Structure evolution and shape phase transitions in odd-mass nuclei

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Quantum phase transitions (QPT) :

Rather extensively studied in **Even-even nuclei**

- Shape phase transitions : abrupt changes in g.s. properties due to competition between different shapes.

- Importance of choice of a right *control parameter: experimental recognition of critical phenomena* only when using an empirical structure property, e.g., ***E(2⁺)***, which varies almost continuously, rather than ***N*** (discontinuous !)   (Casten, Zamfir, Brenner, 1993) .

- *Theoretical*: classes of symmetries as *critical point solutions* in addition to the three IBM dynamical symmetries. Iachello***:*** ***X(5), E(5)***.

- Certain nuclei experimentally recognized as close to a critical point symmetry.

Much less studied in **Odd-mass nuclei**

- *Experimental:* major difficulty: diversity of low-energy excitations, cannot follow the same quantity in many nuclei.
- *Theoretical*: several particular critical point Bose-Fermi symmetries proposed.

?#1: the influence of unpaired fermion on the location and nature of PT
?#2: identify observables related to control and order parameters
?#3: signatures of the QPT; possible critical point nuclei

Empirical approach:
Investigate evolution of level structures based on intruder, or *unique parity orbitals (UPO)*

Extremely pure wave functions (high j-purity) : nearly identical effects for any UPO
→ Apply investigation method (e.g., correlations between certain level structure observables) to different mass regions → cover consistent part of nuclear map

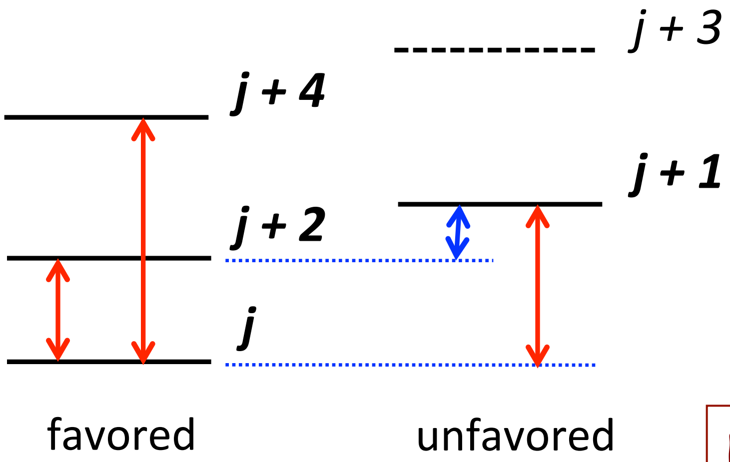
Unique parity orbitals **(UPO)** considered:

Shell **28 – 50** : $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $\mathbf{1g_{9/2}}$

Shell **50 – 82** : $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $\mathbf{1h_{11/2}}$

Shell **82 – 126** : $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, $\mathbf{1i_{13/2}}$

ENSDF: ~ **500** nuclei with $30 \leq Z \leq 95$



Structures based on UPO of spin j

Relative excitation energies: $E(I)=E^*(I)-E^*(j)$

$E(j+2)$, $E(j+4)$, $E(j+1)$

Energy ratios: $R_{j+4/j+2} = E(j+4)/E(j+2)$

Signature splitting index: $R^s_j = [E(j+2)-E(j+1)]/E(j+2)$

Strong coupling:

$R_{j+4/j+2} = (4j+10)/(2j+3)$: ≈ 2.29 (2.333; 2.286; 2.25 for $g_{9/2}$, $h_{11/2}$, $i_{13/2}$)

$R^s_j = (j+2)/(2j+3)$: ≈ 0.54

Particle-plus-Rotor Model

H ≈ s.p. (intrinsic) + rotation of inert core + Coriolis interaction

Limit coupling schemes within the PRM *(axially symm. rotor)*
(*Ring&Schuck, ch.3; Casten, chs. 8,9*)

(i) Weak coupling

When: For *very small core deformations* : up to $\beta_2 \approx 0.14$ ($R_{4/2} = E(4^+)/E(2^+) \approx 2.0$ to ~2.2)
→ favored *states j, j+2, j+4, ⋯* : spacings similar to gsb (0^+, 2^+, 4^+, ⋯) of the core.

(ii) Strong coupling (deformation alignment)

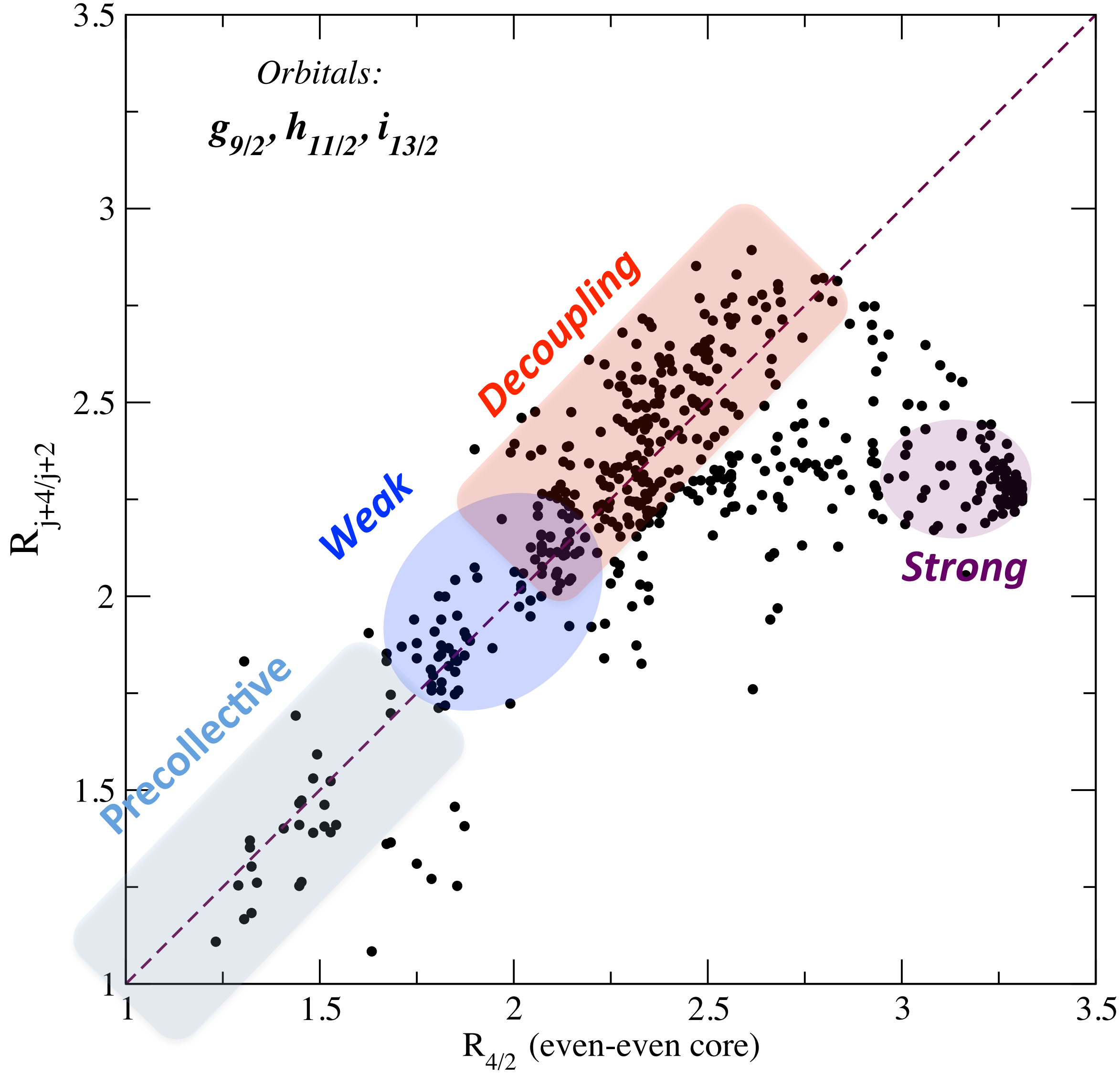
When: Coriolis interaction m.e. are small compared to s.p. energy splittings;
(a) For *large deformations* β_2 > ~0.24 ($R_{4/2} \geq 3.0$)
(b) small Coriolis m.e.; large-j (UPO): when odd particle in high-Ω Nilsson orbitals.
→ favored (j, j+2, j+2, ⋯) and unfavored (j+1, j+3, ⋯) merge into a ***ΔI=1 rotational band***.

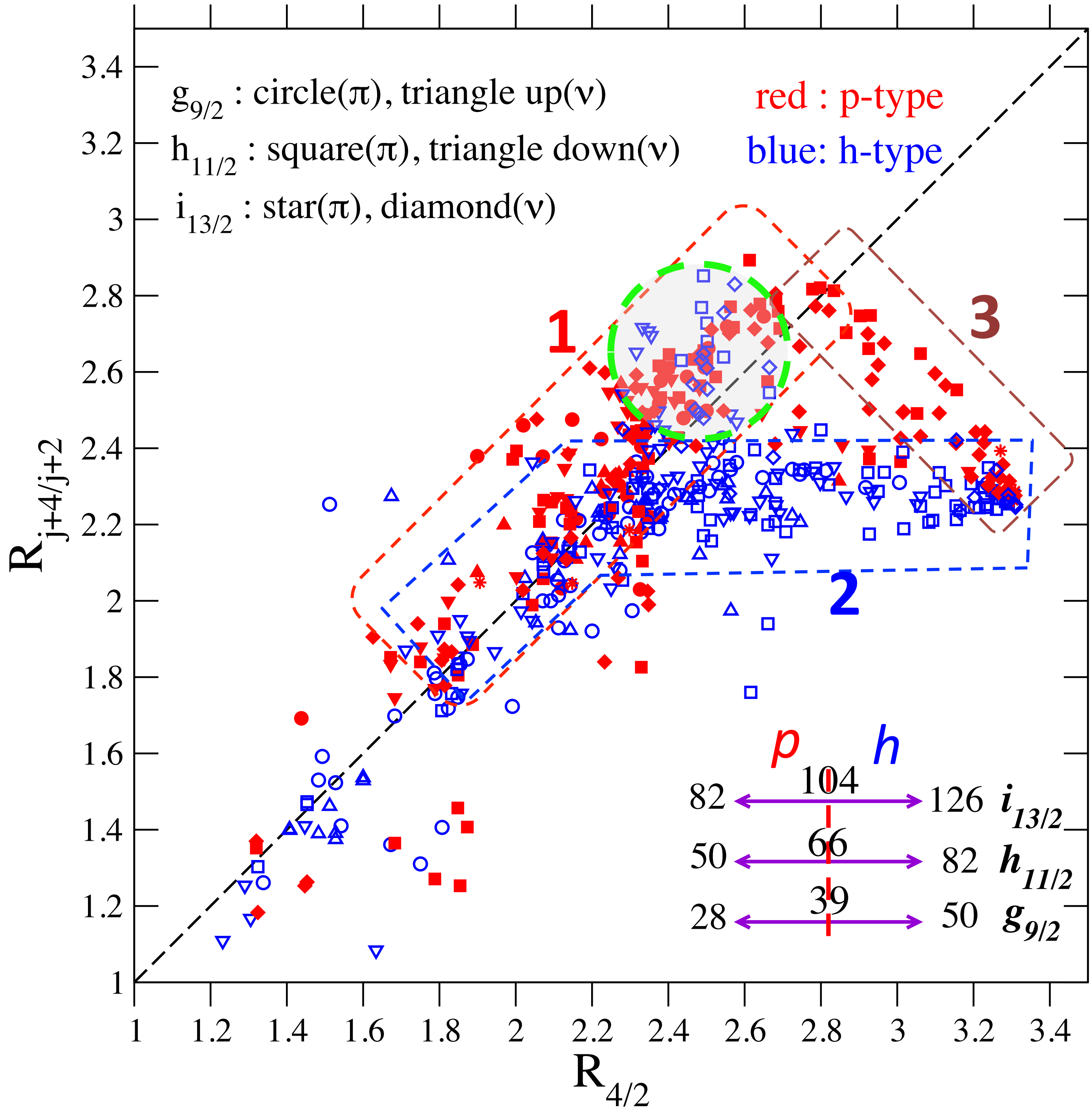
(iii) Decoupling (rotational alignment)

When: Coriolis interaction is strong and cannot be neglected;
for large-j UPO: when odd particle in low-Ω states;
occurs for *intermediate deformations*: β_2 ~0.14 to ~0.23 ($R_{4/2}$ ~2.2 to ~2.7)
→ favored states (j, j+2, j+2, ⋯) spacings similar to gsb of the core;
unfavored states (j+1, j+2, ⋯) lie at higher energies.

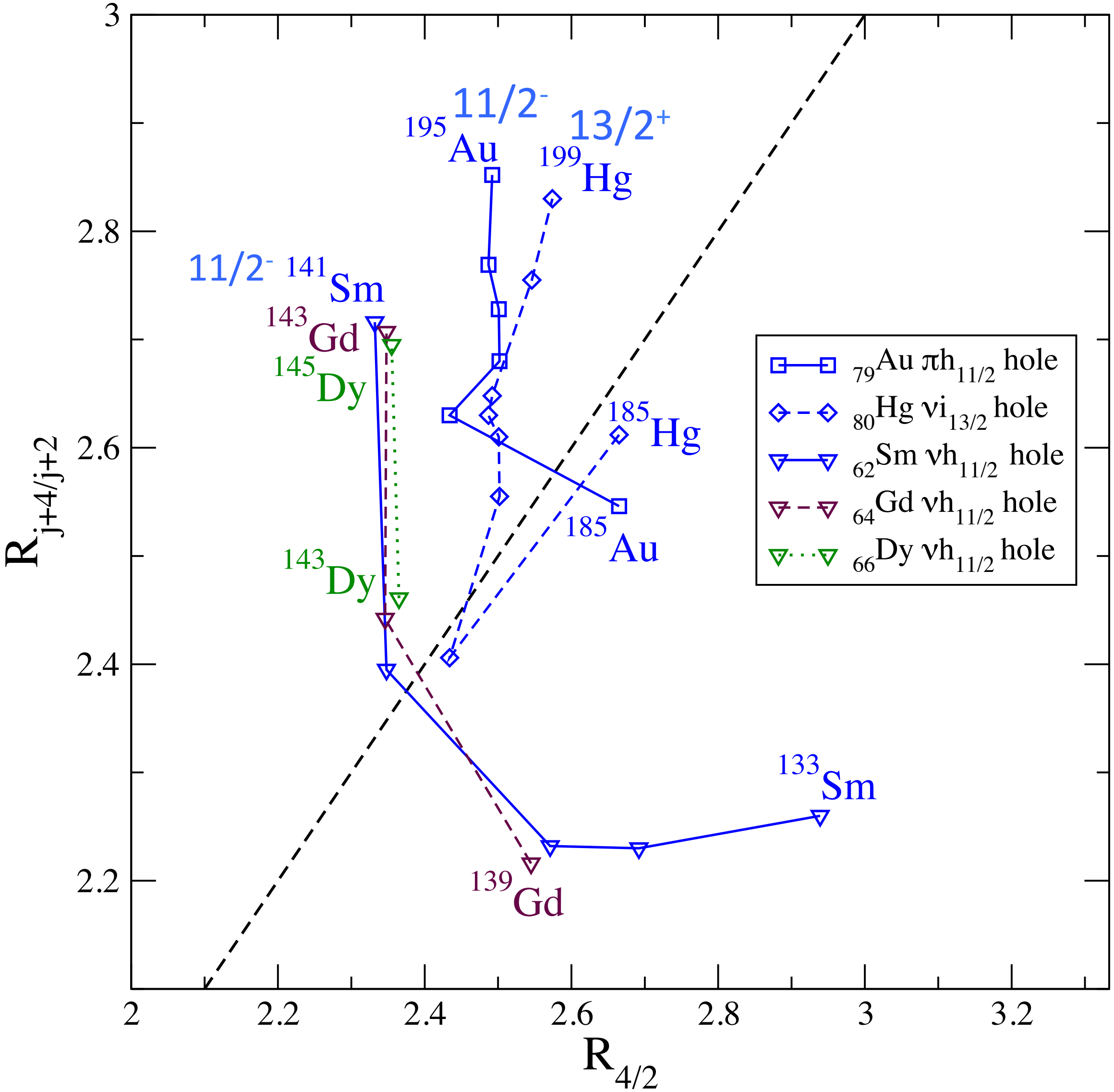
$$< |Cor| > \sim [I(I+1)-K^2)(j(j+1)-\Omega^2)]^{1/2}$$

A first look at evolution of UPO structures





g9/2 : circle(π), triangle up(ν)
h11/2 : square(π), triangle down(ν)
i13/2 : star(π), diamond(ν)
red : p-type
blue: h-type
1
2
3
p
h
82
104
126 i13/2
50
66
82 h11/2
28
39
50 g9/2
R4/2
Rj+4/j+2

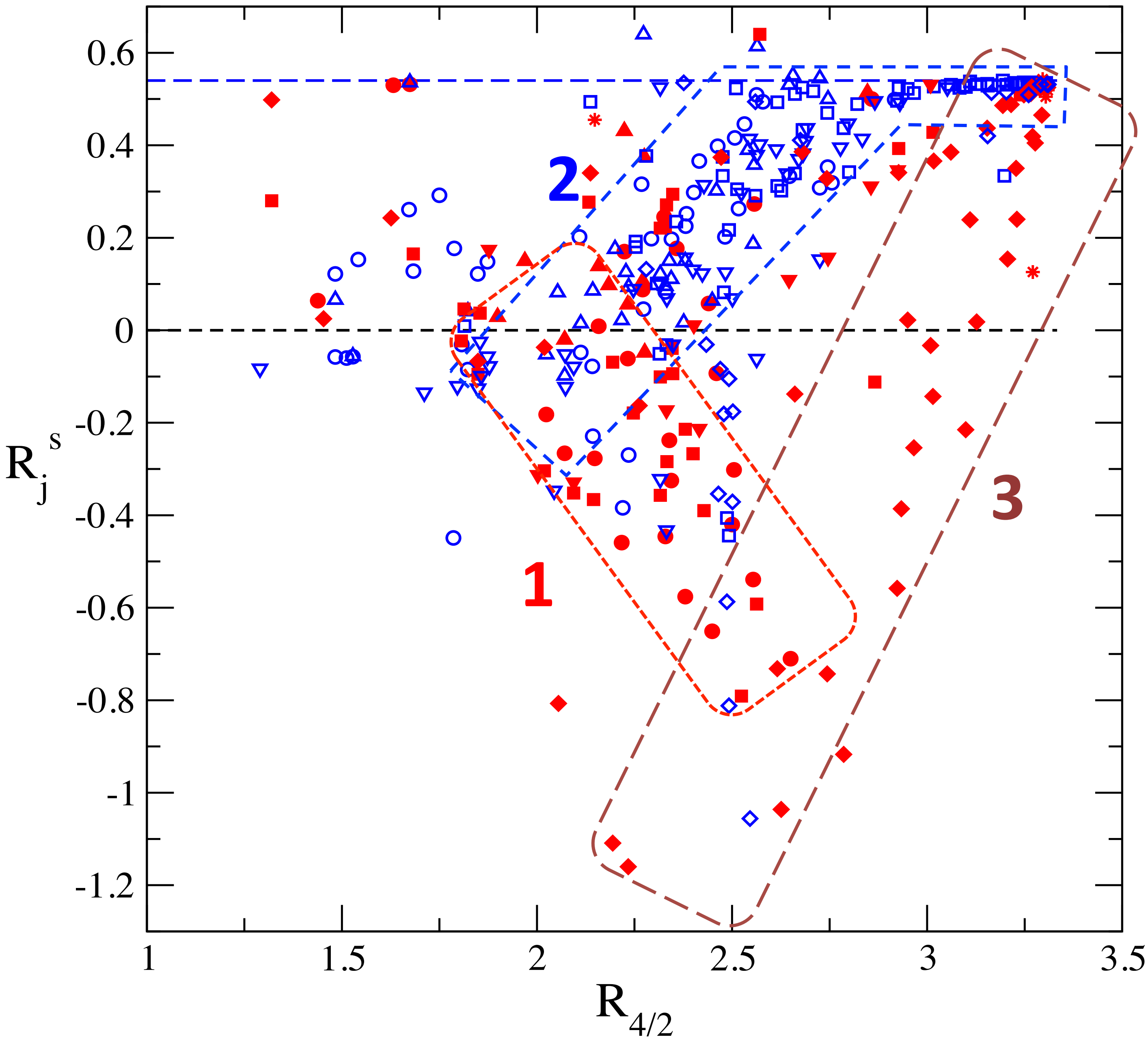


Oblate shapes

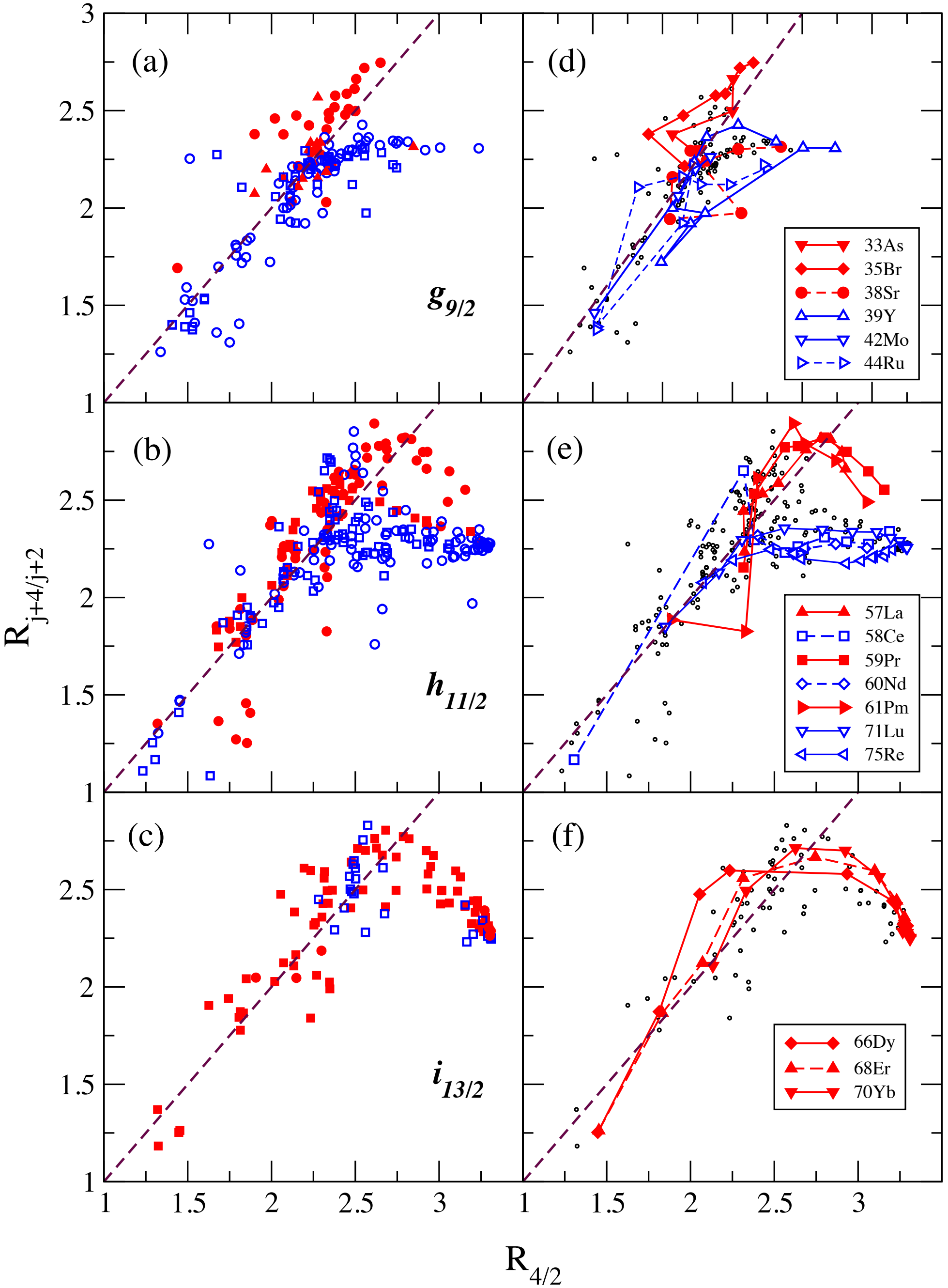
(Au, Hg, ^{141}Sm: N.J. Stone, Table of nuclear electric quadrupole moments, IAEA, INDC-0650, 2013)

Hole states:

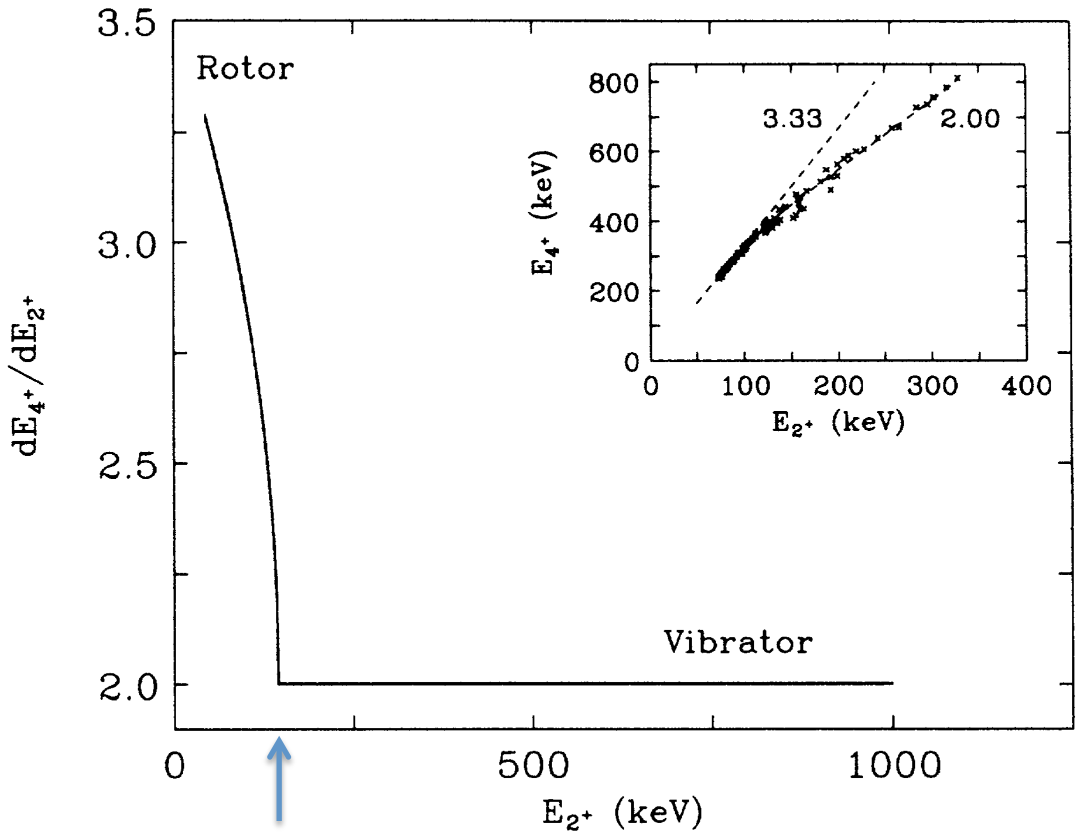
*odd particle in **low-Ω** orbits: decoupling .*



0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
R_j^s
1
1.5
2
2.5
3
3.5
$R_{4/2}$
1
2
3



(a)
$g_{9/2}$
(b)
$h_{11/2}$
(c)
$i_{13/2}$
(d)
33As
35Br
38Sr
39Y
42Mo
44Ru
(e)
57La
58Ce
59Pr
60Nd
61Pm
71Lu
75Re
(f)
66Dy
68Er
70Yb
$R_{j+4/j+2}$
$R_{4/2}$

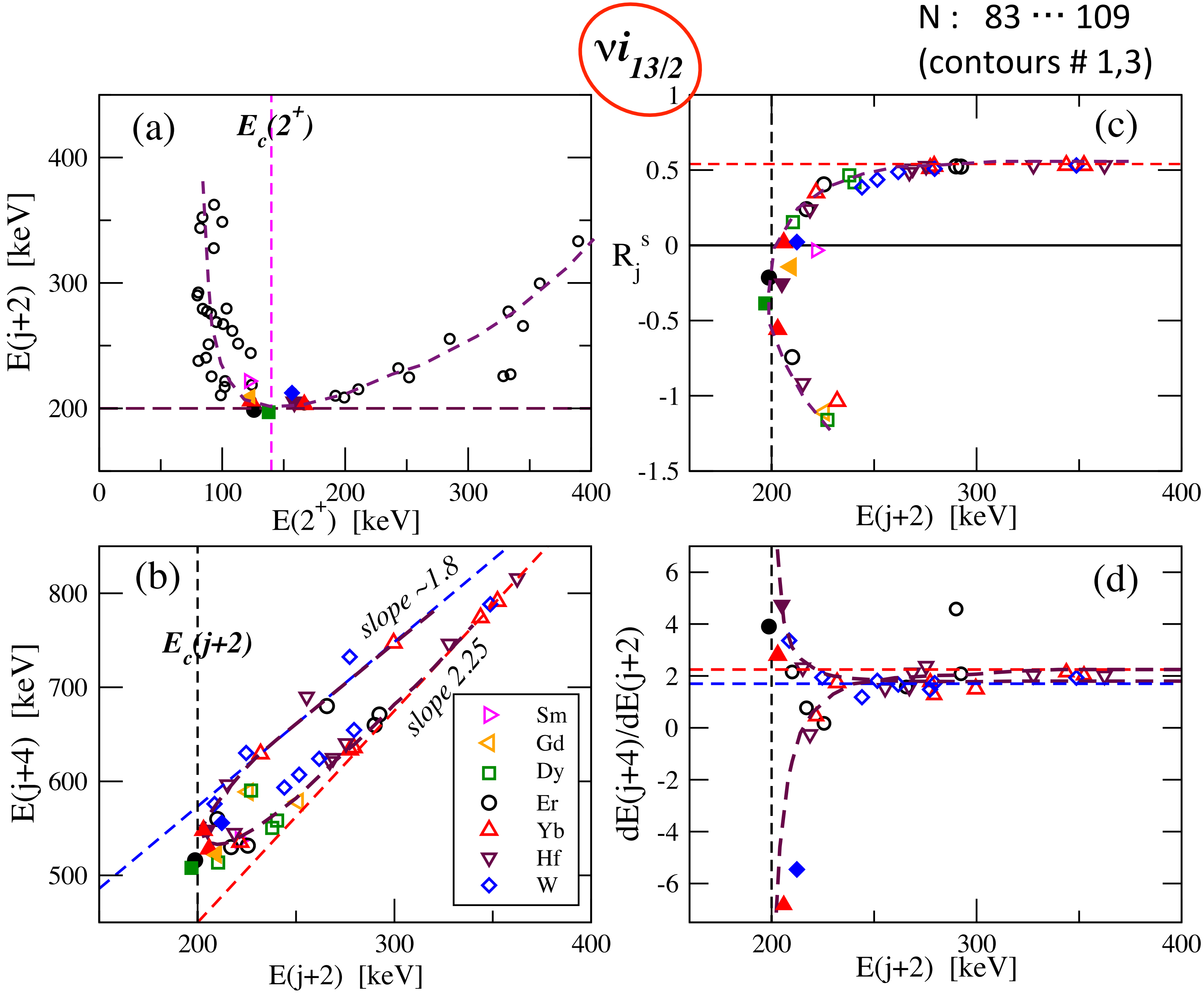


$E_c(2^+) \approx 145$ keV

Critical PST in even –even nuclei (the X(5) critical point)

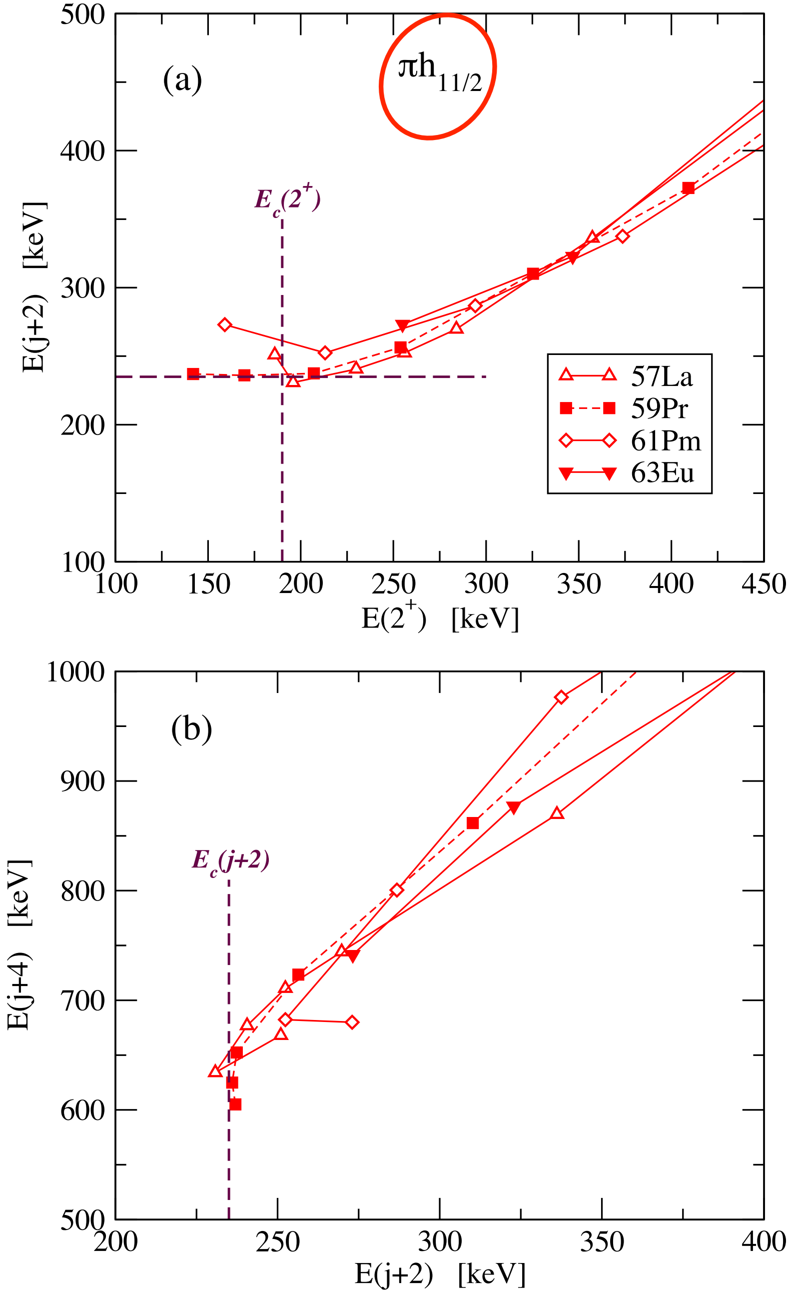
Casten, Zamfir, Brenner, *PRL 71(1993)227*

Z : 62(Sm) ··· 80(Hg)
N : 83 ··· 109
(contours # 1,3)



$\nu i_{13/2}$ structures: candidate nuclei for the critical point of the phase transition *decoupling → strong coupling*

	Core	X(5)	$R^s_j \approx 0$	N
$^{153}Sm^{62}$	^{152}Sm	✓	x	90
$^{155}Gd^{64}$	^{154}Gd	(✓)	~	90
$^{157}Dy^{66}$	^{156}Dy	✓	~	90
$^{161}Er^{68}$	^{160}Er	(✓)		92
$^{163}Yb^{70}$	^{162}Yb	✓		92
$^{165}Yb^{70}$	^{164}Yb		x	94
$^{167}Hf^{72}$	^{166}Hf	✓		94
$^{171}W^{74}$	^{170}W	✓	x	96



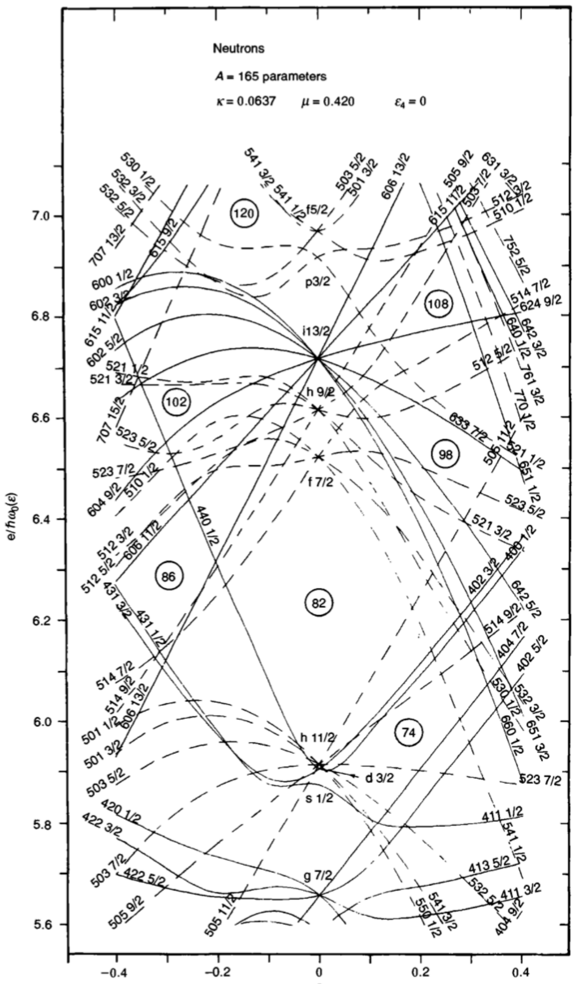
Z : 55(Cs) ··· 75(Tb)
N : 62 ··· 82
(contours # 1,3)

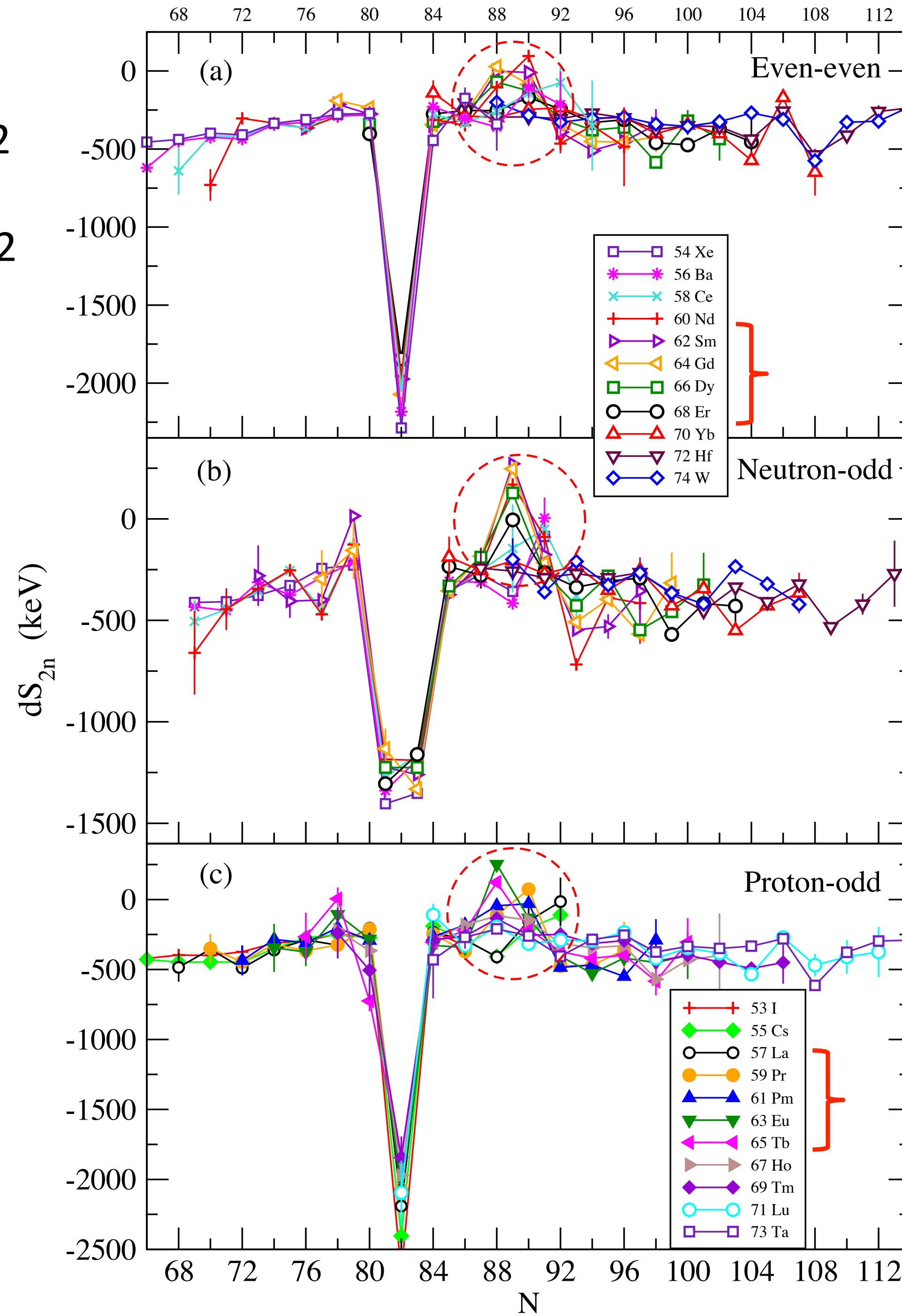
Possible critical SPT at A~130:

	Core	N
$^{125}La^{57}$	^{124}Ba	68
$^{127,129}Pr^{59}$	$^{126,128}Ce$	68,70
$^{133}Pm^{61}$	^{132}Nd	72
$^{135}Eu^{63}$	^{134}Sm	72

Values from Mass Tables – 2012

$$dS_{2n}(Z,N)=[S_{2n}(Z,N+2)-S_{2n}(Z,N)]/2$$





Theoretical

Critical point symmetry models for odd-A nuclei:

- *E(5/4)*: Bose-Fermi symmetry for *j=3/2* particle coupled to *E(5)* core (Iachello *PRL95(2005)052503*)
  some E(5/4) features in ^{135}Ba (Fetea et al, *PRC73(2006)051301(R)*)
- *E(5/12)*: multi-orbit: *j=1/2,3/2,5/2* (Alonso, Arias, Vitturi, *PRC75(2007)064316*)

- *X(5/(2j+1)) j-particle* coupled to *X(5)* core (Zhang, Pan, Liu, Hou, Draayer *PRC82(2010)034327*)
  limited agreement for ^{189}Au (j=1/2), ^{155}Tb (j=5/2); multi-orbit approach needed.

- Recent approach to shape phase transitions in odd-A:
  energy density functional theory + particle-plus-boson core coupling: define possible signatures related to deformations, exc. energies, E2-trans. rates, separation energies (as quantum order parameters).
  Nomura, Ničsić, Vretenar PRC94(2016)064310: Eu, Sm with N~90
  Nomura, Ničsić, Vretenar PRC96(2017)014304: Ba, Xe, La, Cs with A~130, γ-soft

SUMMARY

Correlations between UPO structure observables (energies, energy ratios):

- Interesting structure evolution along the three limit coupling schemes of the PRM (weak coupling, decoupling, strong coupling)

- Evidence for critical PT (*fast transition from **decoupling** to **strong coupling***) for ***νi***$_{13/2}$ structures at N=90-92, and ***πh***$_{11/2}$ structures at N=70-72, correlated with the critical SPT in the even-even core nuclei (X(5) critical point).

  Features of this transition:
  - fast change in the pattern of *E(j+4) versus E(j+2)* at some critical value $E_c(j+2)$
  - discontinuous change in $dE(j+4)/d(Ej+2)$ at $E_c(j+2)$
  - ~ degeneracy of the energies of favored and unfavored sequences at $E_c(j+2)$

- Shape phase transition corroborated by systematics of mass-related quantities: dS_{2n}

- Critical point symmetry model description of these observations, as well as of other low-excitation structure features are welcome.

D. Bucurescu, N.V. Zamfir, PRC95(2017)014329