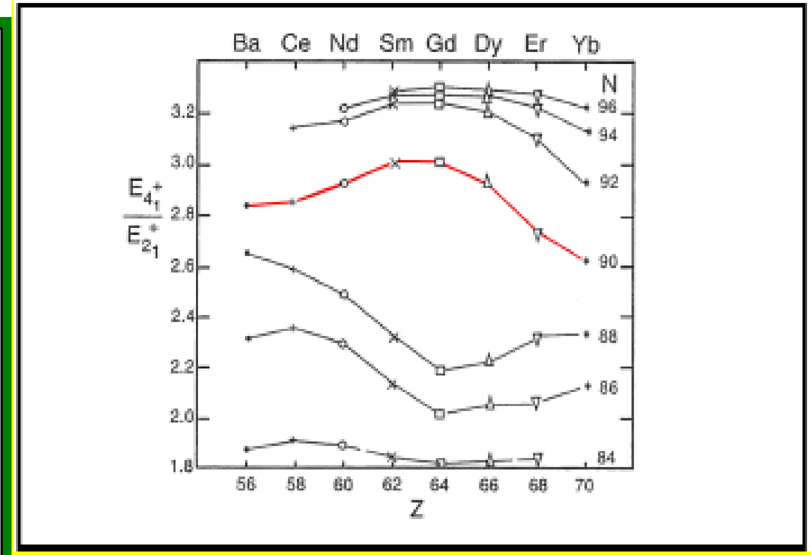
$20 < \omega$ → **closed shell** → Spherical nuclei → U(5)

Property	U(5)	E(5)	O(6)	X(5)	SU(3)
ω	20÷30	11,12	6	3,4	1
$R_r = E_4/E_2$	2.00	2.20	2.50	2.91	3.33
$R_v = E_0/E_2$		3.03		5.67	

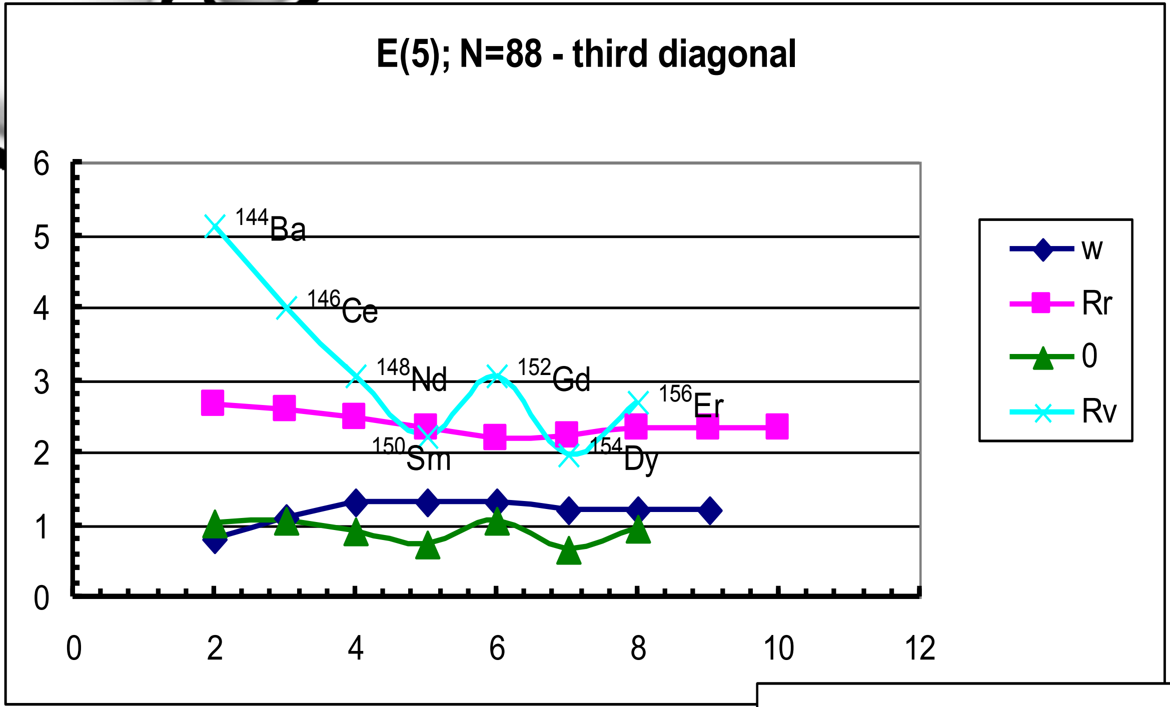
Critical Point Symmetries

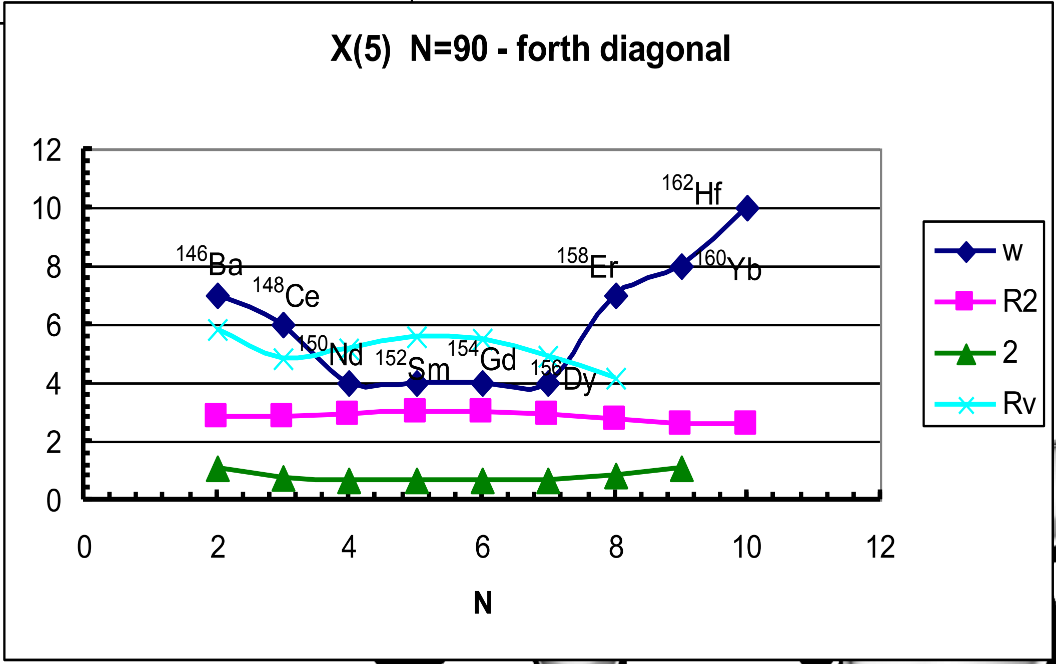
Other important properties
B(E2) transition strengths





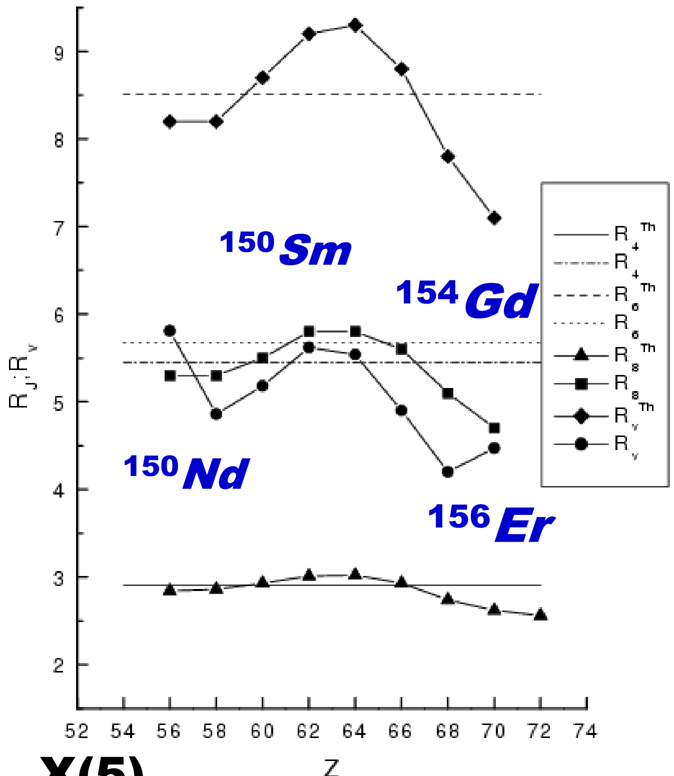
A, I, Georgieva, Tz. Venkova, and A. Aprahamian
“ *Systematics of Nuclei with Critical Point Symmetries in the Rare-Earth Region*”
Proceedings of the XXIII International Workshop on Nuclear Theory (14-19 June , 2004, Rila Mountains, Bulgaria),
ed. S. Dimitrova, Heron Press Ltd., Sofia, Bulgaria (2005), 283-294


E(5); N=88 - third diagonal
^{144}Ba
^{146}Ce
^{148}Nd
^{150}Sm
^{152}Gd
^{154}Dy
^{156}Er
w
Rr
0
Rv


X(5) N=90 - forth diagonal
^{146}Ba
^{148}Ce
^{150}Nd
^{152}Sm
^{154}Gd
^{156}Dy
^{158}Er
^{160}Yb
^{162}Hf
N
w
R2
2
Rv

Theoretical predictions of the critical points signatures and their corresponding experimental values for the nuclei in the:

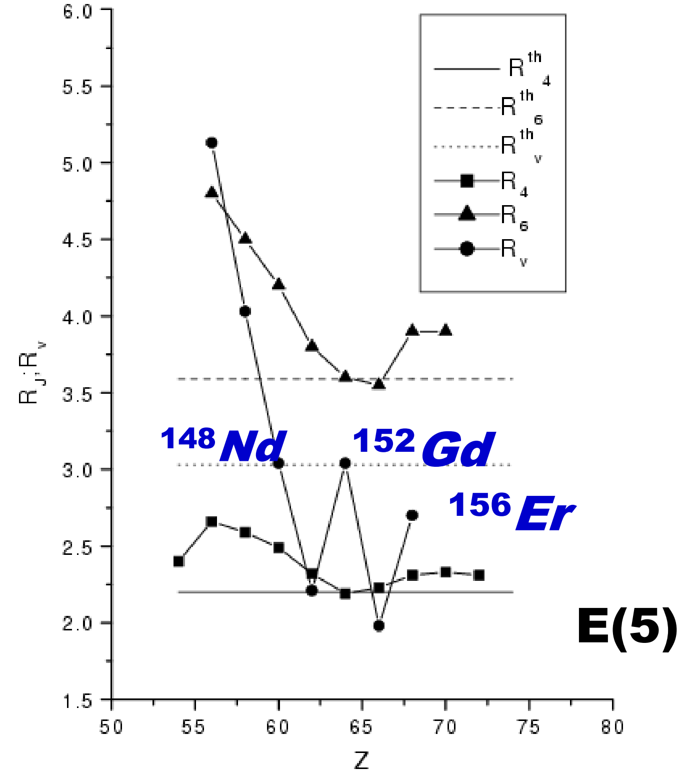
N=90 isotonic chain



X(5)

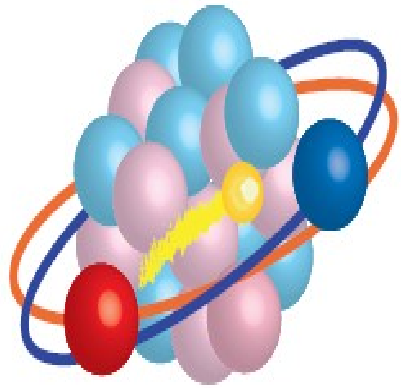
Nuclei	Z	N_p	N_pN_n	P
^{150}Nd	60	10	80	4.44
^{152}Sm	62	12	96	4.80
^{154}Gd	64	14	112	5.09
^{156}Dy	66	16	128	5.33

N=88 isotonic chain



Nuclei	Z	N_p	N_pN_n	P
^{148}Nd	60	10	60	3.75
^{150}Sm	62	12	72	4.00
^{152}Gd	64	14	84	4.20
^{154}Dy	66	16	96	4.36
^{156}Er	68	18	108	4.50

The application of the nuclear systematics for



Evaluation of the microscopic component of the nuclear binding energy

as a function of F-spin and the proton number Z

in the nuclear shells

Semi – empirical Microscopic Mass

$SEM = M_{exp} - E_{macro}$

Isolates the mass effects from the valence space

M_{exp} – experimental ground-state atomic mass-excess

$E_{macro} = M_{th} - E_{mic}$

M_{th} – calculated ground-state atomic mass-excess from FRDM

E_{mic} – calculated ground-state microscopic energy from FRDM– **Finite-Range Droplet Model** ([5] P. MËoller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, Atomic Data and Nuclear Data Tables 59, 185 (1995).)

Comparing the P systematics and F-spin classification

$$P = \frac{N_\pi N_\nu}{N_\pi + N_\nu}$$

average number of proton-neutron interactions per a valence nucleon

The relation between P and F_0 as classification parameters

$$N_\nu = \frac{1}{2}N_t - F_0 = \frac{1}{2}N_n$$

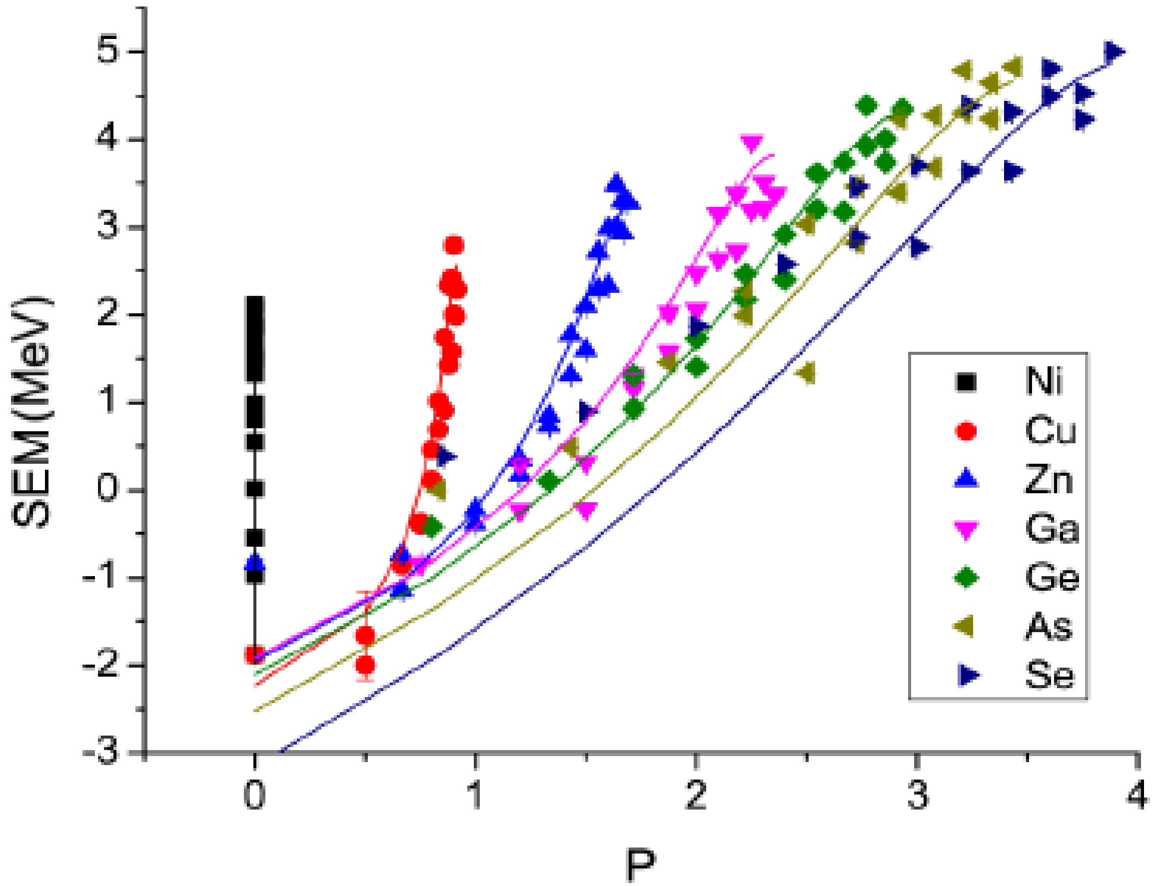
$$N_\pi = \frac{1}{2}N_t + F_0 = \frac{1}{2}N_p$$

$$P = \frac{1}{2N_t}\left(N_t^2 - 4F_0^2\right)$$

$$F_0^2 = \frac{N_t}{2}\left(\frac{N_t}{2} - P\right)$$

A. Teymurazyan, A. Aprahamian, and A. I. Georgieva
***"Prediction of Nuclear Masses in the A=80 Region of Nuclei as a Function of P and F-Spin"*,**
Proceedings of the International Conference on Nuclear Structure, ``Mapping the Triangle'', Wyoming, May 22-25, 2002,
eds. A. Aprahamian, J. Cizewski, S. Pittel and N. Zamfir,
AIP Conference Proceedings 638 (2002), 271 - 273

Fit of SEM as a linear function of P for the even and odd A nuclei in each isotopic chain



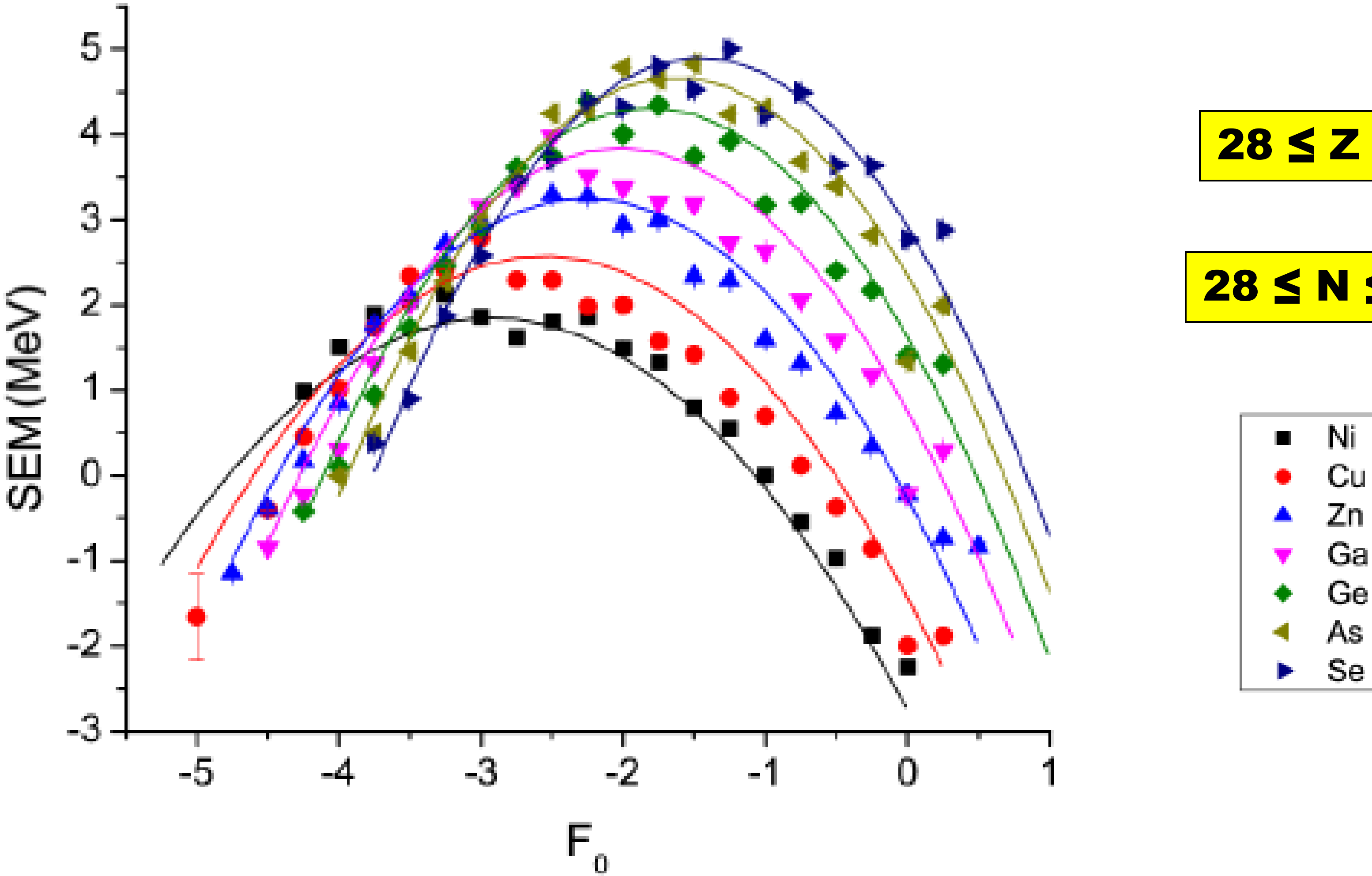
28 ≤ Z ≤ 34

28 ≤ N ≤ 49

$$SEM_i = B_0 + B_1 P_i$$

Result - Set of two coefficients B_0 and B_1 for each isotopic chain

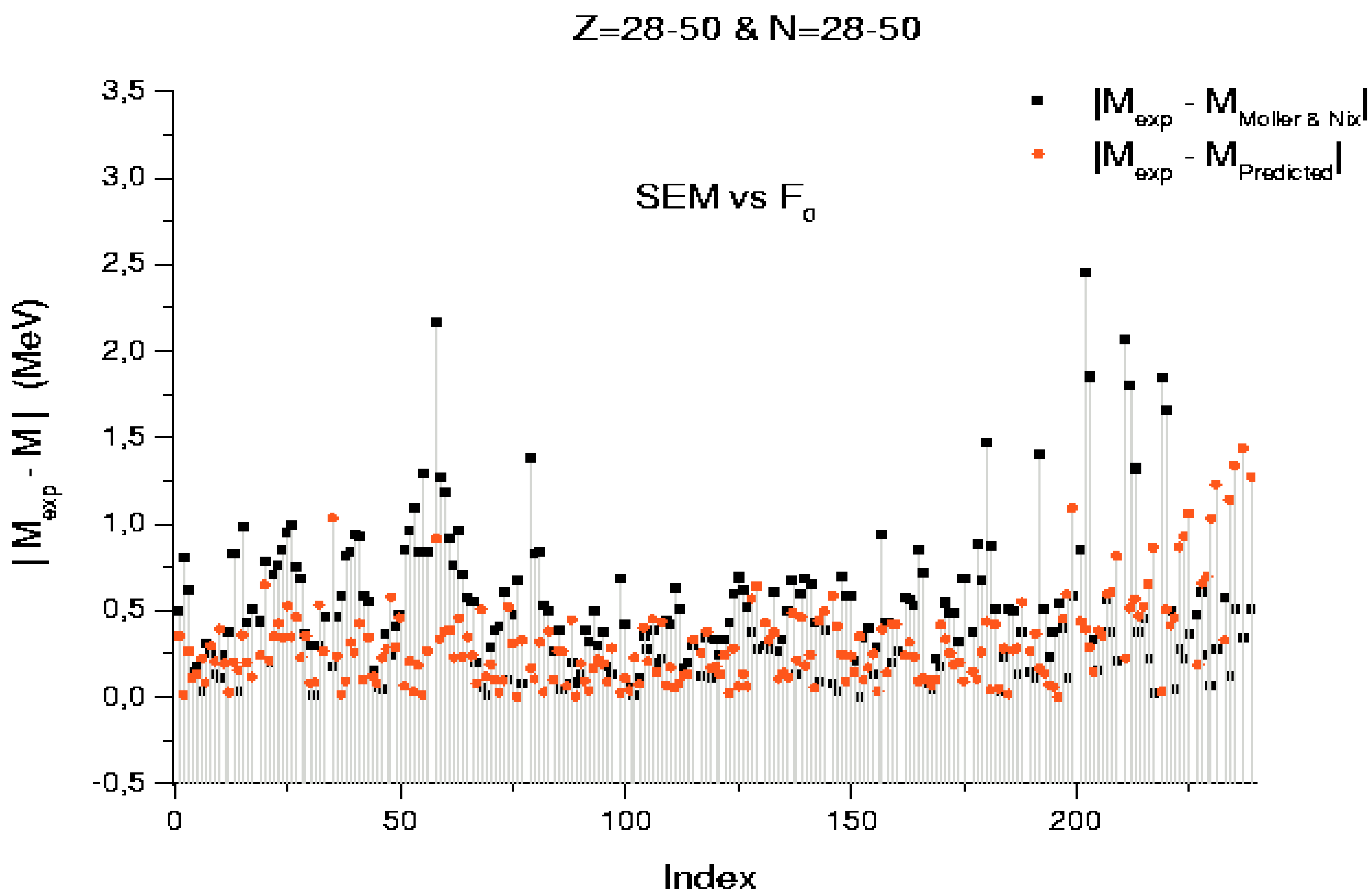
Fit of SEM as a parabolic function of F_0 for the even and odd A nuclei in each isotopic chain



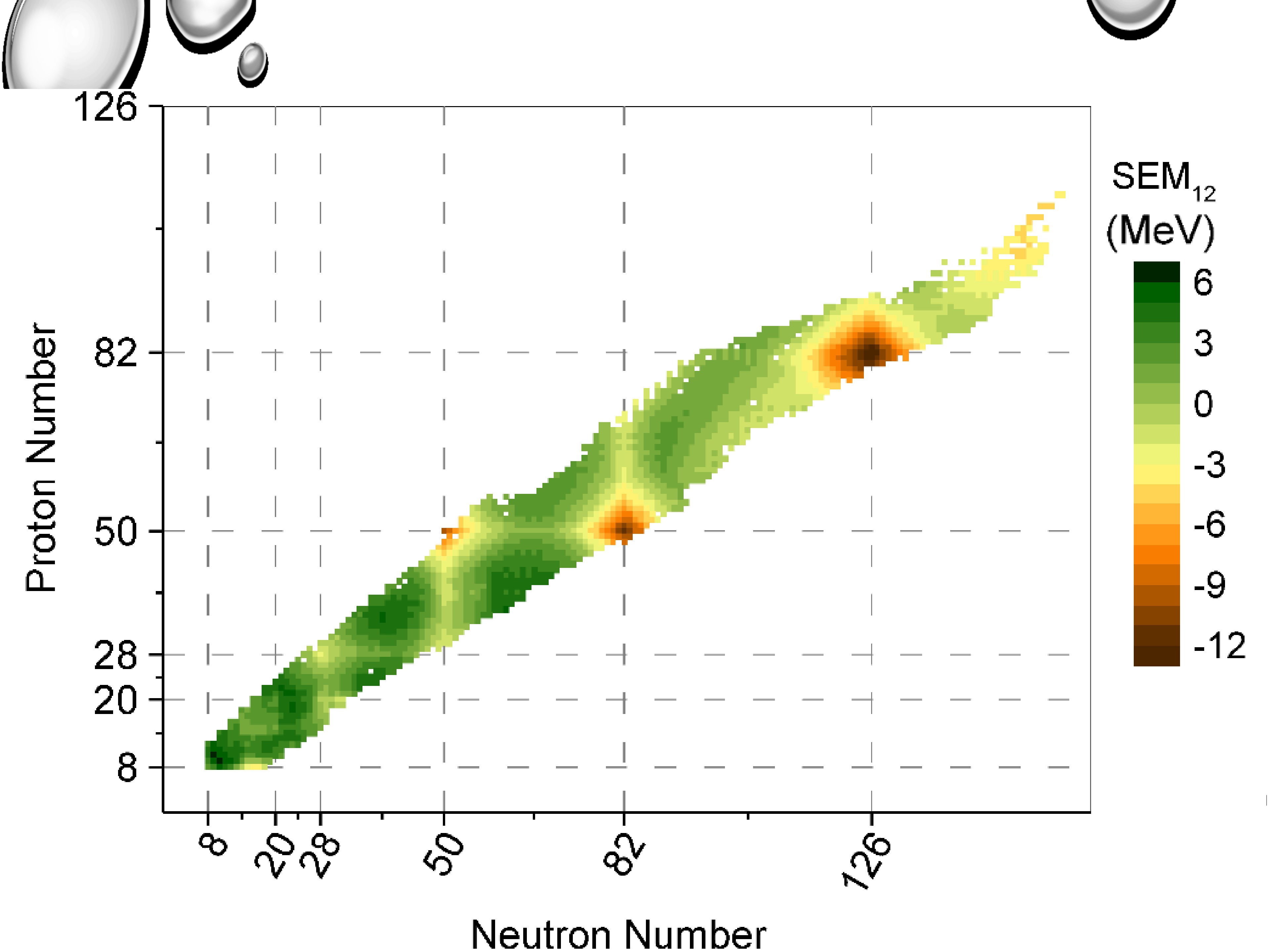
$28 \leq Z \leq 34$

$28 \leq N \leq 49$

$$SEM_i = B_0 + B_1 F_{0i} + B_2 F_{0i}^2$$



Comparison of the differences of the M_{exp} with the evaluation of FRDM and SEM(F_0)



SEM12 (MeV)
6
3
0
-3
-6
-9
-12
Proton Number
126
82
50
28
20
8
Neutron Number
8
20
28
50
82
126

Generalization of the fit for all shells

Taking into account subshells

$$SEM_{F-Spin} = (B_{00} + B_{01}Z + B_{02}Z^2)$$

$$+(B_{10} + B_{11}Z + B_{12}Z^2)F_0$$

$$+(B_{20} + B_{21}Z + B_{22}Z^2)F_0^2.$$

TABLE I. F-Spin$_{12}$ and F-Spin$_{12S}$ Zones

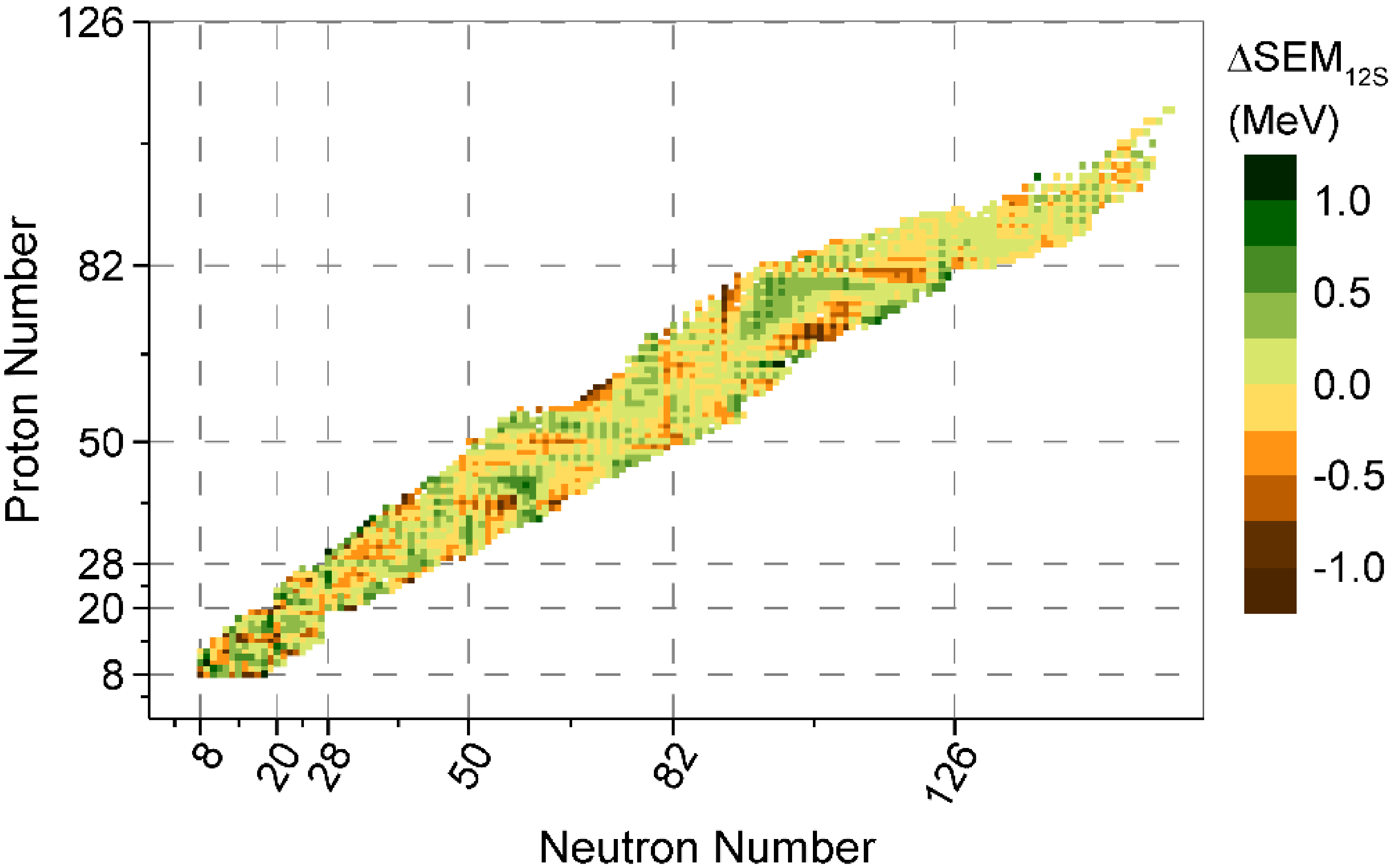
Zone	Z_{min}	Z_{max}	N_{min}	N_{max}	Number in AME$_{12}$
1	8	19	8	19	111
2	8	19	20	27	63
3	20	27	20	27	53
4	20	27	28	49	88
5	28	49	28	49	241
6	28	49	50	81	327
7	50	81	50	81	297
8	50	81	82	125	664
9	82	125	82	125	161
10	82	125	126	169	312
1a	8	19	8	13[a]	42
1b	8	19	14[a]	19	69
8a	50	81	82	89[a]	182
8b	50	63 [a]	90 [a]	125	62
8c	64[a]	81	90 [a]	125	420
10a	82	125	126	139[a]	152
10b	82	125	140[a]	169	160

NEW RESULTS

TABLE II. F-Spin_{12} and F-Spin_{12S} Coefficients and Fit Parameters

Zone	B_{00} x10^{3}	B_{01} x10^{1}	B_{02} x10^{-1}	B_{10} x10^{2}	B_{11} x10^{0}	B_{12} x10^{-1}	B_{20} x10^{1}	B_{21} x10^{0}	B_{22} x10^{-1}	χ^2
1	0.0147900	-0.1452416	0.4285861	0.1163917	-1.6121992	0.5694757	-1.0610863	1.8878219	-0.7900485	0.88
2	-0.0475336	0.7870946	-3.0531705	0.6054538	-8.6012758	3.2322905	-2.4266262	2.9444939	-0.9753579	0.17
3	-0.0741829	0.7367648	-1.7272693	-0.8492452	6.4960763	-1.1991330	-3.4099642	2.4420547	-0.4308491	0.18
4	-0.0863218	0.7171904	-1.4457002	0.7306156	-6.0531135	1.2234365	4.4280193	-3.4529176	0.6652815	0.16
5	-0.0977425	0.5425733	-0.7256019	0.0751298	-0.7473322	0.1320527	0.3513260	-0.2106814	0.0237564	0.12
6	-0.0811950	0.4234495	-0.5236430	0.0831527	-0.6586171	0.1074919	-0.5821897	0.2747766	-0.0346648	0.11
7	-0.2836332	0.9055215	-0.7127516	-0.7886907	2.3032209	-0.1670863	-0.6533741	0.1790153	-0.0125753	0.10
8	-0.1153739	0.3432309	-0.2525416	-0.0865233	0.1532584	-0.0027881	0.6696855	-0.1831231	0.0120209	1.18
9	-1.7106991	3.8622297	-2.1788232	-3.6607297	8.1261257	-0.4526118	-1.7925891	0.3840867	-0.0208027	0.03
10	-0.6157849	1.3006683	-0.6877197	-3.8641512	8.1970697	-0.4339413	-4.9044329	1.0658861	-0.0578951	0.95
1a	-0.0283268	0.6574424	-3.1936856	0.1033252	-3.6748069	2.5282586	-1.4860543	2.5421077	-1.3537825	0.37
1b	-0.0274703	0.4902454	-1.9057283	0.5251396	-8.5107438	3.2539922	-0.0939506	0.8355109	-0.4825100	0.29
8a	-0.3871140	1.2626455	-1.0226552	0.0670634	-0.6962423	0.0953547	0.2086205	-0.0473826	0.0007183	0.04
8b	-0.0261230	0.1187289	-0.1234234	-1.7651592	5.7003851	-0.4617681	6.3602980	-2.0576716	0.1674703	0.06
8c	0.0525327	-0.1335492	0.0857013	0.6845233	-1.8306691	0.1234314	-0.8907357	0.2746192	-0.0209872	0.16
10a	-1.1041726	2.4026863	-1.3075806	0.5074829	-1.7769501	0.1343399	-4.0551190	0.9572352	-0.0569254	0.04
10b	0.0600503	-0.0923226	0.0390522	0.7830141	-1.5869912	0.0793818	-1.9350190	0.3968104	-0.0199591	0.04

Difference between the experimental AME12 and the F-Spin12 fit.



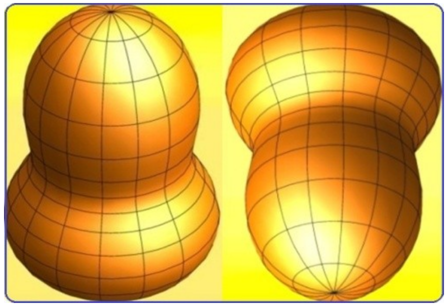
Conclusions

- We have obtained expression describing the relationship between P and F-spin.

- F-spin is generalized for all even and odd A nuclei and to accommodate particle and/or hole interpretations of the valence nucleons.

- A smooth dependence of the microscopic component of the nuclear binding energy has been obtained using a simple quadratic expansion of the third projection of the F-spin and the proton number Z. This allows for the fit of 2317 nuclear masses using 14 common shell zones with an overall standard deviation of 324 keV.

- The predictive power of the new approach is discussed, and tables are included for the predictions of masses which are presently unmeasured, or which have considerable experimental uncertainties.



Thank you