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Algebraic Models for Structure of Heavy N=Z Nuclei: IBM-4 Results for Gamow-Teller and α-transfer Strengths

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Plan of the Talk

- **Introduction to N=Z nuclei**
- **IBM-4:** $U_{sdST}(36) \supset SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ ***limit***
- **GT strengths for** ${}^{4n+2}_{2n+2}X_{2n} \rightarrow {}^{4n+2}_{2n+1}Y_{2n+1}$
- **α-transfer strengths involving N=Z nuclei**
- **Conclusions**

1. INTRODUCTION

N=Z nuclei start from deuteron and stop at ^{100}Sn

Light N=Z nuclei: ^{12}C , ^{16}O , ^{24}Mg , ------

- ^{12}C → Hoyle state at 7.654 MeV and Hoyle band
- ^{16}O → Tetrahedral symmetry
- ^{24}Mg → Rod like structure at high excitation

For N=Z nuclei, protons and neutrons occupy same orbits

Isospin and SU(4) symmetry are important

N=Z nuclei with A>60 are heavy N=Z nuclei - they start from ^{62}Ga

Isoscalar (T=0) and isovector (T=1) pairing

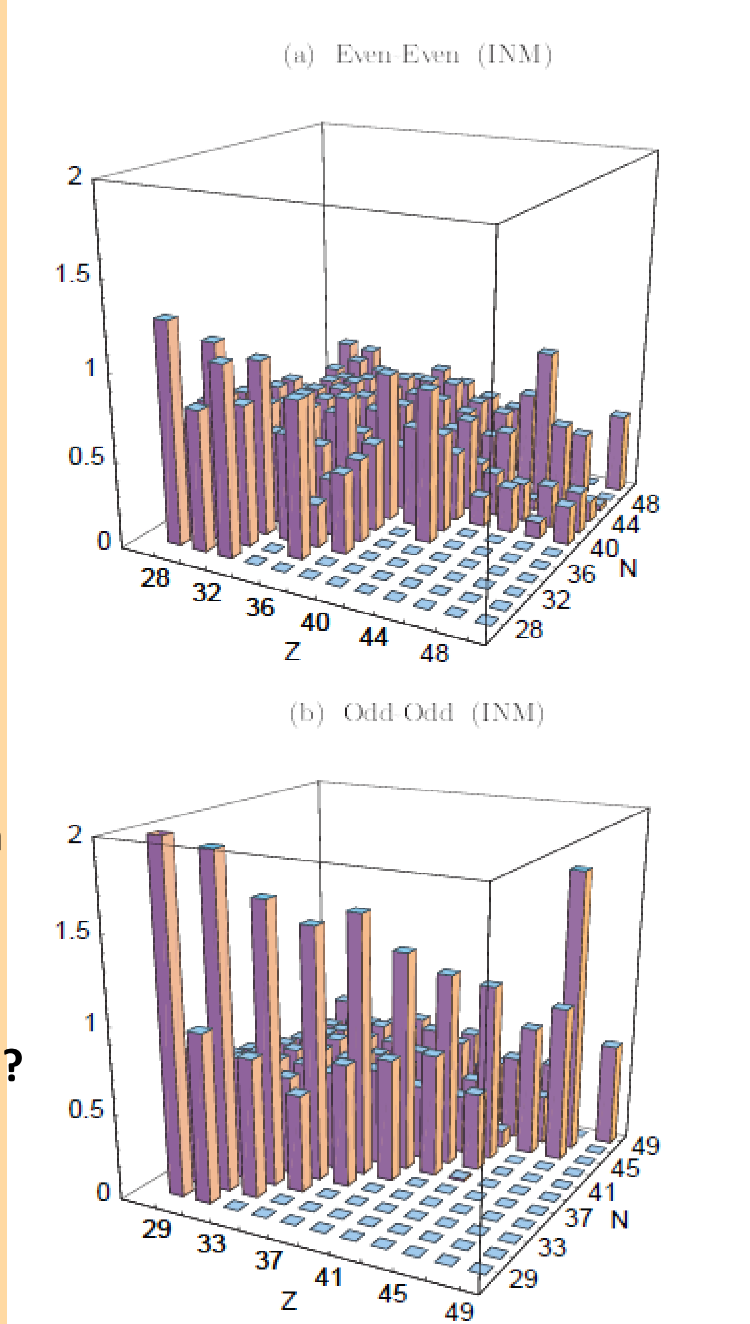
N=Z odd-odd nuclei – new insights into pn (or T=0) superfluidity

SU(4) ⇒ degenerate T=0 and T=1 states in o-o N=Z nuclei

Double binding energy difference

δV_{np}

restoration of SU(4) symmetry In odd-odd N=Z nuclei ?



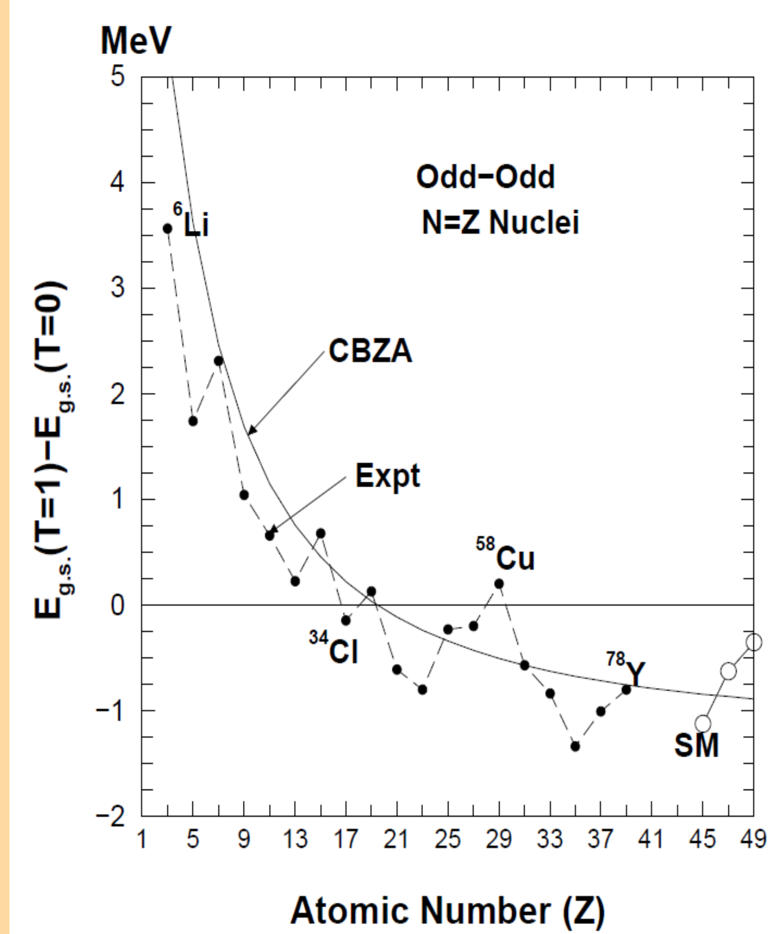
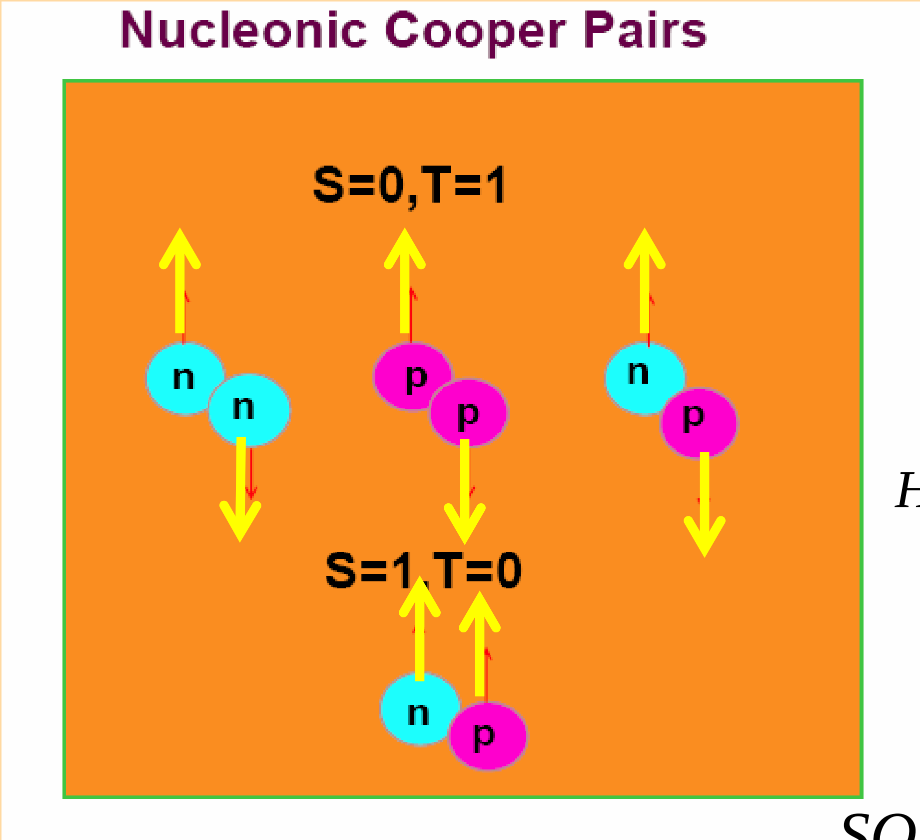


Figure 6.2: Color on-line. Energy of the lowest $T = 1$ level with respect to the lowest $T = 0$ level in N=Z odd-odd nuclei with Z=3-49. The filed circles are experimental data and they are taken from [10]; they are joined by dashed lines to guide the eye. The curve labeled CBZA correspond to the result of Eq. (6.23); see [12]. The open circles for Z=45,47 and 49 are from shell model calculations as reported in [12].

Competition between Wigner energy, symmetry energy and isovector pairing energy



In *LST* coupling with two-particle *L=0*:

isovector pairs $P^{\dagger}_{\mu}$: *(ST)=(01)*

isoscalar pairs $D^{\dagger}_{\mu}$: *(ST)=(10)*

$$H_p(x) = -(1-x)\sum_{\mu} P^{\dagger}_{\mu}(P^{\dagger}_{\mu})^{\dagger} - (1+x)\sum_{\mu} D^{\dagger}_{\mu}(D^{\dagger}_{\mu})^{\dagger}$$

SO(8) algebraic model

$$SO(8) \supset SO_{ST}(6) \supset SO_S(3) \oplus SO_T(3) \quad \mathbf{I}$$

$\chi = 0$: **Wigner's *SU(4)*** (I)

$$SO(8) \supset [SO(5) \supset SO_S(3)] \otimes SO_T(3) \quad \mathbf{II}$$

$\chi = 1$: **Iso vibrational** (II)

$$SO(8) \supset [SO(5) \supset SO_T(3)] \otimes SO_S(3) \quad \mathbf{III}$$

$\chi = -1$: **Iso rotational** (III)

$$U(4\Omega) \supset [U(\Omega) \supset \{SO(\Omega)\ (\mathbf{I}),\ SU(3)\} \supset \text{---}] \otimes [SU(4) \supset SU_S(2) \otimes SU_T(2)]$$

$$U(4\Omega) \supset \{Sp(2\Omega) \supset [SO(\Omega) \supset \text{----}] \otimes SU(2)\} \otimes SU(2) \quad \mathbf{II,III}$$

Kalin Drumev

VKBK and CA, Nucl. Phys. A764, 181 (2006)

A hermitization procedure for the *SO*(8) Hamiltonian + Dyson mapping gives interacting *s*-boson *U(6)* model with the bosons carrying *(ST)* = *(10)* $\oplus$ *(01)* degrees of freedom.

By adding *d*-bosons gives the full IBM-4 with *U*(36) SGA
This will include deformation effects

A group chain of IBM-4 that is proved to useful for heavy N=Z nuclei is [VKBK, Ann. Phys. (N.Y.) 280, 1 (2000)]:

$$U_{sd:ST}(36) \supset SO_{sd:ST}(36) \supset SO_{s:ST}(6) \oplus SO_{d:ST}(30) \supset \ldots$$
$$\supset SO_L(3) \otimes SO_S(3) \otimes SO_T(3)$$

2. IBM-4 for N=Z nuclei:

$U_{sdST}(36) \supset SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ ***limit***

IBM-4: s and d bosons each with *(ST)=(10)+(01)*

$U(36) \to U_{sd}(6) \otimes U_{ST}(6)$

$U(36) \to U_{sdS}(18) \oplus U_{sdT}(18)$

$U(36) \to U_{sST}(6) \oplus U_{dST}(30)$

$U(36) \to SO_{sdST}(36)$

$U_{sd}(6) \otimes U_{ST}(6)$ and $SO_{sdST}(36)$ → **allow for Wigner *SU(4)***

$SO_{sdST}(36)$ → **Soft , shape mixing etc., like *SO(6)* of IBM-1 expected to be good for heavy N=Z nuclei**

$SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ **limit is used in the past and in the present work**

$$\left| \begin{array}{ccccccccc} U_{sdST}(36) & \supset & SO_{sdST}(36) & \supset & SO_{S_sT_s}(6) & \oplus & [SO_{dST}(30) & \supset & \{SO_d(5) \\ \{N\} & & [\omega] & & [\omega_s] & & [\omega_d] & & [\omega_1\omega_2] \end{array} \right.$$

$$\begin{array}{ccccccc} \supset & SO_L(3)\} & \otimes & SO_{S_dT_d}(6)] & \supset & SO_L(3) & \otimes & SO_{ST}(6) \\ & L & & [\sigma_1\sigma_2\sigma_3] & & L & & [\sigma_a\sigma_b\sigma_c] \end{array}$$

$$\left. \begin{array}{ccccccccc} \supset & SO_L(3) & \otimes & [SO_S(3) & \oplus & SO_T(3)] & \supset & SO_J(3) & \otimes & SO_T(3) \\ & L & & S & & T & & \vec{J} = \vec{L} + \vec{S} & & T \end{array} \right\rangle$$

Note: *SO(6) ~SU(4)*

We have not only good Wigner *SU(4)* in the total space but also in the *s* and *d* boson spaces seperately

ω, ω_s and ω_d are seniority quantum numbers, also $\omega = N$ for low-lying states

gs: for o-o N=Z $SO_{ST}(6)$ irrep [1] giving *(ST)=(10) and (01)*
for e-e N=Z $SO_{ST}(6)$ irrep [0] giving *(ST)=(00)*

(a) $(ST) = (00)$

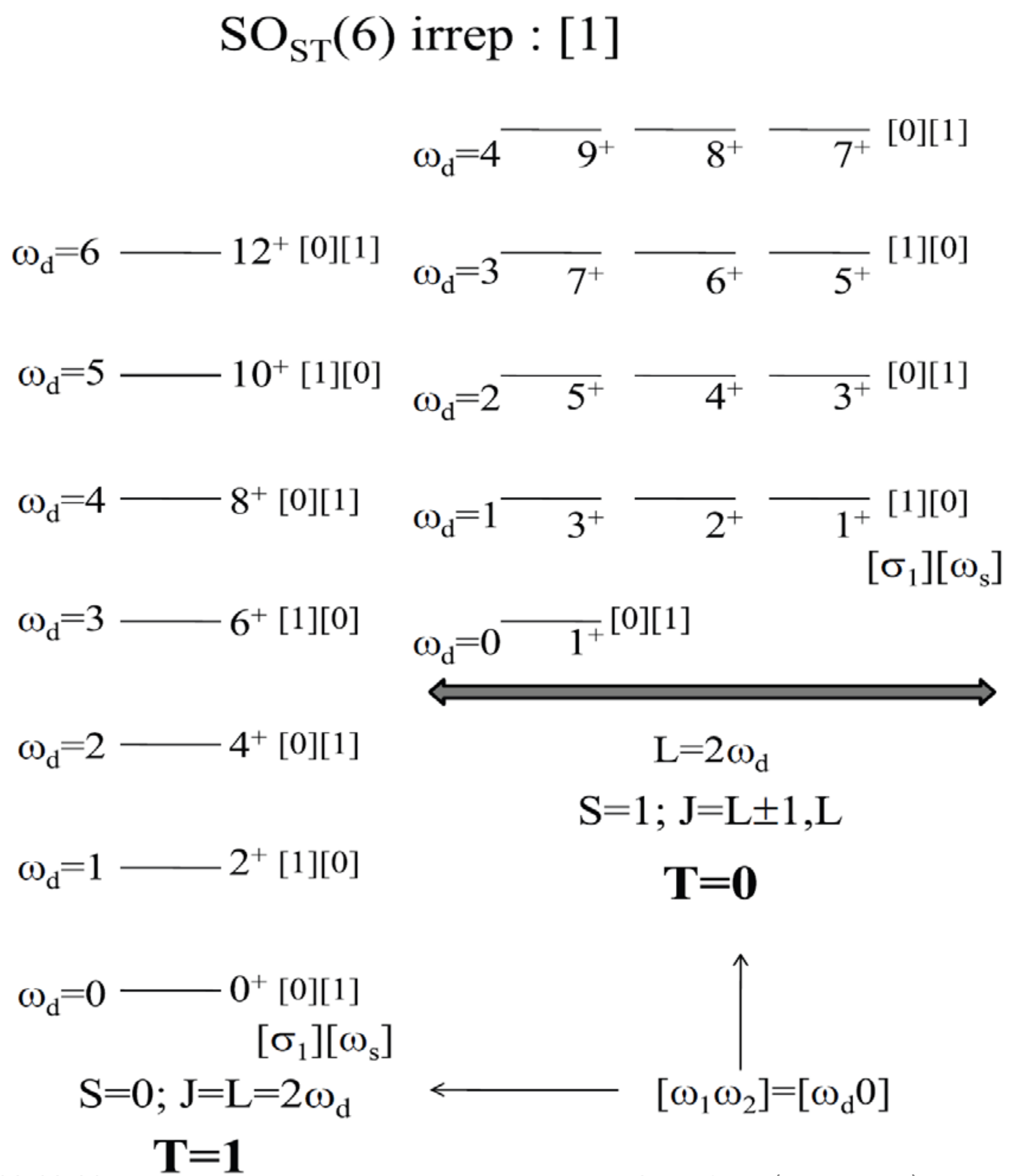
$[\omega_d]$	$[\omega_1\omega_2]$	L^π	$[\sigma_1\sigma_2\sigma_3]$ $[\omega_s]$
[3]	[1]	2^+	[1] [1] , [3] [3]
	[21]	$5^+, 4^+, 3^+, 2^+, 1^+$	[1] [1]
	[3]	$6^+, 4^+, 3^+, 0^+$	[1] [1] , [3] [3]
[2]	[0]	0^+	[2] [2]
	[2]	$4^+, 2^+$	[0] [0] , [2] [2]
[1]	[1]	2^+	[1] [1]
[0]	[0]	0^+	[0] [0]

(b) $(ST) = (10) \oplus (01)$

$[\omega_d]$	$[\omega_1\omega_2]$	L^π	$[\sigma_1\sigma_2\sigma_3]$ $[\omega_s]$
[3]	[1]	2^+	[1] [0] , [1] [2] , [3] [2] , [3] [4] , [21] [2
	[21]	$5^+, 4^+, 3^+, 2^+, 1^+$	[1] [0] , [1] [2] , [21] [2]
	[3]	$6^+, 4^+, 3^+, 0^+$	[1] [0] , [1] [2] , [3] [2] , [3] [4]
[2]	[0]	0^+	[2] [1] , [2] [3]
	[11]	$3^+, 1^+$	[11] [1]
	[2]	$4^+, 2^+$	[0] [1] , [2] [1] , [2] [3]
[1]	[1]	2^+	[1] [0] , [1] [2]
[0]	[0]	0^+	[0] [1]

Energy formula will have many terms

Group theory is more involved



Known results in the exact symmetry limit:

(1) B(E2; $L \rightarrow L$-2) exhibit a $\Delta L=4$ staggering

(2) Formulas for B(E2)'s involving low-lying states are derived

(3) Some of the observed properties of low-lying levels in o-o N=Z nucleus ^{74}Rb are described including aligned spin *1* in the lowest *T=0* band just as in CSM

(4) Predictions for deuteron transfer strengths are available

Simple Hamiltonian and a basis for incorporating the competition between *T=1* and *T=0* pairing

$$\left|N,\omega_s,(ST)\right\rangle = \left|N,\omega = N,\omega_s,\omega_d = 0,(ST)\right\rangle;$$

$$\omega_s = N, N-2, \ldots, 0 \text{ or } 1$$

mixing term

$$H_{mix} = \alpha\left\{C_2(SO_{s:S\,T}(6)) + (\beta/\alpha)\hat{n}_{s:S} + (\gamma/\alpha)\hat{n}_d\right\}$$

$$\left|N,\omega_s,(ST)\right\rangle = \sum_{n_s(n_d)} C^{N,\omega=N,\omega_s,\omega_d=0}_{n_s,n_d}(6,30)\left|N,n_s,n_d,\omega_s,\omega_d = 0,(ST)\right\rangle$$

$$\left|n_s,\omega_s,(ST)\right\rangle = \sum_{n_{sS}(n_{sT})} C^{n_s,\omega_s,(ST)}_{n_{sS},n_{sT}}(3,3)\left|n_s,n_{sS},n_{sT},(ST)\right\rangle;\; n_s = n_{sS} + n_{sT}$$

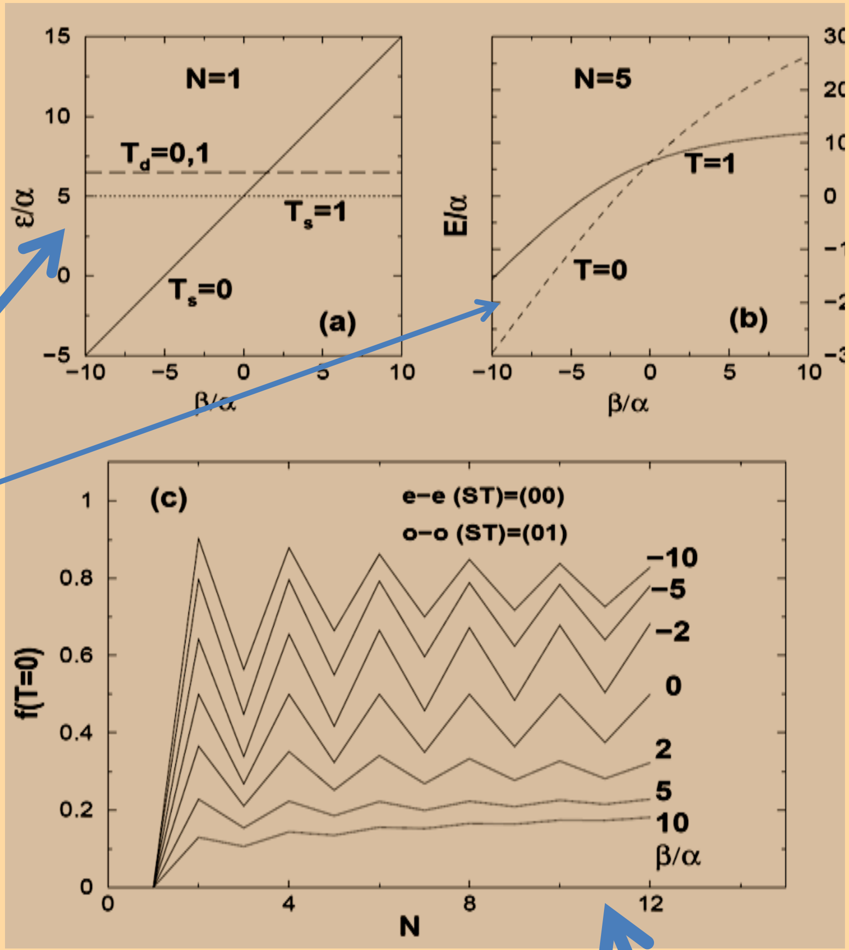
VKBK, J. Math. Phys. **38,** 6639 (1997)

$$\left|N,\alpha,(ST)\right\rangle = \sum_{\omega_s} \mathbb{C}^{N,\alpha,(ST)}_{\omega_s}\left|N,\omega_s,(ST)\right\rangle$$

Significance of the β/α term

Single boson energies:

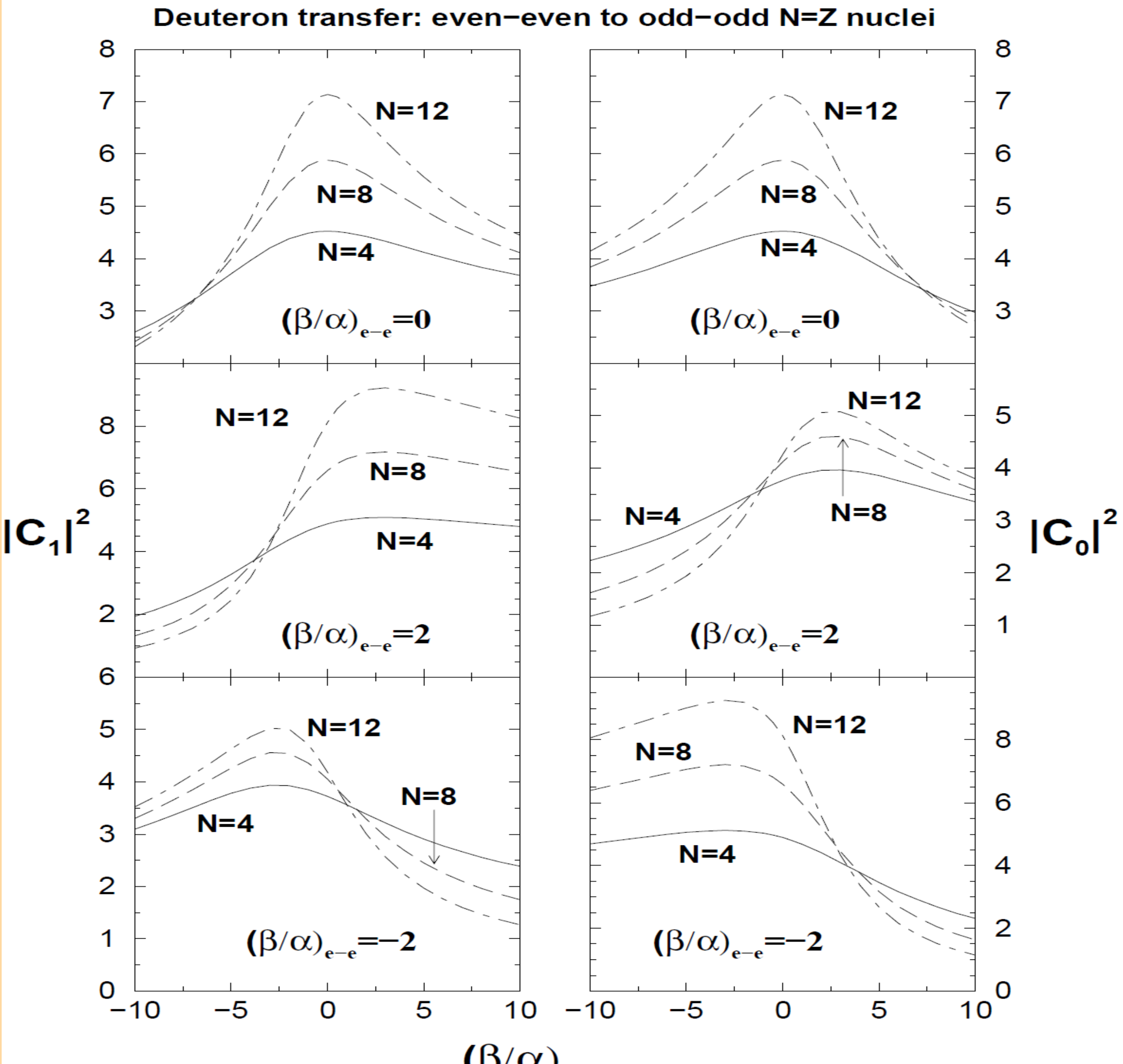
$$\varepsilon(T_s = 0)/\alpha = 5 + \beta/\alpha,$$
$$\varepsilon(T_s = 1)/\alpha = 5,$$
$$\varepsilon(T_d = 0,1)/\alpha = 5 + \gamma/\alpha$$



For a *N* boson system with *N* odd
$\beta/\alpha<0$ gives *T=0* gs
$\beta/\alpha>0$ gives *T=1* gs
they will be degenerate for $\beta/\alpha=0$.
Thus β/α term generates competition between *T=0* and *T=1* pairing correlations.

$f(T=0)$ exhibits odd-even staggering in number of *T=0* pairs in the gs of N=Z nuclei just as in the shell model and the staggering is maximum for $|\beta/\alpha|<2$.

VKBK, Prog. Theo. Phys. 118 (2007) 893.



e-e to o-o
N→N+1

operator:
$S^{\dagger}_{ST}$

Results close to those obtained using *SO(8)* model

3. GT strengths

$$ {}^{4n+2}_{2n+2}X_{2n} \rightarrow {}^{4n+2}_{2n+1}Y_{2n+1} \Leftrightarrow (ST) = (01) \rightarrow (10);\ N \text{ is odd for both} $$

$$ T^{GT} = g_A^{eff}(s)\left[s^\dagger_{10}\tilde{s}_{01} + s^\dagger_{01}\tilde{s}_{10} \right]^{L=0,S=1,T=1}_{\mu_s,\mu_t} $$

generator of $SO(6)$ in s boson space

For β^+ decay we have $\mu_t = +1$

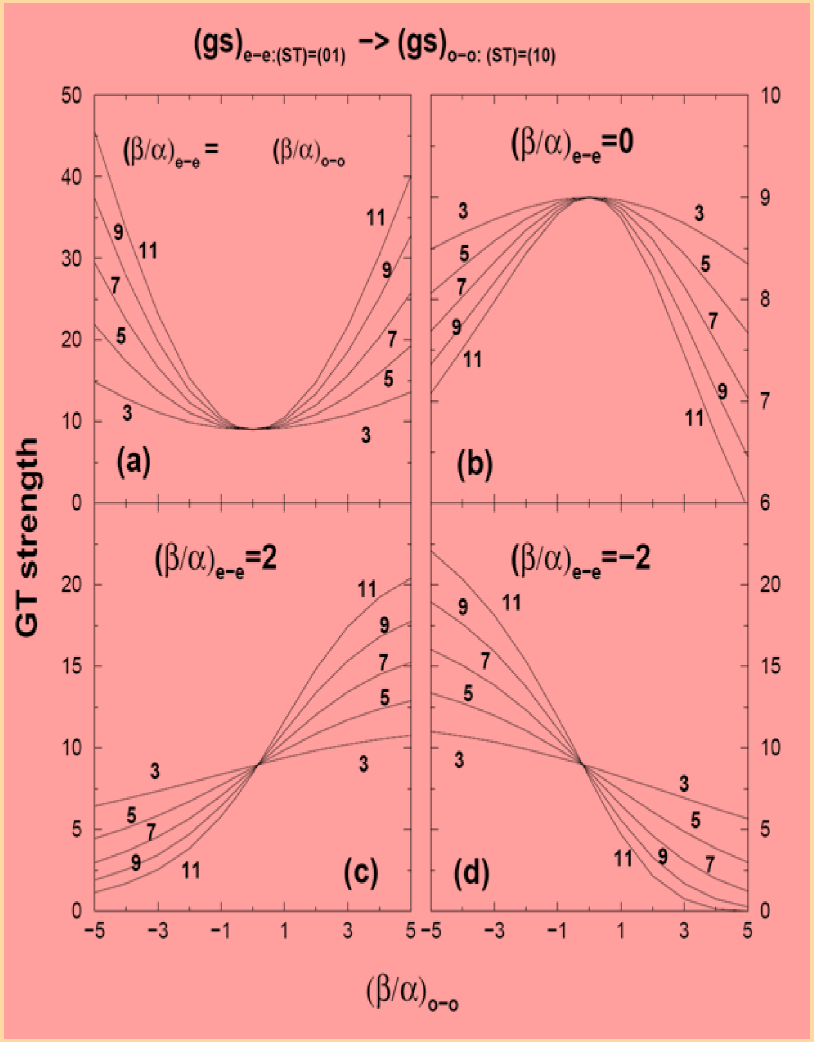
$$ M(GT) = \left| \left\langle Y, \alpha, (S_f T_f) = (10) \left\| T^{GT} \right\| X, \alpha^{gs}, (S_i T_i) = (01) \right\rangle \right|^2 ; $$

$$ \left\langle \left\| T^{GT} \right\| \right\rangle = g_A^{eff}(s) \sum_{\omega_s} \mathbb{C}^{N,\alpha^{gs},(S_iT_i)}_{\omega_s} \mathbb{C}^{N,\alpha,(S_fT_f)}_{\omega_s} $$

$$ \times \left\{ (2S_f+1)(2T_f+1) \left\langle C_2(SO_s(6)) \right\rangle^{\omega_s} \right\}^{1/2} \left\langle \begin{matrix} [\omega_s] & [11] \\ (S_iT_i) & (11) \end{matrix} \right\| \left. \begin{matrix} [\omega_s] \\ (S_fT_f) \end{matrix} \right\rangle $$

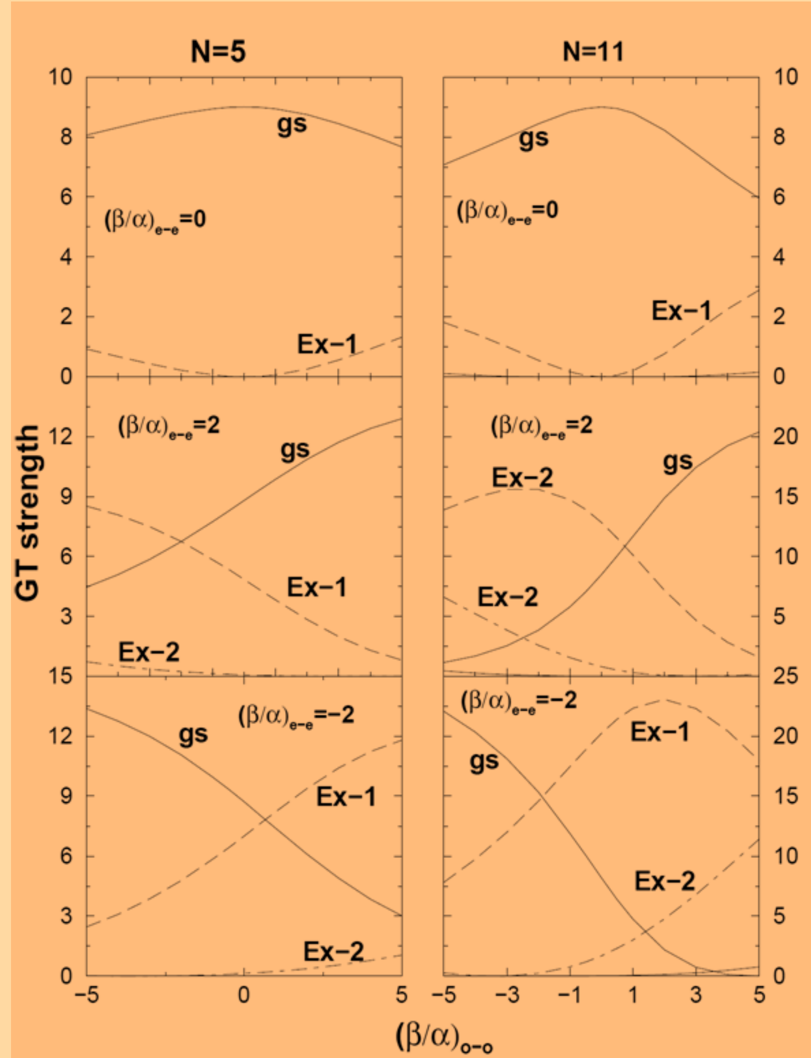
$$ = g_A^{eff}(s) \sum_{\omega_s} \mathbb{C}^{N,\alpha^{gs},(S_iT_i)}_{\omega_s} \mathbb{C}^{N,\alpha,(S_fT_f)}_{\omega_s} (\omega_s + 2) $$

Depending on β/α it is possible that gs to excited state strength can be larger than gs to gs



(a) GT strength increases as *N* increases and also as $|\beta/\alpha|$ increases

(b)-(d) for e-e and o-o involved, the $|\beta/\alpha|$ need not be same



β/α -ve means *T=0* pairing is stronger
+ve means *T=1* pairing stronger
-- these explain the results

The GT strengths calculated are all only with *L=0*
– thus they are only spin excited

For $^{62}Ge \rightarrow {}^{62}Ga$, data is now available in PRL 113, 092501 (2014)

low-lying 1^{+} states have more complex structure [*SU(4)* broken]

in IBM-4, the *L* and *S* need not be real (but *T* must be)

Therefore we need terms such as

$$\left[d^{\dagger}_{01}\tilde{s}_{10} + s^{\dagger}_{10}\tilde{d}_{01} \right]^{L=2,S=1;J=1,T=1} \;,\; \left[d^{\dagger}_{10}\tilde{s}_{01} + s^{\dagger}_{01}\tilde{d}_{10} \right]^{L=2,S=1;J=1,T=1}$$

Also we need to include $\omega_d \neq 0$ states in the basis and also perhaps some more *SU(4)* breaking terms (*SU(3)* Casimir?)

4. α-transfer strengths

$$T_\alpha = \kappa\left[s^\dagger_{10} \bullet s^\dagger_{10} + s^\dagger_{01} \bullet s^\dagger_{01} \right]$$

generator of *SU(1,1)* complimentary to *SO*$_{sST}$*(6)*

$$S_\alpha(A:N \to B:N+2) = \left| \left\langle B:N+2, \beta^{gs}, (ST) \| T_\alpha \| A:N, \delta^{gs}, (ST) \right\rangle \right|^2$$

$$\left\langle B:N+2, \beta^{gs}, (ST) \| T_\alpha \| A:N, \delta^{gs}, (ST) \right\rangle = \kappa \sqrt{(2S+1)(2T+1)}$$

$$\times \sum_{n_s(n_d),\omega_s} \mathbb{C}^{N,\delta^{gs},(ST)}_{\omega_s} \mathbb{C}^{N+2,\beta^{gs},(ST)}_{\omega_s} C^{N,N,\omega_s,\omega_d=0}_{n_s,n_d}(6,30) C^{N+2,N+2,\omega_s,\omega_d=0}_{n_s+2,n_d}(6,30)$$

$$\times \left\langle n_s+2, \omega_s, (ST) \| T_\alpha \| n_s, \omega_s, (ST) \right\rangle$$

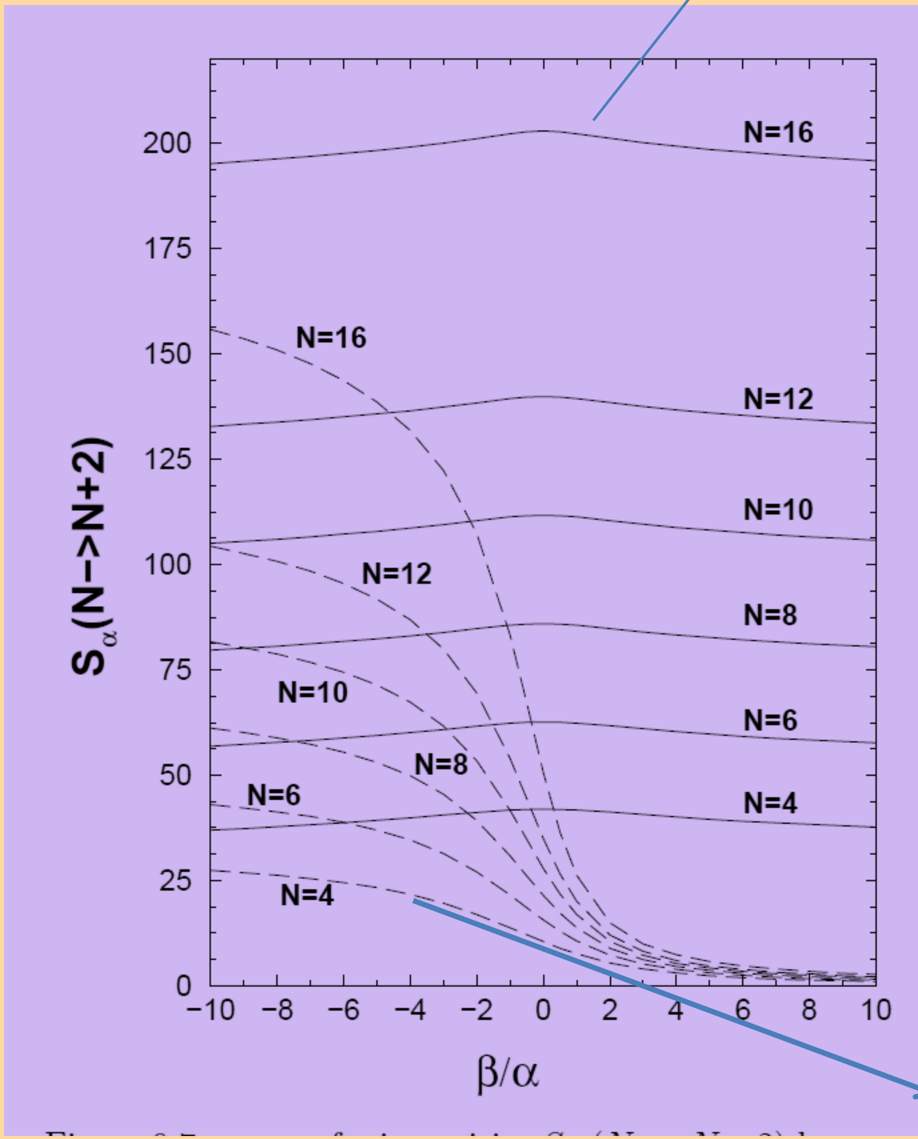
$$\sqrt{(n_s - \omega_s + 2)(n_s + \omega_s + 6)}$$

For $\beta/\alpha=0$:

$$S_\alpha = \kappa^2 (2S+1)(2T+1) \left[\frac{(N+6+\omega_s)(N+30-\omega_s)(N+34+\omega_s)(N+2-\omega_s)}{4(N+17)(N+18)} \right]$$

Note that ω_s = 0 for e-e and 1 for o-o

using T_α



Variation in the transfer strength is quite small as β/α changes. There is a small peak at $\beta/\alpha = 0$

The structures seen in the figure are same as those obtained in the fermionic *SO(8)* model

Strengths have ~ N^2 scaling

possible to consider

$$T_\alpha^{(eff)} = \kappa'(s_{10}^\dagger \bullet s_{10}^\dagger) + \kappa''(s_{01}^\dagger \bullet s_{01}^\dagger)$$

$$T_\alpha^{(d)} = \kappa' \; (s_{10}^\dagger \bullet s_{10}^\dagger)$$

$$\left\langle B:N+2,\beta^{gs},(ST)\left\| T_{\alpha}^{(d)} \right\| A:N,\delta^{gs},(ST)\right\rangle =$$

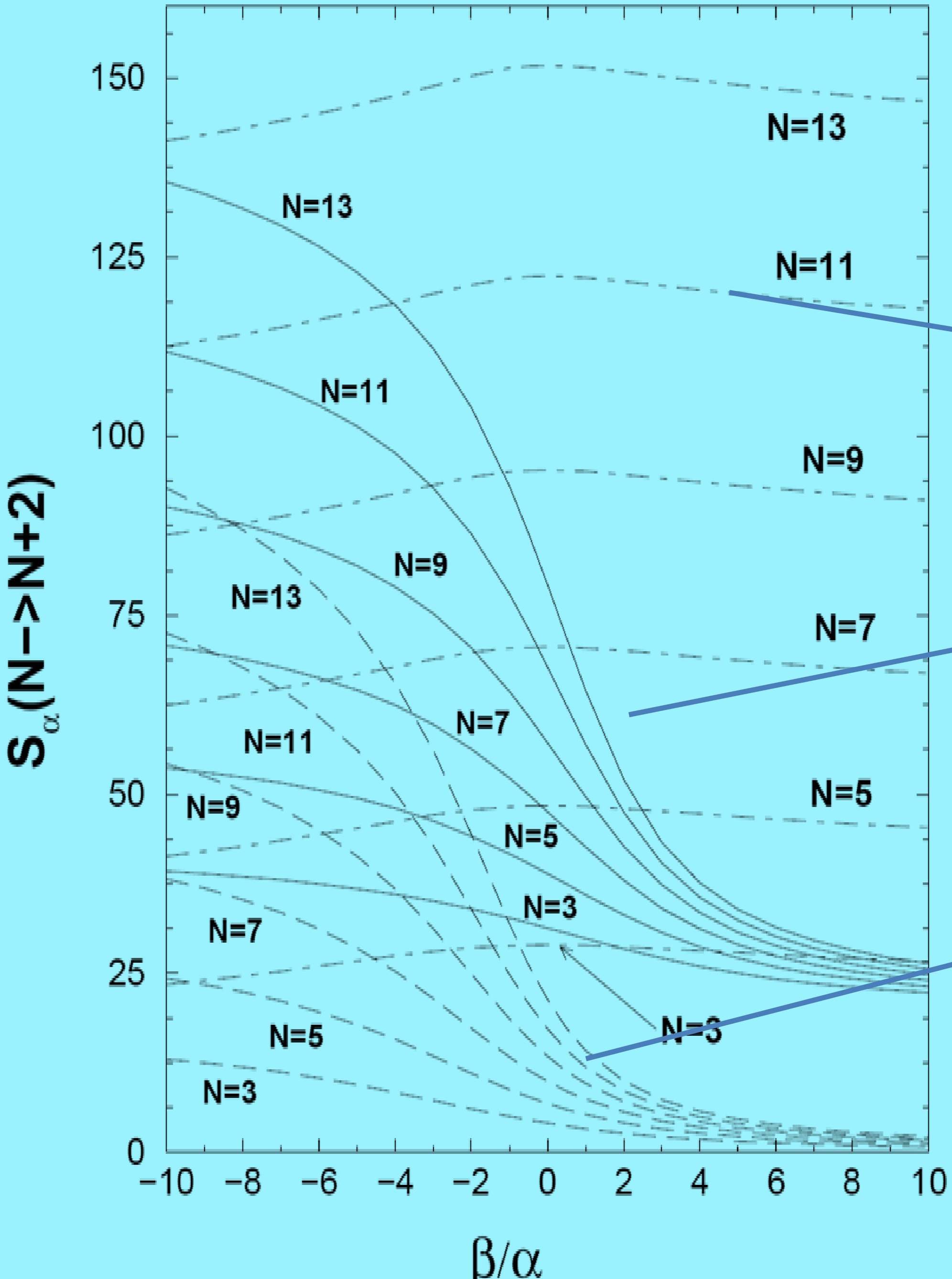
$$\kappa'\sqrt{(2S+1)(2T+1)}$$

$$\times \sum_{n_s(n_d),\omega_s^i,\omega_s^f} \mathbb{C}_{\omega_s^i}^{N,\delta^{gs},(ST)}\ \mathbb{C}_{\omega_s^f}^{N+2,\beta^{gs},(ST)}\ C_{n_s,n_d}^{N,N,\omega_s^i,\omega_d=0}(6,30)$$

$$\times C_{n_s+2,n_d}^{N+2,N+2,\omega_s^f,\omega_d=0}(6,30) \sum_{n_{sS}+n_{sT}=n_s} C_{n_{sS},n_{sT}}^{n_s,\omega_s^i,S,T}(3,3)\ C_{n_{sS}+2,n_{sT}}^{n_s+2,\omega_s^f,S,T}(3,3)$$

$$\times \sqrt{(n_{sS}-S+2)(n_{sS}+S+3)}$$

note that here $\omega_s^i \neq \omega_s^f$ is allowed



for odd-odd N=Z nuclei
N **is odd, gs → gs**

using the operator T_α
(ST)=(01) or (10)

using the operator $T_\alpha^{(d)}$
(ST)=(10)
results shifted by 20

using the operator $T_\alpha^{(d)}$
(ST)=(01)

****general structure for e-e and o-o is same**

5. Conclusions

- **Formulation and first numerical results for GT strengths and α-transfer strengths are presented**

- **These are in addition to deuteron transfer reported before**

- **Employed a simple basis within IBM-4 and a one parameter mixing Hamiltonian**

- **Many results are close to those given by the *SO(8)* model**

- **Experimental tests ??**

- **possible extensions of the formulation [inclusion of $\omega_d \neq 0$ states, $SU(6) \supset SU(3)$ term in *H*, more general transition operators]**