# Exact Solutions of the Extended Pairing Interactions in Bose and Fermi Many-Body Systems 

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#### Abstract

An $E_{2}$ algebra is realized based on local boson or fermion operators, with which the extended pairing Hamiltonian of both Bose and Fermi many-body systems is established. A solvable extended Hamiltonian that includes multi-pair interactions among $s$ - and $d$-bosons up to infinite order within the framework of the interacting boson model is proposed to gain a better description of $\mathrm{E}(5)$ model results for finite-N systems. Numerical fits to low-lying energy levels and reduced E2 transition rates within this extended version of the theory are presented for various N values. It shows that the extended Hamiltonian within the IBM provides a better description of the $\mathrm{E}(5)$ model results for small N cases, while the results of the model in the large-N cases are close to those of the $\mathrm{E}(5)-\beta^{2 n}$ type models studied previously. As an application of the theory, the extended pairing Hamiltonian to describe pairing interactions among valence nucleon monopole pairs up to infinite order in a spherical mean-field, such as the spherical shell model, is proposed based on the local E2 algebraic structure, which includes the extended pairing interaction model within a deformed mean-field theory [F. Pan, V. G. Gueorguiev, and J. P. Draayer, Phys. Rev. Lett. 92, 112503 (2004)] as a special case. The advantage of the model lies in the fact that numerical solutions of the model can be obtained more easily with less CPU time than the standard pairing model. Thus, large scale with open shell calculations within the model becomes feasible. As an example of the application, pairing contributions to the binding energies of ${ }^{12-28} \mathrm{O}$ in the model with 11 j -orbits up to the fifth major shell are estimated. The results show that the pairing interaction energy per particle in ${ }^{12-28} \mathrm{O}$ ranges from 0.7-42.23 MeV/A, and the strongest pairing interaction seems in ${ }^{20} \mathrm{O}$.


