Various aspects of the Deformation Dependent Mass Model Of Nuclear Structure

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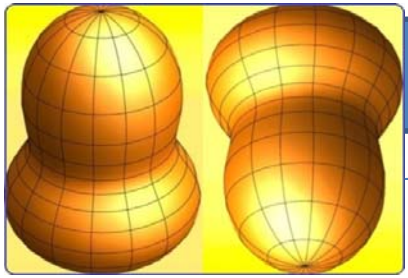


SDANCA, 8-10 October 2015, Sofia, Bulgaria

Bohr-Motelson geometric collective model

$$R(\theta,\varphi) = R_0\left[1+\sum_{\mu=-2}^{2}\alpha_\mu Y^*_{2,\mu}(\theta,\varphi)\right]$$

α_μ → Shape coordinates

Transformation to principal axes

$$a_\nu = \sum_{\mu=-2}^{2}\alpha_\mu D^*_{\mu,\nu}(\theta_i) \Rightarrow \begin{cases} a_2 = a_{-2} \\ a_1 = a_{-1} = 0 \end{cases}$$

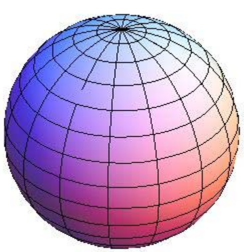
$$a_0 = \beta\cos\gamma$$

$$a_2 = \beta\sin\gamma\big/\sqrt{2}$$

β,γ Intrinsic (shape) $\beta \geq 0,\ 60^\circ \geq \gamma \geq 0^\circ$

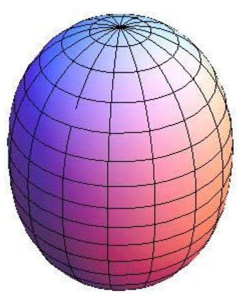
$\theta_i, (i=1,2,3)$ Collective (orientation)

Spherical



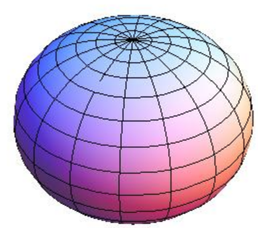
$\beta = 0$

Prolate



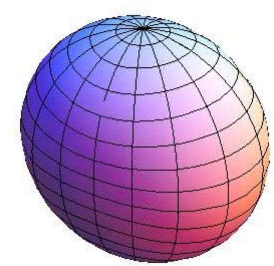
$\beta \neq 0, \gamma = 0^\circ$

Oblate



$\beta \neq 0, \gamma = 60^\circ$

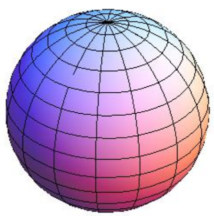
Triaxial



$\beta \neq 0, \gamma \neq 0^\circ, 60^\circ$

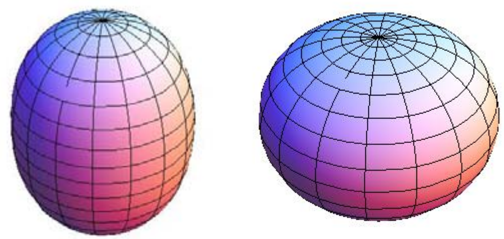
“exact” limits ←→ equilibrium shapes

Spherical



$\beta_0 = 0$

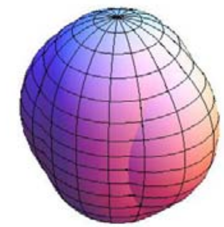
Axially deformed



$\beta_0 \neq 0$

$\gamma_0 = 0^\circ$ $\gamma_0 = 60^\circ$

γ- unstable



$\beta_0 \neq 0$

$V(\beta)$

Energy minima at

A new version of the Bohr Hamiltonian

Dennis Bonatsos, P. E. Georgoudis, D. Lenis, N. Minkov, and C. Quesne, Phys. Rev. C 83, 044321 (2011).

- Mass depending on deformation: $B_0 \to B(\beta) = \frac{B_0}{(f(\beta))^2}$

  Position dependent effective mass: O. von Roos, Phys. Rev. B 27, 7547 (1983).

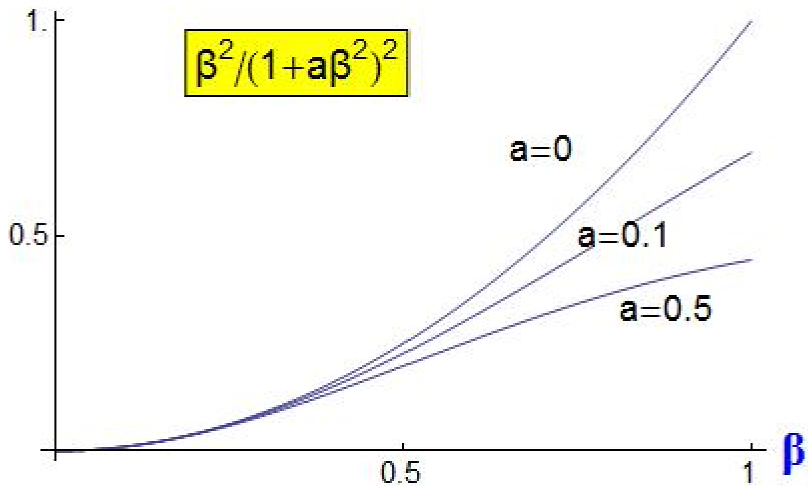
- Solved using SUSYQM

  F. Cooper, A. Khare, and U. Sukhatme, *Supersymmetry in Quantum Mechanics (World Scientific, Singapore, 2001).*

- Moderates increase rate of MOI with $\boldsymbol{\beta}$

$$\beta^2 \to \frac{\beta^2}{(f(\beta))^2}$$

new parameter **a**

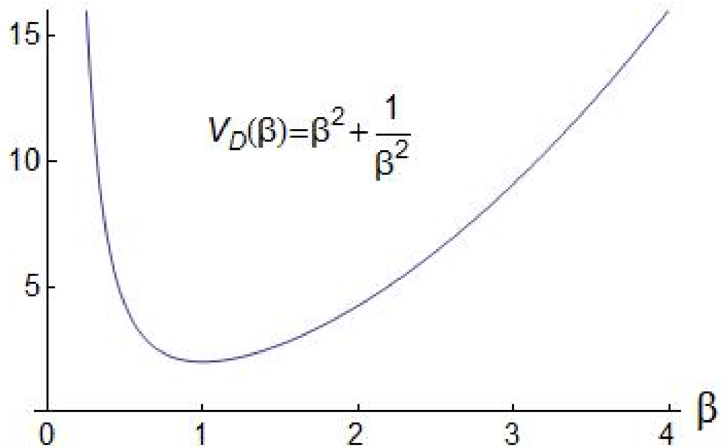


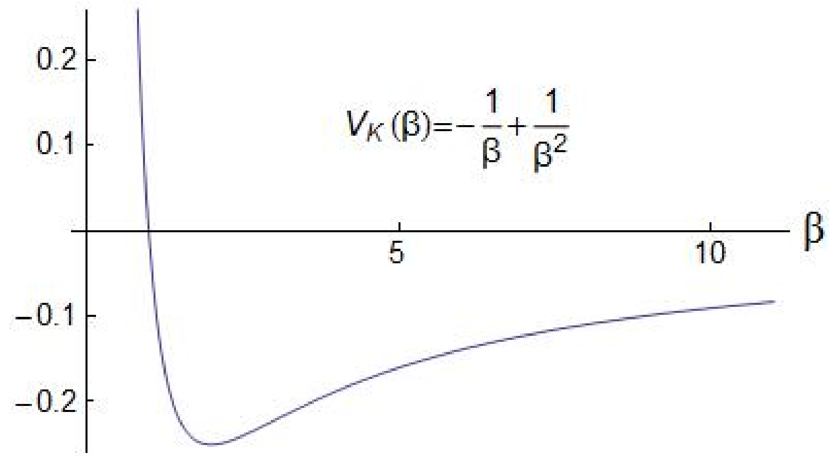
Another approach to the same problem was taken by Diamond and Stephens, who had first fitted such bands with the triaxial model plus beta vibrations, and then discovered, in collaboration with Swiatecki, that an equivalent interpretation of the spectrum is obtained by means of a "beta-stretching model," in which the deformation, just as the interatomic distance in diatomic molecules, is taken to increase with angular momentum. This latter approach, just as Harris', leads to a two parameter expression for the energy levels, but it requires in addition an assumption for the dependence of $\mathcal{J}$ on the deformation β, i.e. a "hidden" third parameter.

theorist, Brian Buck[7]). We started out with the beta stretching model, but found that it was preferable to eliminate the deformation parameter β and to determine $\mathcal{J}$ by minimizing E with respect to $\mathcal{J}$, rather than to β. Hence we named the new model "Variable Moment of Inertia (VMI)" model. Subsequent comparison with Harris' model

from
G. S. Goldhaber in
A. L. Goodman, G. S. Goldhaber, A. Klein and R. A. Sorensen (eds.),
Band Structure and Nulear Dynamics, Tulane, 1980

potential ←→mass dependence on deformation





$f_D(\beta)=1+a\beta^2$

$f_K(\beta)=1+a\beta$

Davidson:

Dennis Bonatsos, P. E. Georgoudis, D. Lenis, N. Minkov, and C. Quesne, Phys. Rev. C 83, 044321 (2011).

Kratzer:

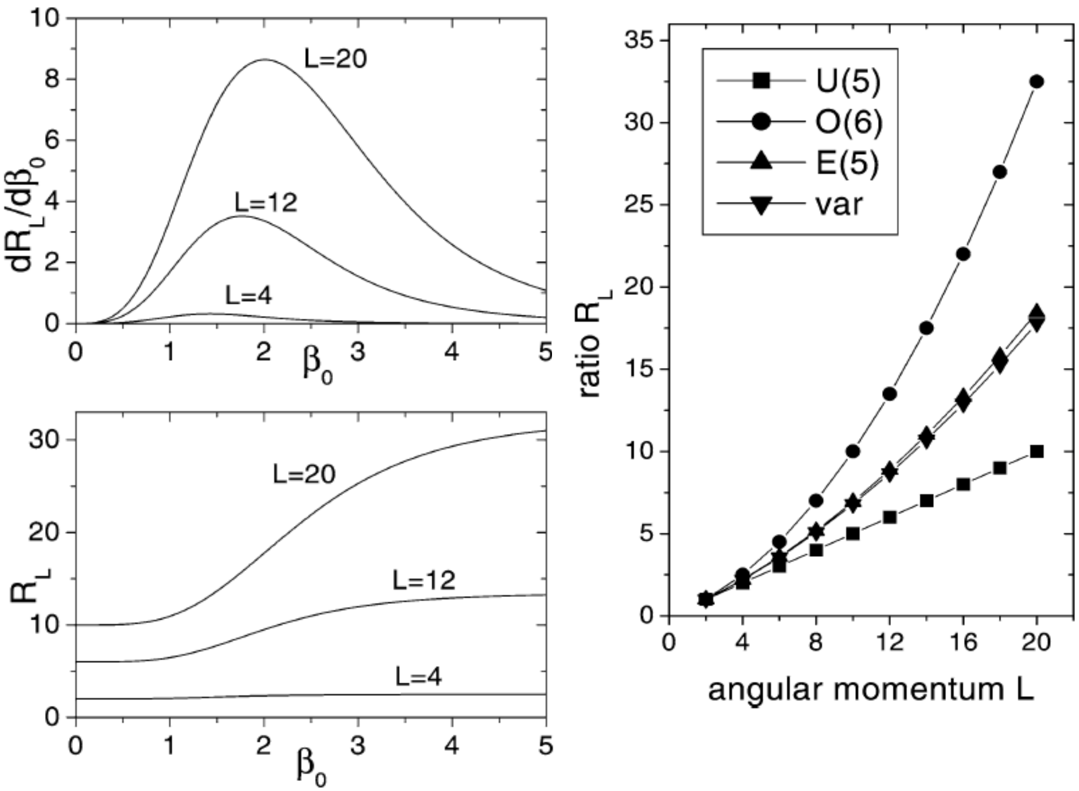
Dennis Bonatsos, P. E. Georgoudis, N. Minkov, D. Petrellis and C. Quesne, Phys. Rev. C 88, 034316 (2013)

Physical meaning of the "a" parameter

- comparison with the classical limit of the IBM
  - U(5): a→d-boson pairing
  - SU(3): a→ quadrupole – quadrupole
  - O(6): a→ (pairing among s & d-bosons) – (d-boson pairing)
- geometric interpretation
  - curvature of 5D Bohr space → classical limit of IBM

Dennis Bonatsos, N. Minkov and D. Petrellis, J. Phys. G: Nucl. Part. Phys. 42, 095104 (2015)
P.E. Georgoudis, Phys. Lett. B 731, 122 (2014)

Bohr with Davidson + "variational" (extremum) method



$$R_L = R_L(\beta_0)$$

Seek β_0 that maximize $\dfrac{dR_L}{d\beta_0}$

$$R_L = \frac{E_L - E_0}{E_2 - E_0}$$

for g.s. band

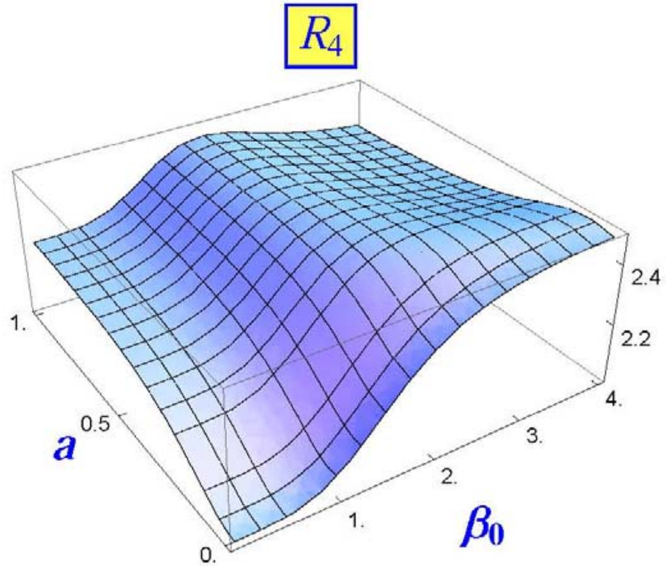
Dennis Bonatsos, D. Lenis, N. Minkov, D. Petrellis, P.P. Raychev, P.A. Terziev, Phys. Lett. B 584, 40 (2004)

V. Werner, P. von Brentano, R.F. Casten, J. Jolie, Phys. Lett. B 527 (2002) 55.

R_L ratios for g.s. band
from DDM-var compared with E(5) and X(5)
for fixed values of "a"

L	a=0.011	E(5)	a=0.0035	X(5)
4	2.19663	2.199	2.90566	2.904
6	3.58597	3.59	5.43696	5.43
8	5.16385	5.169	8.49561	8.483
10	6.92715	6.934	12.0404	12.027
12	8.87378	8.881	16.051	16.041
14	11.0025	11.009	20.5167	20.514
16	13.3126	13.316	25.4323	25.437
18	15.8041	15.799	30.7955	30.804
20	18.477	18.459	36.6061	36.611

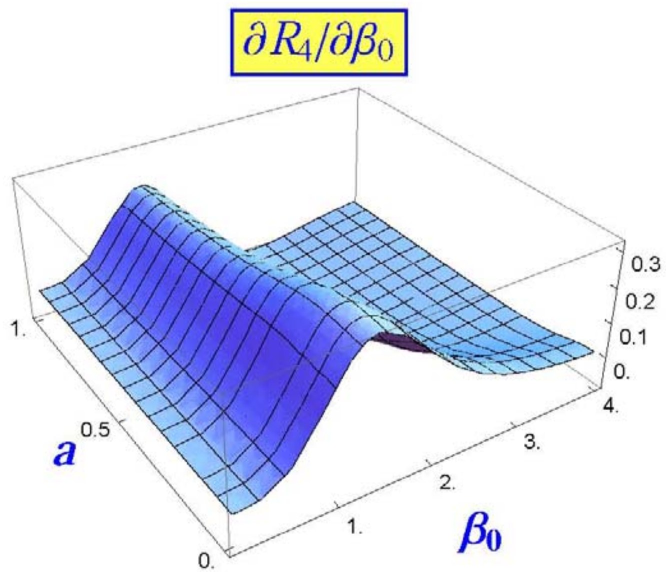
Extension of the "variational" method to the DDM +Davidson



for <u>varying</u> "a"

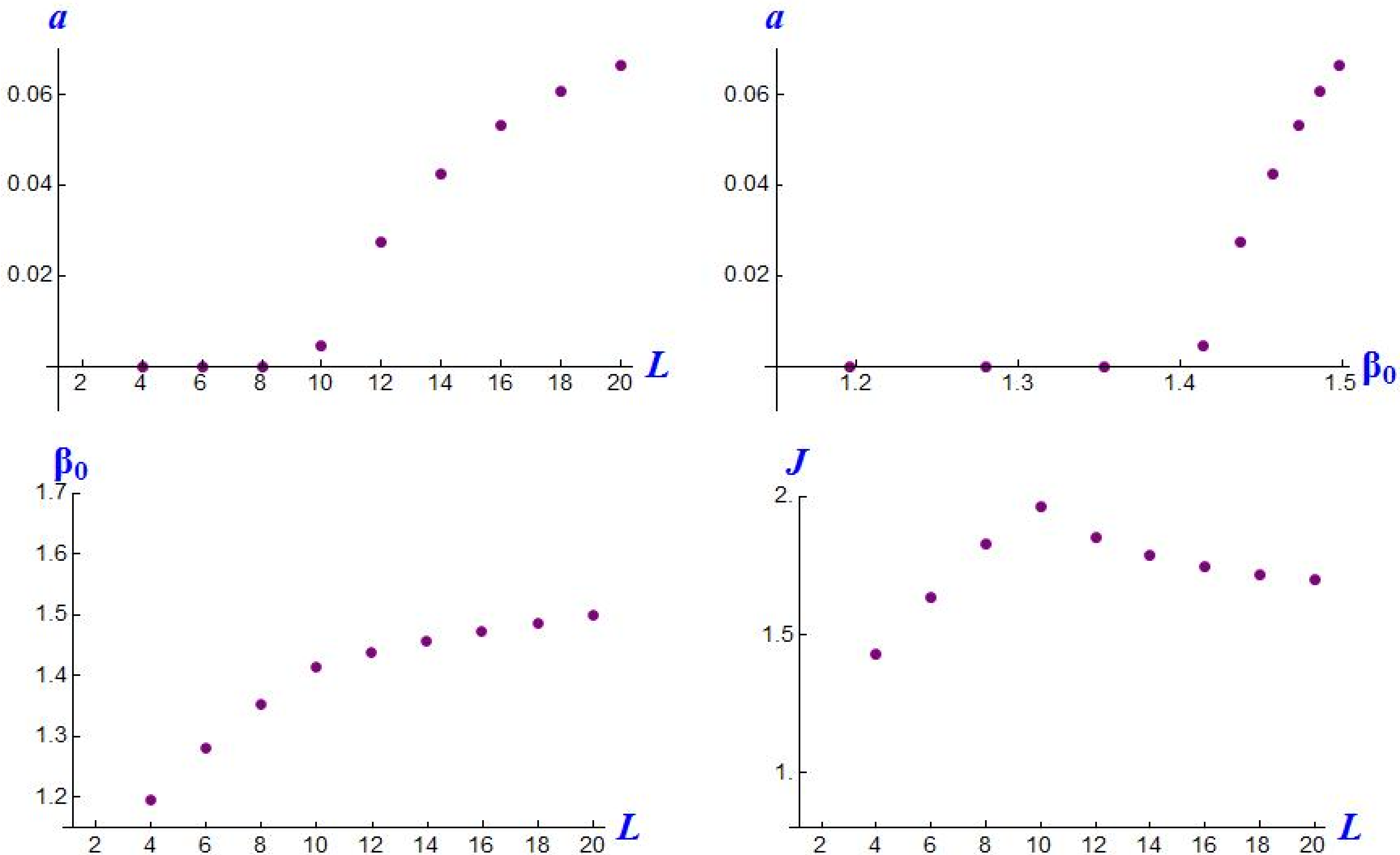
$$R_L(\beta_0) \to R_L(a, \beta_0)$$

line → surface

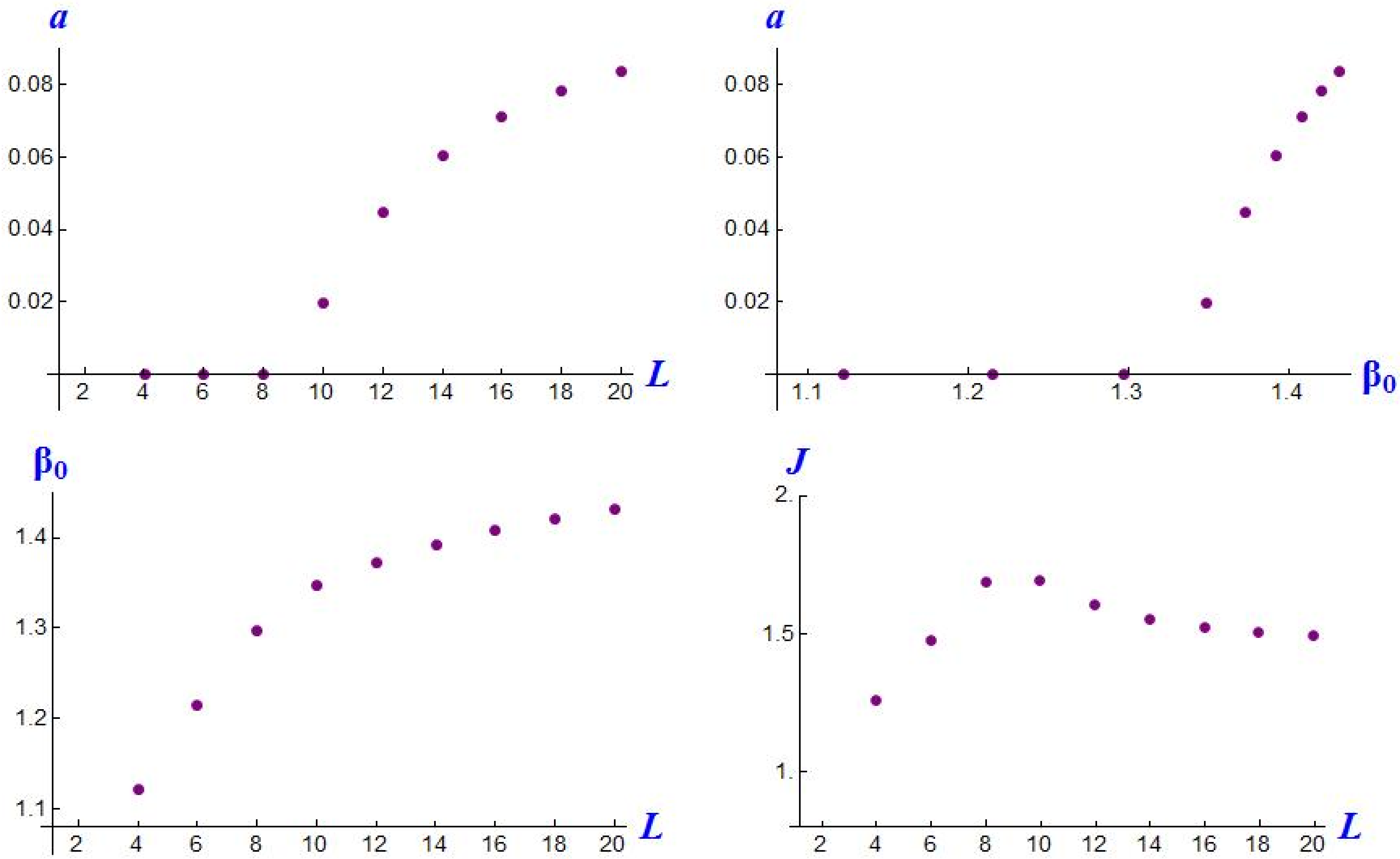


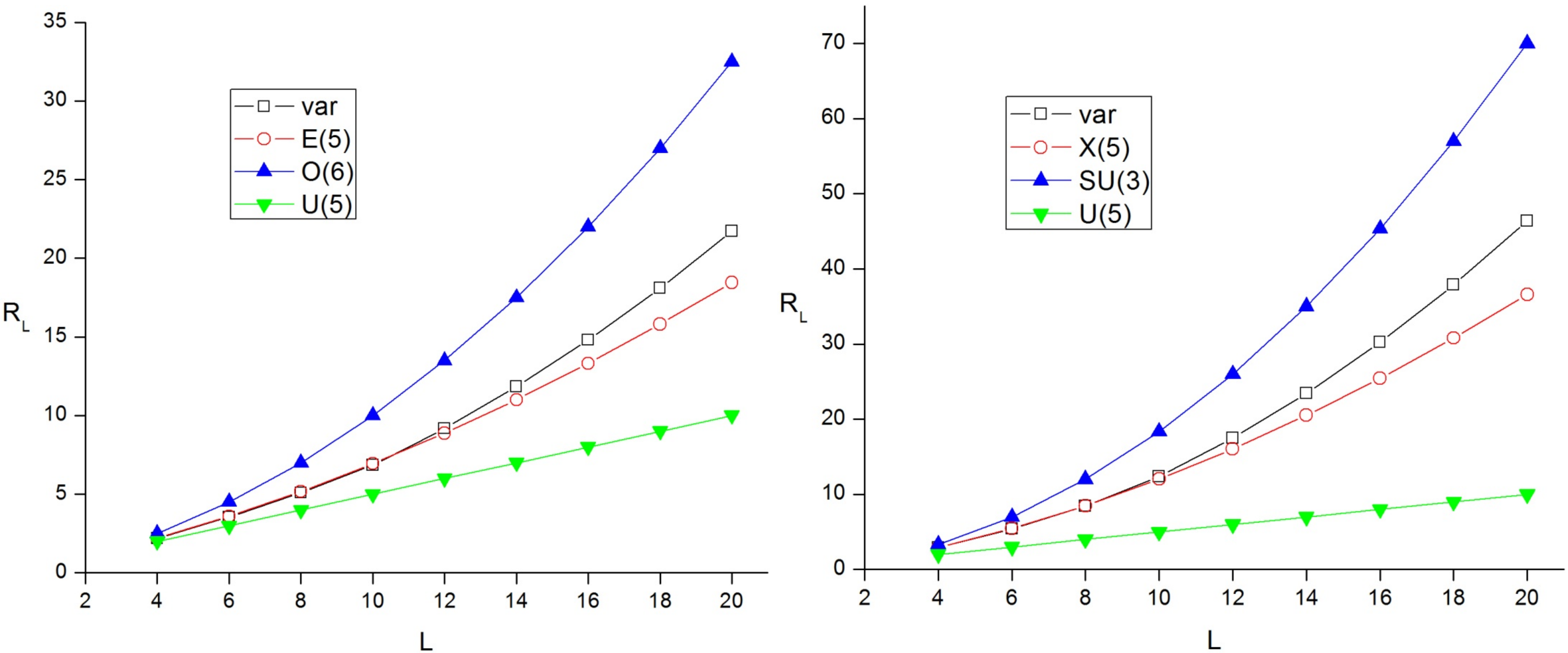
Seek (a, β_0) that maximize $\dfrac{\partial R_L}{\partial \beta_0}$

Spherical to γ-unstable



Spherical to deformed





var
E(5)
O(6)
U(5)
R_L
L
var
X(5)
SU(3)
U(5)
R_L
L

Summary

- Bohr Hamiltonian with B(β) solved with SUSYQM (Deformation dependent mass model)
  - solved for Davidson and Kratzer potentials
  - remedies the β^2 –dependence problem for m.o.i.
  - reproduces features of E(5) and X(5)
- open questions

  role of "a" parameter?

  "downbending"?

  …