PREDICTIVE POWER OF NUCLEAR MEAN FIELDS FROM TWO-BODY INTERACTIONS

SDANCA-15
8-10 october 2015, Sofia, Bulgaria

Hervé MOLIQUE and Jerzy DUDEK

IPHC/DRS and Université de Strasbourg, France

Introduction

→ Nuclear mean field techniques are common approaches to the nuclear many-body problem.

→ Often used approaches are Nilsson or Woods-Saxon non self-consistent mean fields.

→ Self-consistent techniques are also widely employed: Hartree-Fock with Skyrme interaction etc.

QUESTIONS

- How many terms (central + spin-orbit + tensor + ... + ... ???)
- How many parameters for each term ???
- What about correlations between parameters ??? → control via regularisation methods (Singular Value Decomposition for ex.)
- What is the predictive power of the theory ???

Two-body interactions allowed by symmetry

- A systematic way to construct the two-body interactions allowed by symmetry considerations is provided by the spin-tensor decomposition.

- With the help of the spin operators for two nucleons one constructs the following 6 tensors:

$$S_1^{(0)} = 1, \qquad S_2^{(0)} = [\vec{\sigma}^a \otimes \vec{\sigma}^b]^{(0)}, \qquad S_3^{(1)} = \vec{\sigma}^a + \vec{\sigma}^b$$
$$S_4^{(2)} = [\vec{\sigma}^a \otimes \vec{\sigma}^b]^{(2)}, \qquad S_5^{(1)} = [\vec{\sigma}^a \otimes \vec{\sigma}^b]^{(1)}, \qquad S_6^{(1)} = \vec{\sigma}^a - \vec{\sigma}^b$$

- They are coupled with tensors of the same rank in configuration space to a scalar interaction ($P_{T=0}$ and $P_{T=1}$ are projectors on the states $T = 0$ and $T = 1$):

$$V(a,b) = \sum_{\mu=1}^{6} \left\{ [X_\mu^{(k)} \otimes S_\mu^{(k)}]^{(0)} P_{T=0} + [Y_\mu^{(k)} \otimes S_\mu^{(k)}]^{(0)} P_{T=1} \right\}$$

Tensors in configuration space $X_\mu^{(k)}$

Postulating translational invariance of the NN interaction, Galilean invariance; assuming that $V(1,2)$ should be symmetric under particle exchange, and retaining only hermitian, scalar, time even terms, one obtains the following possibilities:

$$\{\vec{r}\otimes\vec{p}\}^{(0)}+\{\vec{p}\otimes\vec{r}\}^{(0)}=-\tfrac{1}{3}(\vec{r}\cdot\vec{p}+\vec{p}\cdot\vec{r}),\ \{\vec{r}\otimes\vec{p}\}^{(1)}=\tfrac{i}{\sqrt{2}}\vec{L},\ \{\vec{r}\otimes\vec{p}\}^{(2)},$$
$$\{\vec{r}\otimes\vec{r}\}^{(0)}=-\tfrac{1}{3}r^2,\ \{\vec{r}\otimes\vec{r}\}^{(2)},\ \{\vec{p}\otimes\vec{p}\}^{(0)}=-\tfrac{1}{3}p^2\{\vec{p}\otimes\vec{p}\}^{(2)}.$$

Alternative combinations are:

- The scalars L^2, r^2 and p^2.
- The vector $\vec{L}$.
- The second rank tensors $\{\vec{r}\otimes\vec{r}\}^{(2)}$, $\{\vec{p}\otimes\vec{p}\}^{(2)}$ and $\{\vec{L}\otimes\vec{L}\}^{(2)}$.

Examples of realisations

Combining different terms of the Spin-Tensor Decomposition using projection operators Π onto the singlet (s), triplet (t), even (e), odd (o) states leads to central forces*:

$$V(\mathbf{1},\mathbf{2}) = V_{\mathbf{0}} f(r/r_{\mathbf{0}}) \left(a_{et} \Pi_e^r \Pi_t^\sigma + a_{es} \Pi_e^r \Pi_s^\sigma + a_{ot} \Pi_o^r \Pi_t^\sigma + a_{os} \Pi_o^r \Pi_s^\sigma \right)$$

Different combinations of coefficients (a_{et}, a_{es}, a_{ot}, a_{os}) are encountered in the literature:

Wigner (1, 1, 1, 1), Kurath (1, 0.6, -0.6, -1), Serber (1, 1, 0 0), Rosenfeld (1, 0.6, -0.34, -1.78) . . .

*P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, Springer, New York (1980)

Shell-Model framework

In the Shell-Model, Spin Tensor Decomposition has been used to extract scalar (central) V_0, vector (LS and ALS) V_1, and tensor V_2 contributions out of the total two-body matrix elements V.

LS matrix elements of V_k can be given in function of the total LS matrix elements:

$$\langle AB; LSJT|V_k|CD; L'S'JT\rangle = (-)^J(2k+1)\left\{\begin{array}{ccc} L & S & J \\ S' & L' & k \end{array}\right\} \times \sum_{J'}(-)^{J'}(2J'+1)\left\{\begin{array}{ccc} L & S & J' \\ S' & L' & k \end{array}\right\}\langle AB; LSJ'T|V|CD; L'S'J'T\rangle \,,$$

where $A \equiv (n_a, l_a)$.

B.A. Brown, W.A. Richter and B.H. Wildenthal, J. Phys. G: Nucl. Phys. **11** (1985) 1191

B.A. Brown, W.A. Richter, R.E. Julies and B.H. Wildenthal, Ann. Phys. **182** (1988) 191

N.A. Smirnova, B. Bally, K. Heyde, F. Nowacki and K. Sieja, Phys. Lett. **B686** (2010) 109

Applications to the Shell-Model

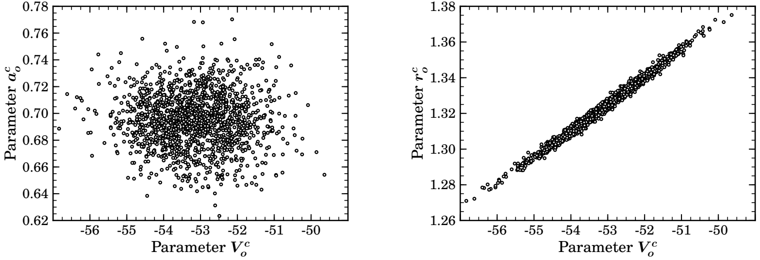
In turn, the total LS matrix elements can be expressed in terms of the jj matrix elements as

$$\begin{aligned}\langle AB; LSJ'T|V|CD; L'S'J'T\rangle &= [(1+\delta_{AB})(1+\delta_{CD})]^{-1/2} \\ &\times \sum_{j_a,\, j_b,\, j_c,\, j_c} \left\{ \begin{array}{ccc} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L & S & J' \end{array} \right\} \left\{ \begin{array}{ccc} l_c & 1/2 & j_c \\ l_d & 1/2 & j_d \\ L' & S' & J' \end{array} \right\} \\ &\times [(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)]^{1/2} \\ &\times [(1+\delta_{ab})(1+\delta_{cd})]^{1/2} \langle ab; J'T|V|cd; J'T\rangle \ ,\end{aligned}$$

where $a \equiv (n_a, l_a, j_a)$.

LEFT: no correlation between the central potential-depth V_0^c and diffusivity parameter a_0^c.

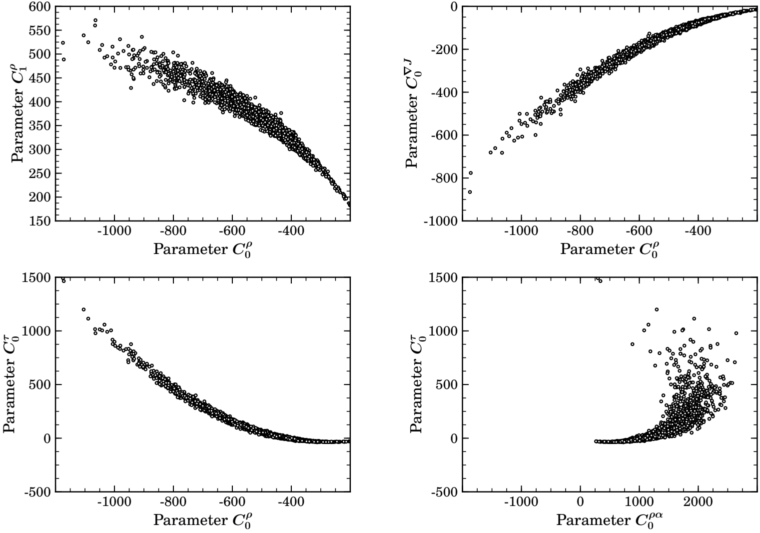
RIGHT: strong correlation between the central potential-depth V_0^c and radius parameter r_0^c.



(Monte-Carlo analysis for ^{208}Pb).

From J. Dudek, B. Szpak, A. Dromard, M.G. Porquet, B. Fornal and A. Gozdz, Int. J. Mod. Phys. **E21** (2012) 1250053

Parametric correlations: Skyrme-Hartree-Fock



From J. Dudek, B. Szpak, A. Dromard, M.G. Porquet, B. Fornal and A. Gozdz, Int. J. Mod. Phys. **E21** (2012) 1250053

Singular Value Decomposition (SVD)

- One sometimes deals with more parameters (e.g. single-particle energies and two-body matrix elements) than data.
- Parameters may not be well determined by data, or may show correlations.
- Way out: in the least square fit, one restricts the number of relevant parameters to the "orthogonal parameters". The parameters are ordered according to they accuracy and not well defined parameters can be selected out.
- Method known as the Linear Combination† or Diagonal Correlation Matrix* Method.
- In this same spirit, we make use, in the Mean-Field approach, of the Singular Value Decomposition method.

† W. Chung, Ph. D Thesis, Michigan State University (1976), B.A. Brown, W.A. Richter, R.E. Julies and B.H. Wildenthal, Ann. Phys. **182** (1988) 191

* P.J. Brussaard and P.W.M. Glaudemans, *Shell-Model Applications in Nuclear Spectroscopy,* North-Holland, Amsterdam (1977)

Least Square Problem and SVD

$\rightarrow$ Consider the least-square problem (W_j are some weights factors):

$$\chi^2(\vec{p}) = \frac{1}{m-n} \sum_{j=1}^{m} W_j \left[e_j^{\text{Theor.}}(\vec{p}) - e_j^{\text{Exp.}}\right]^2$$

where there are m data points and n parameters.

$\rightarrow$ We consider the problem referred to as overdetermined: $m > n$.

$\rightarrow$ Linearization of the problem via Taylor expansion:

$$e_j^{\text{Theor.}}(\vec{p}) \simeq e_j^{\text{Theor.}}(\vec{p}_0) + \sum_{i=1}^{n} \left(\frac{\partial e_j^{\text{Theor.}}}{\partial p_i}\right)_{\vec{p}=\vec{p}_0} (p_i - p_{0,i})$$

$\rightarrow$ Minimising $\chi^2(\vec{p})$ with respect to the parameters p_i leads to the set of so-called normal equations:

$$(J^T J)\,(\vec{p} - \vec{p}_0) = J^T \vec{y} \quad \textbf{or} \quad (J^T J)\,\vec{p} = J^T \vec{b}$$

with $y_j \equiv \sqrt{W_j}[e_j^{\text{Exp.}} - e_j^{\text{Theor.}}(p_0)]$ and $\vec{b} \equiv J\vec{p}_0 + \vec{y}$.

Least Square Problem and SVD

The design matrix or generalised Jacobian matrix $J_{m\times n}$ reads

$$J_{ji} \equiv \sqrt{W_j}\; I_{ji}, \quad I_{ji} \equiv \left(\frac{\partial e_j^{\text{Theor.}}}{\partial p_i}\right)_{\vec{p}=\vec{p}_0}.$$

Defining the residual vector $\vec{r} \equiv J\vec{p} - \vec{b}$, the merit function can be expressed as

$$\chi^2(\vec{p}) = \frac{1}{m-n}\sum_{j=1}^{m} r_i^2 = \frac{1}{m-n}\|J\vec{p} - \vec{b}\|^2$$

$\rightarrow$ Minimising chi-square means solving the normal equations, or finding p that minimises the norm of the residual $\|J\vec{p} - \vec{b}\|$.

J. Toivanen, J. Dobaczewski, M. Kortelainen and K. Mizuyama, Phys. Rev. **C78**, 034306 (2008)

Least Square Problem and SVD

$\rightarrow$ The difficulty lies in the (im)possibility of finding $(J^T J)^{-1}$.

$\rightarrow$ Singular Value Decomposition helps !

$\rightarrow$ For a (real) matrix $J_{m\times n}$ of rank r, there exist two orthogonal matrices $U_{m\times m}$, $V_{n\times n}$, and $S_{m\times n} = \mathrm{diag}(w_1, w_2, \ldots, w_r, 0\ldots 0)$ (w_i singular values $w_1 \geq w_2 \geq \ldots \geq w_r > w_{r+1} = \ldots = w_n = 0$):

$$\boxed{J = USV^T}$$

$\rightarrow$ Formal least-square solution of minimal norm is given by:

$$\boxed{\vec{p} = J^\dagger\ \vec{b}}$$

where $J^\dagger \equiv V\ S^\dagger\ U^T$ denotes the Moore-Penrose pseudo-inverse of J, with $S^\dagger_{n\times m} = \mathrm{diag}(1/w_1, 1/w_2, \ldots, 1/w_r, 0\ldots 0)$.

Least Square Problem and SVD

- The Moore-Penrose pseudo-inverse verifies the four conditions:

$$\begin{aligned} J\, J^{\dagger}\, J &= J \\ J^{\dagger}\, J\, J^{\dagger} &= J^{\dagger} \\ (J\, J^{\dagger})^T &= J\, J^{\dagger} \\ (J^{\dagger}\, J)^T &= J^{\dagger}\, J \end{aligned}$$

- If J is of full rank (i.e. $r = n < m$), then $(J^T J)$ can be inverted, and $J^{\dagger} = (J^T J)^{-1} J^T$.

- Singular values = square-roots of the eigenvalues of $(J^T J)$.

- The rank of the matrix J is equal to the number of non-zero singular values.

- Columns of U are orthonormal eigenvectors of (JJ^T).
- Columns of V are orthonormal eigenvectors of $(J^T J)$.

Regularization Procedure: Truncated SVD

In practice:

$\rightarrow$ There might be some poorly determined parameters or correlated parameters, leading to small singular values, and thus the inverse problem gets ill-posed !

$\rightarrow$ The remedy proposed in the SVD procedure is to replace the corresponding large inverse in $S^\dagger$ by zero, forgetting about the associated parameter !

$\rightarrow$ The least-square minim. of $\|J\vec{p} - \vec{b}\|^2$ reads $\|USV^T\vec{p} - \vec{b}\|^2$.

Introducing new independent parameters $\vec{x} \equiv V^T\vec{p}$ and data $\vec{z} \equiv U^T\vec{b}$, one minimises

$$\|S\vec{x} - \vec{z}\|^2 = \sum_{i=1}^{r} |w_i x_i - z_i|^2$$

which is solved with $x_i = z_i/w_i$ for $i = 1, \ldots, r$ and arbitrary for $i = r+1, \ldots, r$.

Regularisation Procedure: Truncated SVD

$\rightarrow$ Finally, one can construct the solution having a minimal norm, which, after ignoring contributions from small singular values:

$$\vec{p} = \sum_{i=1}^{\eta} \frac{\vec{u}_i \cdot \vec{b}}{w_i} \, \vec{v}_i$$

where $\vec{u}_i$ and $\vec{v}_i$ denote columns vectors of U and V, and η represents the regularisation parameter.

This Truncated Singular Value Decomposition (TSVD) constitutes one possible regularisation procedure of the ill-posed inverse problem.

M. Kern, *Problèmes Inverses*, Ecole Supérieure d'Ingénieurs Léonard de Vinci, 2002-2003

Mean-Field from effective phenomenological NN interactions

Within the Skyrme-Hartree-Fock formalism, the spin-orbit, the central and the tensor parts of the interaction contribute to the spin-orbit potential:

$$\hat{V}_{SO} \sim \frac{1}{r} W_q(r)\,(\vec{l} \cdot \vec{\sigma})$$

($\rho =$ particle density, $J =$ vector spin-orbit density).

$$W_q = \underbrace{\frac{W_0}{2}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right)}_{\text{from NN spin-orbit interaction}} \quad + \quad \underbrace{\left(\alpha J_q + \beta J_{q'}\right)}_{\text{from NN central \& NN tensor interaction}}$$

D. Vautherin and D.M. Brink, Phys. Rev. **C5** (1972) 626

F. Stancu, D.M. Brink and H. Flocard, Phys. Lett. **B68** (1977) 108

Effective self-consistent spin-orbit potential

Effective self-consistent spin-orbit potentials can be used in spherical Woods-Saxon mean field calculations* (q stand for proton and neutron form factors):

$$W_q = \left(\lambda^{qq}\frac{d\rho_q}{dr} + \lambda^{qq'}\frac{d\rho_{q'}}{dr}\right) + \left(\alpha J_q + \beta J_{q'}\right)$$

with

$$\lambda^{qq} = \lambda^{qq'} = \lambda > 0$$

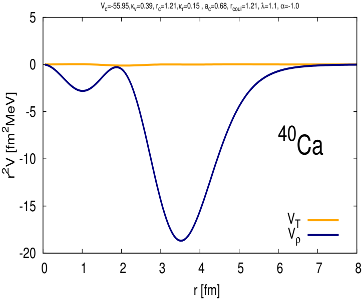
and

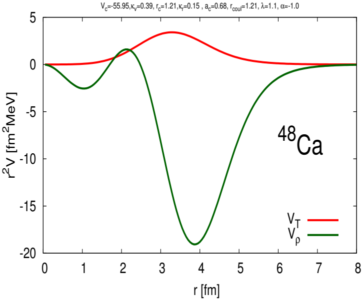
$$\alpha + \beta \simeq 0; \qquad \alpha < 0; \quad \beta > 0$$

*H.M., J. Dudek, K. Rybak and M.G. Porquet, Acta Phys. Pol. **B40** (2009) 597

Effective self-consistent spin-orbit potential

Total “ordinary” and “tensor” spin-orbit potentials for protons in spin-saturated ^{40}Ca (left) and spin-unsaturated ^{48}Ca (right) nuclei:





- → $V^p_\rho = r^2(1/r)\lambda[d\rho_p/dr + d\rho_n/dr]$; $V^p_T = r^2(1/r)[\alpha J_p + \beta J_n] = r^2(1/r)\alpha[J_p - J_n]$
- → For neutrons we would have $V^n_\rho = V^p_\rho$ and $V^n_T = -V^p_T$
- → The proton “tensor” potential is almost zero in ^{40}Ca nucleus, but significant and positive in ^{48}Ca. This leads to an “abnormal” proton spin-orbit splitting†.

†Wong, Nucl. Phys. **A108** (1968) 481

Two-body NN spin-orbit interaction

Consider a two-body nucleon-nucleon spin-orbit interaction:

$$\hat{V}^{SO} = V(\|\vec{r} - \vec{r}\,'\|)\,(\vec{r} - \vec{r}\,') \wedge (\vec{p} - \vec{p}\,') \cdot (\vec{\sigma} + \vec{\sigma}\,')$$

Expanding, we obtain the sum of 8 terms:

$$\begin{aligned}
\hat{V}^{SO} \;=\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r} \wedge \vec{p}\,) \cdot \vec{\sigma} && \rightarrow T_1 \\
-\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r}' \wedge \vec{p}\,) \cdot \vec{\sigma} && \rightarrow T_2 \\
-\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r} \wedge \vec{p}') \cdot \vec{\sigma} && \rightarrow T_3 \\
+\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r}' \wedge \vec{p}') \cdot \vec{\sigma} && \rightarrow T_4 \\
+\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r} \wedge \vec{p}\,) \cdot \vec{\sigma}\,' && \rightarrow T_5 \\
-\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r}' \wedge \vec{p}\,) \cdot \vec{\sigma}\,' && \rightarrow T_6 \\
-\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r} \wedge \vec{p}') \cdot \vec{\sigma}\,' && \rightarrow T_7 \\
+\;& V(\|\vec{r} - \vec{r}'\|)\,(\vec{r}' \wedge \vec{p}') \cdot \vec{\sigma}\,'. && \rightarrow T_8
\end{aligned}$$

Hartree potentials for the spin-orbit interaction

Formulation of the Hartree-Fock equations in coordinate space:

$$-\frac{\hbar^2}{2m}\Delta\phi_i(q) + U_H\ \phi_i(q) - \int U_F\ \phi_i(q')\ dq' = \varepsilon_i\phi_i(q).$$

Contribution of the 8 terms in function of the particle density by $\rho(\vec{r})$, the current density by $\vec{\jmath}(\vec{r})$, the spin density by $\vec{s}(\vec{r})$ and the vector spin current density by $\vec{J}_q(\vec{r})$:

$$\hat{U}^{SO}_{H,T_1} = \left[\int d^3\vec{r}'\, V(\|\vec{r}-\vec{r}'\|)\rho(\vec{r}')\right]\vec{\ell}\cdot\vec{\sigma}$$

$$U^{SO}_{H,T_2} = \left[-\int d^3\vec{r}'\, V(\|\vec{r}-\vec{r}'\|)\,\rho(\vec{r}')\,\vec{r}'\right]\wedge\vec{p}\cdot\vec{\sigma}$$

$$\hat{U}^{SO}_{H,T_3} = \left\{\frac{\hbar}{i}\int d^3\vec{r}'\, V(\|\vec{r}-\vec{r}'\|)\left[\tfrac{1}{2}\vec{\nabla}'\rho(\vec{r}') + i\vec{\jmath}(\vec{r}')\right]\right\}\cdot(\vec{r}\wedge\vec{\sigma})$$

$$U^{SO}_{H,T_4} = \frac{\hbar}{i}\left\{\int d^3\vec{r}'\, V(\|\vec{r}-\vec{r}'\|)\,\vec{r}'\wedge\left[\tfrac{1}{2}\vec{\nabla}'\rho(\vec{r}') + i\vec{\jmath}(\vec{r}')\right]\right\}\cdot\vec{\sigma}$$

Hartree potentials for the spin-orbit interaction

$$\hat{U}^{SO}_{H,T_5} = \left[\int d^3\vec{r}\,' V(\|\vec{r} - \vec{r}\,'\|)\; \vec{s}(\vec{r}\,') \right] \cdot \vec{\ell}$$

$$\hat{U}^{SO}_{H,T_6} = \left[\int d^3\vec{r}\,' V(\|\vec{r} - \vec{r}\,'\|)\; \vec{r}\,' \wedge \vec{s}(\vec{r}\,') \right] \cdot \vec{p}$$

$$\hat{U}^{SO}_{H,T7} = -\left\{ \int d^3\vec{r}\,' V(\|\vec{r} - \vec{r}\,'\|) \left[\hbar \vec{J}(\vec{r}\,') + \tfrac{1}{2}\vec{p}\,' \wedge \vec{s}(\vec{r}\,') \right] \right\} \cdot \vec{r}$$

$$\hat{U}^{SO}_{H,T_8} = \int d^3\vec{r}\,' V(\|\vec{r} - \vec{r}\,'\|)\, \vec{r}\,' \cdot \left\{ \hbar\, \vec{J}(\vec{r}\,') + \tfrac{1}{2}\left[\vec{p}\,' \wedge \vec{s}(\vec{r}\,') \right] \right\}$$

Systems preserving spherical symmetry and time-reversal

For systems preserving spherical symmetry and time-reversal, only terms T_1 and T_2 survive, as well as T_7 and T_8:

$$\hat{U}^{SO}_{H,T_1} + \hat{U}^{SO}_{H,T_2} = \underbrace{\left[\int d^3\vec{r}'\, V(\|\vec{r} - \vec{r}'\|)\rho(r')\left(1 - \frac{\vec{r}' \cdot \vec{r}}{r^2}\right)\right]}_{\text{"full" form factor } F(r)} \vec{\ell} \cdot \vec{\sigma}$$

$$\hat{U}^{SO}_{H,T7} + \hat{U}^{SO}_{H,T8} = \hbar \int d^3\vec{r}\,'\, V(\|\vec{r} - \vec{r}\,'\|)\vec{J}(\vec{r}') \cdot (\vec{r}' - \vec{r})$$

NB: The latter expression contributes to a different Hartree mean-field than the spin-orbit, and is not discussed here.

Blin-Stoyle* approximation

The form factor $F(r)$ can be expressed as

$$F(r) = -\int d^3\vec{r}\,' V(\|\vec{r} - \vec{r}\,'\|)\rho(\vec{r}, \vec{r}\,').$$

Writing $\rho(\vec{r}, \vec{r}\,') = \rho'(\vec{s}, \vec{r})$ with $\vec{s} = \vec{r}\,' - \vec{r}$, and performing a Taylor expansion of $\rho'(\vec{s}, \vec{r})$ about $\vec{s} = 0$ one obtains, with the help of the mean particle density in closed shell orbitals $\rho_{nl}(r)$:

$$F_{\mathrm{BS}}(r) = K \frac{1}{r} \frac{d\rho(r)}{dr},$$

where

$$K = -\frac{4\pi}{3} \int_0^{+\infty} V(s) s^4 ds.$$

*R.J. Blin-Stoyle (1955) *Phil. Mag.* **46** 973.

Fermi shape density distribution

⋆ We consider a Fermi shape density distribution:

$$\rho(r) = \rho_\circ \frac{1}{1 + \exp[(r - R_\circ)/a]}.$$

where $R_\circ = r_\circ A^{1/3}$ with $r_\circ = 1.3$ fm, $\rho_\circ = A/[\frac{4}{3}\pi R_\circ^3]$ ($A =$ mass number) and $a = 0.7$ fm.

⋆ The spin-orbit form factor corresponding to a two pion exchange is expressed as:

$$V(s) = C_{SO}\left[\frac{e^{-\mu s}}{\mu s}\right]^2$$

⋆ In this case K can be given in analytical form:

$$K = -\frac{\pi}{3}\frac{C_{SO}}{\mu^5}$$

and

$$F_{\rm BS}(r) = -\frac{\pi}{3}\frac{C_{SO}}{\mu^5}\left[-\frac{1}{a\rho_\circ}\rho^2(r)\exp[(r - R_\circ)/a]\right].$$

Parameter determination

⋆ We take $\mu = (m_\pi c^2)/(\hbar c)$, the inverse of the Compton wavelength of the pion: $\mu = 1/1.4\ \mathrm{fm}^{-1}$.

⋆ Therefore, the only free parameter is C_{SO}.

⋆ A rough estimate can be obtained with the help of the Woods-Saxon spin-orbit form factor

$$F_{\mathrm{WS}}(r) = \lambda_{SO}\left[\frac{\hbar}{2mc}\right]^2 \frac{1}{r}\frac{d}{dr}\left[\frac{-V_\circ}{1+\exp[(r-R_{\mathrm{SO}})/a_{\mathrm{SO}}]}\right],$$

with

$$V_\circ = V\left[1 \pm \kappa\left(\frac{N-Z}{N+Z}\right)\right]$$

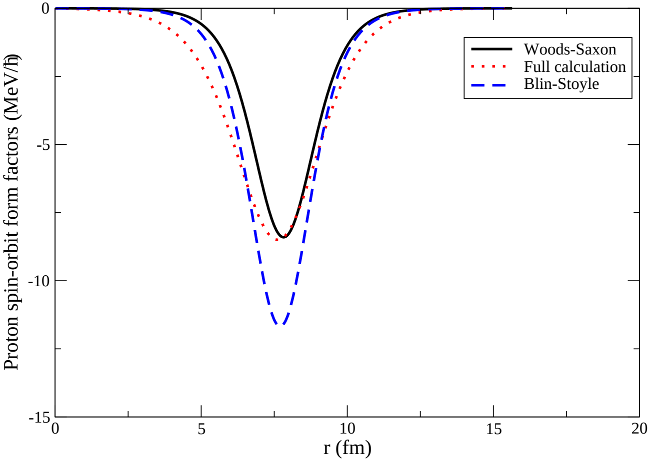
("+" for protons, and "−" for neutrons).

→ "Full" form factor $F(r)$ is calculated such as to coincide with the value of $F_{\mathrm{WS}}(r = R_{\mathrm{SO}})$. For given $\mu = 1/1.4\ \mathrm{fm}^{-1}$ one gets $C_{SO} = -53.4$ MeV.

Proton spin-orbit form factors

COMPARISON OF WOODS-SAXON, $F(r)$ AND BLIN-STOYLE FORM FACTORS

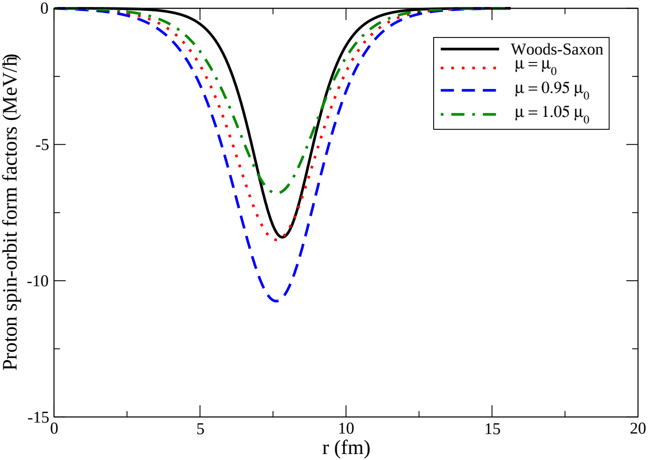
$\mu = 1/1.4\ \text{fm}^{-1}$ FIXED; obtained value $C_{SO} = -53.4$ MeV



Proton $F(r)$ spin-orbit form factor

SENSITIVITY OF $F(r)$ WITH RESPECT TO μ

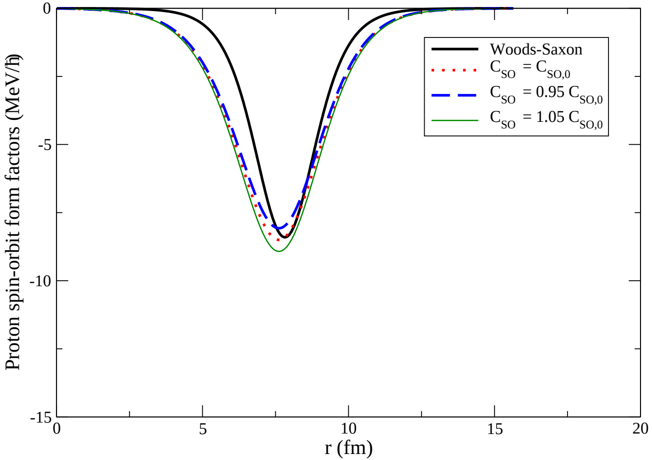
$C_{SO} = -53.4$ MeV FIXED; $\mu_\circ = 1/1.4$ fm^{-1}



Proton $F(r)$ spin-orbit form factor

SENSITIVITY OF $F(r)$ WITH RESPECT TO C_{SO}

$\mu = 1/1.4$ fm^{-1} FIXED; $C_{SO,0} = -53.4$ MeV



CONCLUSIONS AND PERSPECTIVES

- The issue of reducing the number of parameters in the mean-field parametrizations has been addressed.
- Construction of a mean-field spin-orbit potential from the nucleon-nucleon two-body spin-orbit interaction is discussed.
- Complete form factor calculations have been compared to approximate Blin-Stoyle and phenomenological Woods-Saxon potentials, and coupling constant C_{SO} has been determined.
- Exchange (Fock) terms have to be considered.
- Isospin dependent spin-orbit mean-field interaction must be added.
- Investigations pushed further to include tensor interaction.
- Extension to deformed systems.

THANK YOU FOR YOUR ATTENTION !