Structure evolution and shape phase transitions in odd-mass nuclei

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Quantum phase transitions (QPT):

Rather extensively studied in **Even-even nuclei**

- **Shape phase transitions**: abrupt changes in g.s. properties due to competition between different shapes.

- Importance of choice of a right *control parameter*: experimental recognition of *critical phenomena* only when using an empirical structure property, e.g., $E(2^+)$, which varies almost continuously, rather than $N$ (discontinuous !) (Casten, Zamfir, Brenner, 1993).

- **Theoretical**: classes of symmetries as *critical point solutions* in addition to the three IBM dynamical symmetries. Iachello: $X(5), E(5)$.

- Certain nuclei experimentally recognized as close to a critical point symmetry.

Much less studied in **Odd-mass nuclei**

- **Experimental**: major difficulty: diversity of low-energy excitations, cannot follow the same quantity in many nuclei.

- **Theoretical**: several particular critical point Bose-Fermi symmetries proposed.
?#1: the influence of unpaired fermion on the location and nature of PT
?#2: identify observables related to control and order parameters
?#3: signatures of the QPT; possible critical point nuclei

**Empirical approach:**
*Investigate evolution of level structures based on intruder, or unique parity orbitals (UPO)*

**Extremely pure wave functions** (high j-purity): nearly identical effects for any UPO

→ Apply investigation method (e.g., correlations between certain level structure observables) to different mass regions → cover consistent part of nuclear map

Unique parity orbitals (UPO) considered:

Shell 28 – 50 : $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$

Shell 50 – 82 : $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$

Shell 82 – 126 : $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, $1i_{13/2}$

ENSDF: ~ 500 nuclei with $30 \leq Z \leq 95$
Relative excitation energies: \( E(I) = E^*(I) - E^*(j) \)

\( E(j+2), E(j+4), E(j+1) \)

Energy ratios:

\( R_{j+4/j+2} = E(j+4)/E(j+2) \)

Signature splitting index:

\( R_s^j = \frac{[E(j+2)-E(j+1)]}{E(j+2)} \)

Strong coupling:

\( R_{j+4/j+2} = \frac{4j+10}{2j+3} : \approx 2.29 \quad (2.333; 2.286; 2.25 \text{ for } g_{9/2}, h_{11/2}, i_{13/2}) \)

\( R_s^j = \frac{j+2}{2j+3} : \approx 0.54 \)
Particle-plus-Rotor Model

\[ H \approx \text{s.p. (intrinsic)} + \text{rotation of inert core} + \text{Coriolis interaction} \]

**Limit coupling schemes within the PRM** *(axially symm. rotor)*

*(Ring\&Schuck, ch.3; Casten, chs. 8,9)*

(i) **Weak coupling**

*When:* For **very small core deformations** : up to \( \beta_2 \approx 0.14 \) \((R_{4/2} = E(4^+)/E(2^+) \approx 2.0 \text{ to } ~2.2)\)

→ **favored states** \( j, j+2, j+4, \cdots \) : spacings similar to gsb \((0^+, 2^+, 4^+, \cdots)\) of the core.

(ii) **Strong coupling** *(deformation alignment)*

*When:* Coriolis interaction m.e. are small compared to s.p. energy splittings;

(a) For **large deformations** \( \beta_2 > \sim 0.24 \) \((R_{4/2} \geq 3.0)\)

(b) **small Coriolis m.e.; large-j (UPO):** when odd particle in high-\( \Omega \) Nilsson orbitals.

→ favored \((j, j+2, j+2, \cdots)\) and unfavorable \((j+1, j+3, \cdots)\) merge into a \( \Delta l=1 \) rotational band.

(iii) **Decoupling** *(rotational alignment)*

*When:* Coriolis interaction is strong and cannot be neglected;

for large-j UPO: when odd particle in low-\( \Omega \) states;

occurs for **intermediate deformations**: \( \beta_2 \sim 0.14 \text{ to } \sim 0.23 \) \((R_{4/2} \sim 2.2 \text{ to } \sim 2.7)\)

→ favored states \((j, j+2, j+2, \cdots)\) spacings similar to gsb of the core;

unfavorable states \((j+1, j+2, \cdots)\) lie at higher energies.

\[ < |\text{Cor}| > \sim [I(I+1)-K^2](j(j+1)-\Omega^2)]^{1/2} \]
A first look at evolution of UPO structures

Orbitals:
$g_{9/2}, h_{11/2}, i_{13/2}$

$R_{j+4/j+2}$

$R_{4/2}$ (even-even core)

Strong

Decoupling

Weak

Precollective
$g_{9/2}$ : circle(π), triangle up(ν)  
$h_{11/2}$ : square(π), triangle down(ν)  
$i_{13/2}$ : star(π), diamond(ν)

red : p-type  
blue: h-type

82 → 104 → 126 $i_{13/2}$
50 → 66 → 82 $h_{11/2}$
28 → 39 → 50 $g_{9/2}$
Oblate shapes
(Au, Hg, $^{141}$Sm: N.J. Stone, Table of nuclear electric quadrupole moments, IAEA, INDC-0650, 2013)

Hole states: odd particle in low-$\Omega$ orbits: decoupling.
\[ E_c(2^+) \approx 145 \text{ keV} \]

Critical PST in even–even nuclei (the X(5) critical point)

$\nu i_{13/2}$

- $Z: 62(\text{Sm}) \cdots 80(\text{Hg})$
- $N: 83 \cdots 109$
- (contours # 1,3)

**Figure (a)**
- $E_c(2^+)$ vs. $E(2^+)$ [keV]
- Slope 2.25

**Figure (b)**
- $E_c(j+2)$ vs. $E(j+4)$ vs. $E(j+2)$ [keV]
- Slopes: ~1.8, ~2.25

**Figure (c)**
- $R^s_j$ vs. $E(j+2)$ [keV]

**Figure (d)**
- $dE(j+4)/dE(j+2)$ vs. $E(j+2)$ [keV]
**vi$_{13/2}$ structures:** candidate nuclei for the critical point of the phase transition \textit{decoupling $\rightarrow$ strong coupling}

<table>
<thead>
<tr>
<th>Core</th>
<th>X(5)</th>
<th>R$^s_j\approx$0</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{153}\text{Sm}^{62}$</td>
<td>$^{152}\text{Sm}$</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>$^{155}\text{Gd}^{64}$</td>
<td>$^{154}\text{Gd}$</td>
<td>✓</td>
<td>~</td>
</tr>
<tr>
<td>$^{157}\text{Dy}^{66}$</td>
<td>$^{156}\text{Dy}$</td>
<td>✓</td>
<td>~</td>
</tr>
<tr>
<td>$^{161}\text{Er}^{68}$</td>
<td>$^{160}\text{Er}$</td>
<td>✓</td>
<td>~</td>
</tr>
<tr>
<td>$^{163}\text{Yb}^{70}$</td>
<td>$^{162}\text{Yb}$</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>$^{165}\text{Yb}^{70}$</td>
<td>$^{164}\text{Yb}$</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>$^{167}\text{Hf}^{72}$</td>
<td>$^{166}\text{Hf}$</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>$^{171}\text{W}^{74}$</td>
<td>$^{170}\text{W}$</td>
<td>✓</td>
<td>x</td>
</tr>
</tbody>
</table>
Possible critical SPT at $A \sim 130$:

Z : $55(Cs) \cdots 75(Tb)$
N : $62 \cdots 82$

(Contours # 1,3)

<table>
<thead>
<tr>
<th>Core</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{125}_{57}$La</td>
<td>68</td>
</tr>
<tr>
<td>$^{127,129}_{59}$Pr</td>
<td>$^{126,128}_{61}$Ce</td>
</tr>
<tr>
<td>$^{133}_{61}$Pm</td>
<td>$^{132}_{72}$Nd</td>
</tr>
<tr>
<td>$^{135}_{63}$Eu</td>
<td>$^{134}_{72}$Sm</td>
</tr>
</tbody>
</table>
Values from Mass Tables – 2012

$$dS_{2n}(Z,N) = [S_{2n}(Z,N+2) - S_{2n}(Z,N)]/2$$
Critical point symmetry models for odd-A nuclei:


- **$X(5/(2j+1))$ $j$-particle** coupled to $X(5)$ core (Zhang, Pan, Liu, Hou, Draayer *PRC*82(2010)034327) limited agreement for $^{189}$Au ($j=1/2$), $^{155}$Tb ($j=5/2$); multi-orbit approach needed.

- Recent approach to shape phase transitions in odd-A:
  energy density functional theory + particle-plus-boson core coupling: define possible signatures related to deformations, exc. energies, E2-trans. rates, separation energies (as quantum order parameters).

  Nomura, Ničsić, Vretenar *PRC*94(2016)064310: Eu, Sm with $N\sim 90$

  Nomura, Ničsić, Vretenar *PRC*96(2017)014304: Ba, Xe, La, Cs with $A\sim 130$, $\gamma$-soft
SUMMARY

Correlations between UPO structure observables (energies, energy ratios):

- Interesting structure evolution along the three limit coupling schemes of the PRM (weak coupling, decoupling, strong coupling)

- Evidence for critical PT (*fast transition from decoupling to strong coupling*) for $\nu_{i_{13/2}}$ structures at $N=90-92$, and $\pi h_{11/2}$ structures at $N=70-72$, correlated with the critical SPT in the even-even core nuclei ($X(5)$ critical point).

Features of this transition:
- fast change in the pattern of $E(j+4)$ versus $E(j+2)$ at some critical value $E_c(j+2)$
- discontinuous change in $dE(j+4)/d(Ej+2)$ at $E_c(j+2)$
- $\sim$ degeneracy of the energies of favored and unfavored sequences at $E_c(j+2)$

- Shape phase transition corroborated by systematics of mass-related quantities: $dS_{2n}$

- Critical point symmetry model description of these observations, as well as of other low-excitation structure features are welcome.

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