# *Neutron Transfer Reactions for Deformed Nuclei*

## Sturmian Basis Approach

***to the P($J^{\pi}$, E) distribution in $^{156}$Gd* from $^{157}$Gd($^{3}$He,$^{4}$He)$^{156}$Gd****


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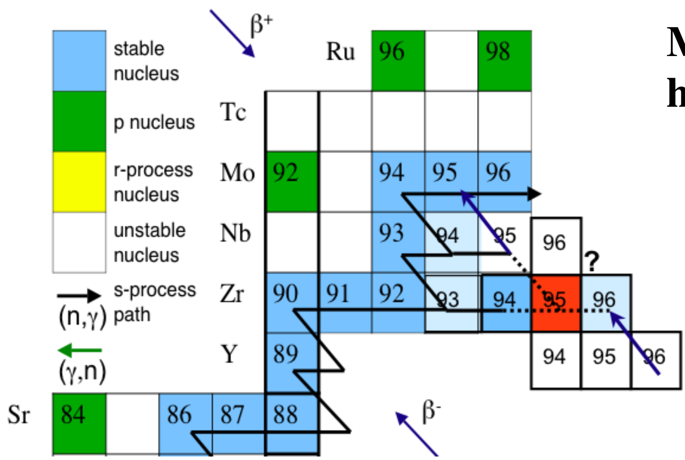
# Outline of the Talk

- **Introduction and Motivation**
- **Surrogate Reactions**
- **Strongly Deformed Nuclei**
  - **Single particle states**
  - ***$^{156}Gd^{*}$ spectrum*** (Bohr-Mottelson rotor plus particle model)
  - **Sturmian basis approach for reaction cross sections**
  - **$J^{\pi}$ distribution estimates for *$^{157}Gd+^{3}He \rightarrow {}^{4}He+^{156}Gd^{*}$***
- **Outlook**

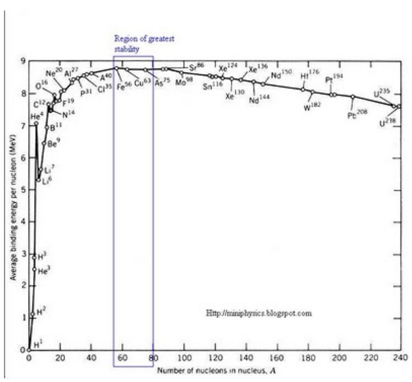
# Origin of the Elements

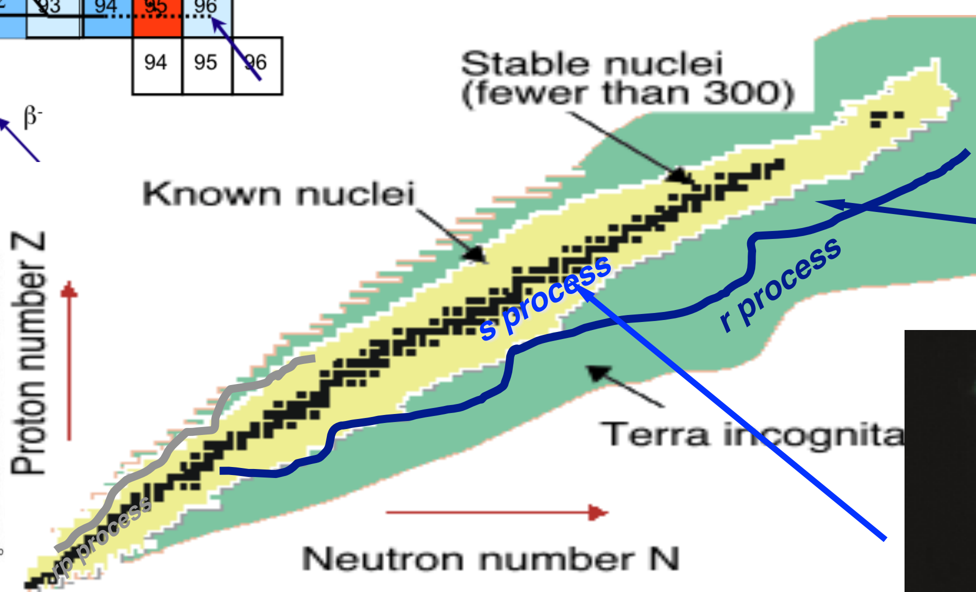
## s-process branch point nuclei are unstable

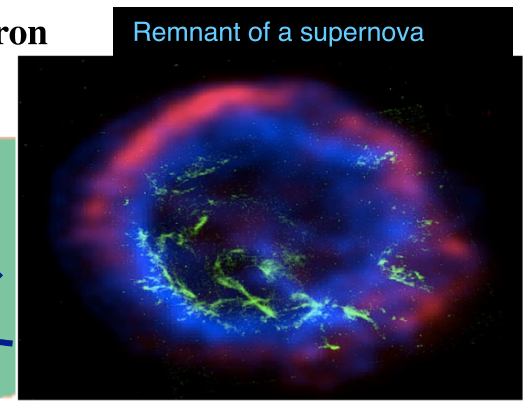


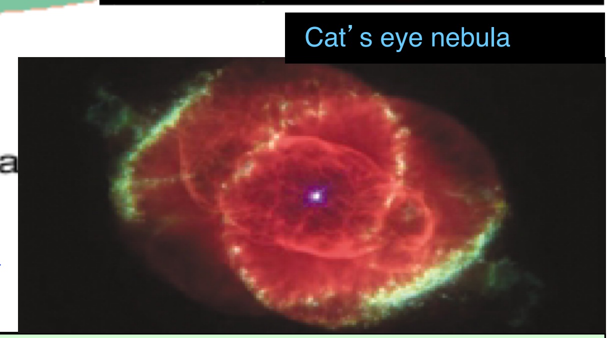

**Many of the actors (*nuclei*) in the play (*nucleosynthesis*) have very important, albeit, episodic roles (*short lifetime*).**

**slow- / rapid- neutron capture process**



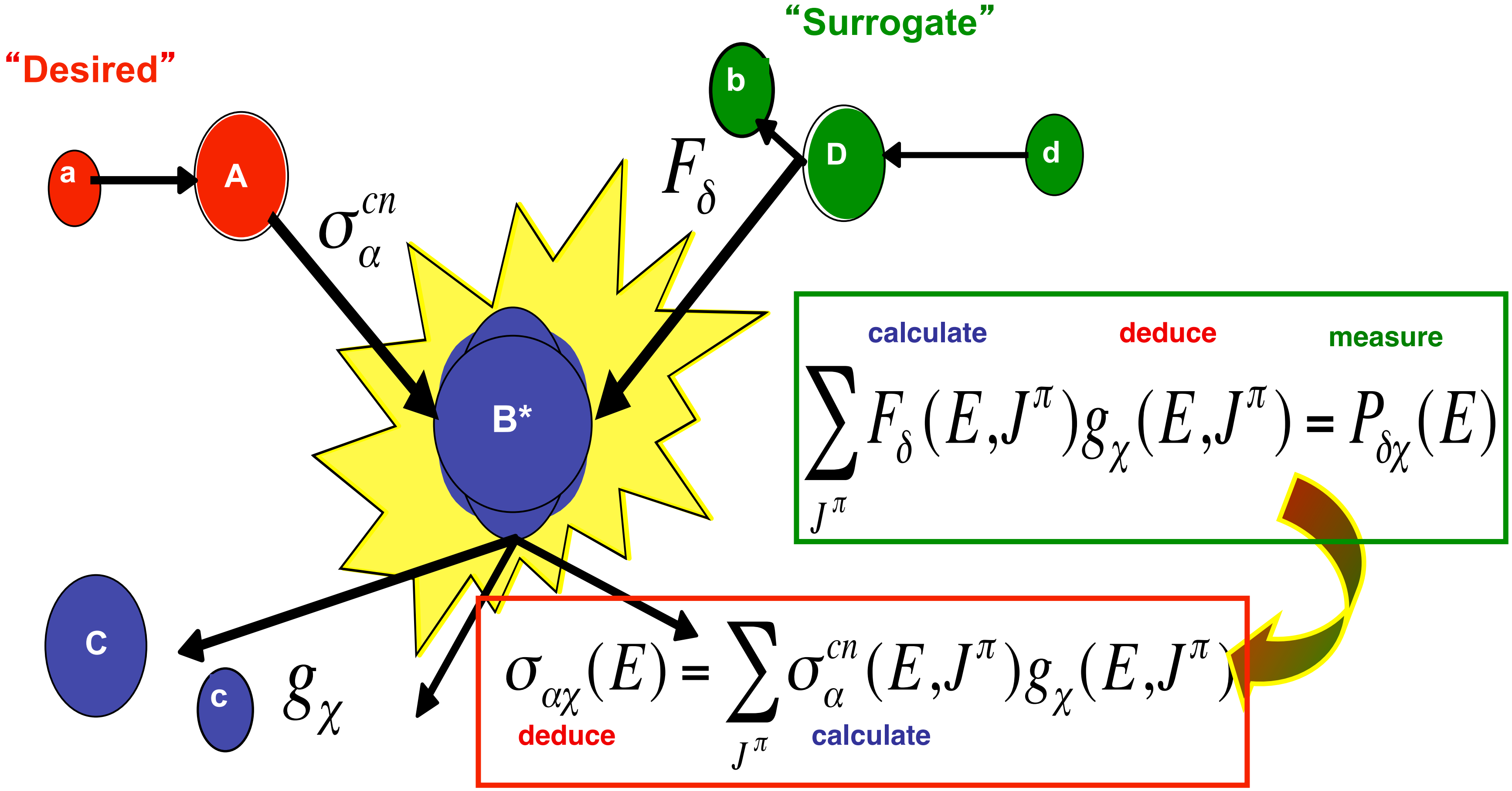




- **All elements starting with carbon (C) are products of stellar evolution.**
- **Most light to intermediate nuclei are products of various nuclear burning-cycles.**
- **Heavier nuclei along the valley of stability are produced via neutron capture reactions.**
- **Elements away from the valley of stability are produced in explosive events.**

# The Surrogate Method

## deducing reaction cross sections on unstable nuclei from reactions on stable nuclei




$$\sum_{J^\pi} F_\delta(E,J^\pi)\, g_\chi(E,J^\pi) = P_{\delta\chi}(E)$$

$$\sigma_{\alpha\chi}(E) = \sum_{J^\pi} \sigma_\alpha^{cn}(E,J^\pi)\, g_\chi(E,J^\pi)$$

**If two reactions proceed via formation of the same intermediate equilibrated compound system $B^*$ then the cross section for the desired reaction $a+A \rightarrow B^* \rightarrow C+c$, which might involve an *unstable* target A, may be deduced using theoretical modeling and experimental data for the *surrogate* reaction $d+D \rightarrow b+B^* \rightarrow C+c$ for a *stable target* D.**

# Reactions with Deformed Nuclei

## Computational Strategy and Methods

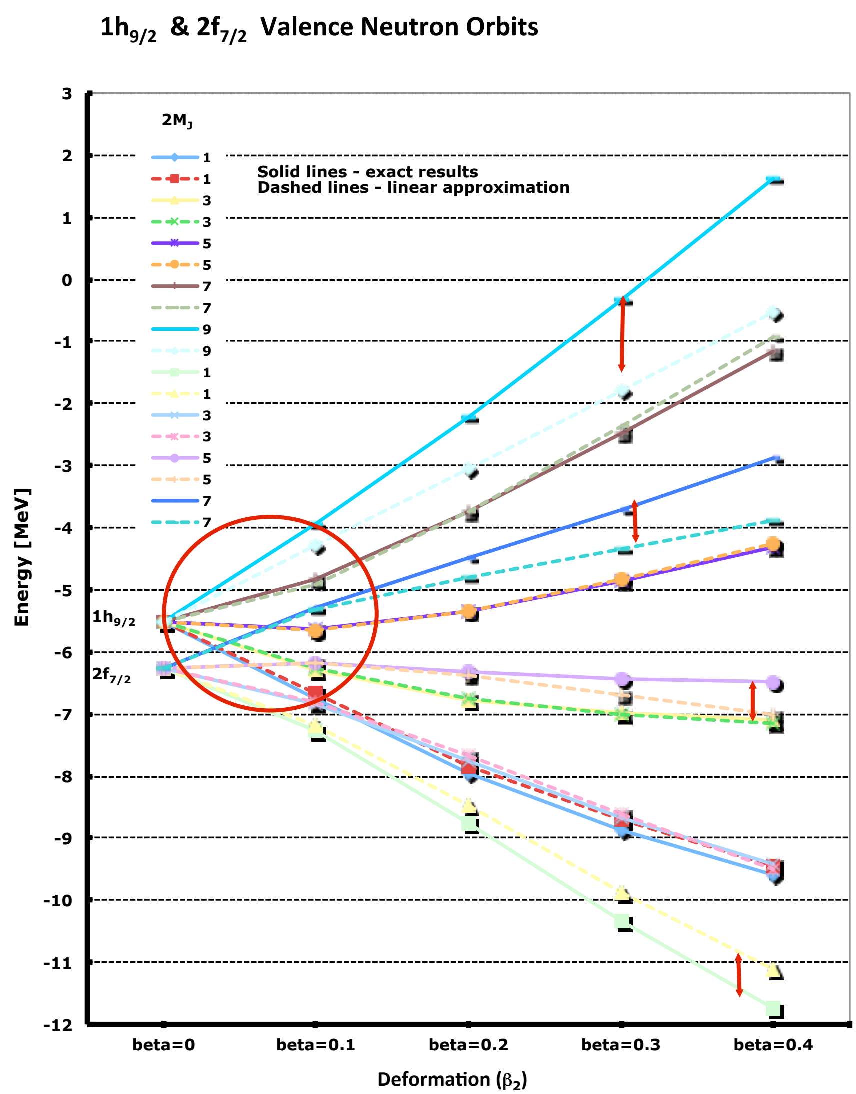
- **Main assumptions:**
  - Direct particle transfer to/from a *deformed single particle state* (sps).
  - *Axially deformed Woods-Saxon potential* defines the *deformed sps*.
  - Reaction cross section can be computed with the available reaction codes by using superposition of *Sturmian spherical single particle states.*
- **Computational steps where coding or scripting is completed:**
  - **WSBETA** code to obtain *energies and deformed states* ($\Psi_E$) for a given *deformed potential, big deformations* ***careful*** *numerical treatment*.
  - **DWUCK*** code generates the set of *spherical WS potentials* and ***non-orthogonal Sturmian basis*** *wave functions* $\Phi_{Enlj}$ for the desired particle binding energy $E$.
  - **WSBETA** code uses DWUCK's *spherical WS potentials* to compute the *Sturmian basis wave functions* in the same basis where the *deformed state* $\Psi_E$ is given.
  - **CalculateCij** code computes the overlaps $C_{nlj}=\langle\Phi_{Enlj}|\Psi_E\rangle$ using WSBETA wave functions.
  - **CHUCK*** CC-code uses $\Psi_E(r)=\sum C_{nlj}\Phi_{Enlj}$ to compute the reaction cross sections.
- **Auxiliary codes**
  - **AddStates** code uses $C_{nlj}$ and $\Phi_{Enlj}(r)$ to verify that the *Sturmian basis* is sufficiently big and to construct $\Psi_E(r)$ as superposition of *Sturmian basis states.*
  - **Various python scripts and Mathematica notebooks** to prepare inputs, run the codes, collect, organize and visualize the results….

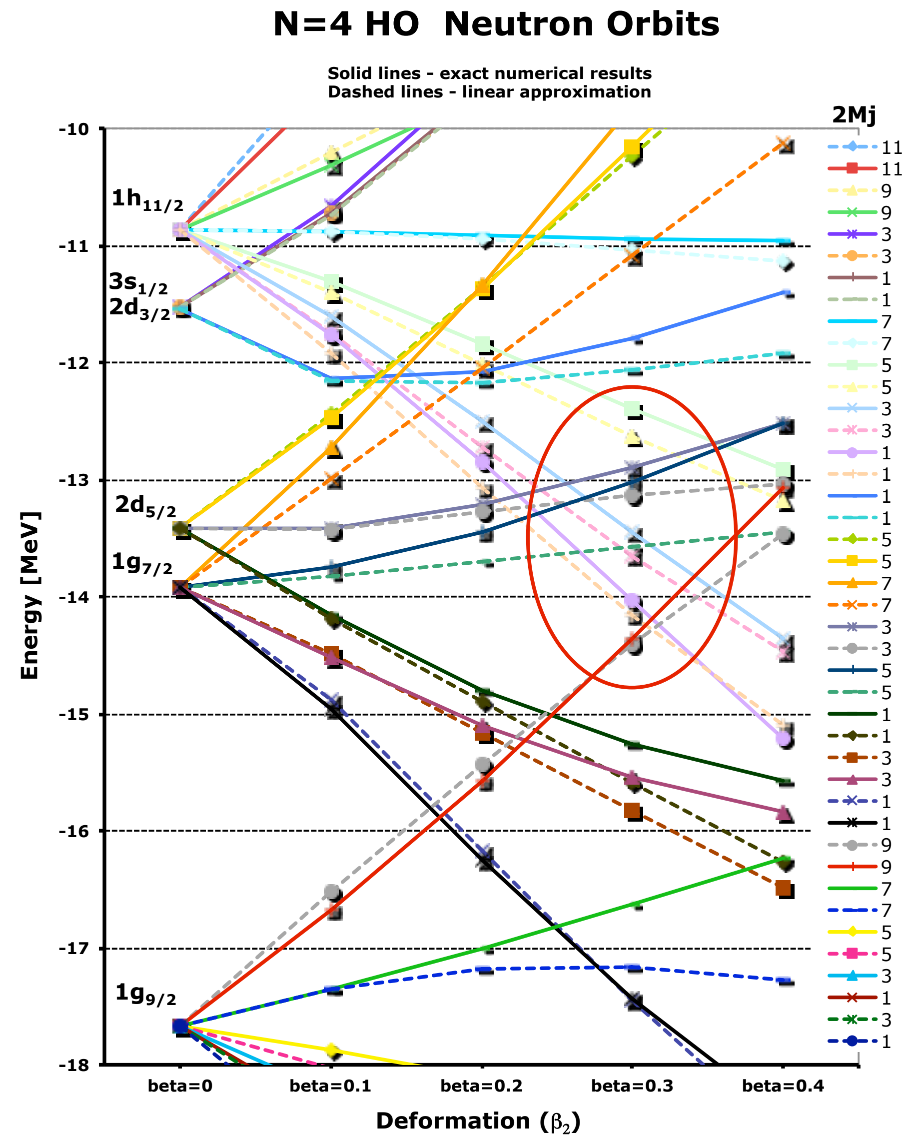
* Essential help from P. D. Kunz, Dept. of Physics &Astrophysics, U. of Colorado, Boulder, CO

# Need for Careful Numerical Treatment

$$V(r) = \frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

$$R(\theta,\phi) = R_0\left(1+\beta_2 Y_{20}(\theta,\phi)+\beta_4 Y_{40}(\theta,\phi)\right)$$



# Neutron Single Particle States

| n | Energy | 2Mj | Parity | n | Energy | 2Mj | Parity | n | Energy | 2Mj | Parity |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -39.706 | 1 | 1 | 20 | **-18.202** | **5** | **1** | 40 | -7.932 | 1 | -1 |
| 2 | -36.096 | 1 | -1 | 21 | -18.096 | 5 | -1 | 41 | -7.867 | 1 | 1 |
| 3 | -34.408 | 3 | -1 | 22 | -17.785 | 3 | -1 | 42 | -7.607 | 3 | -1 |
| 4 | -33.782 | 1 | -1 | 23 | -16.681 | 1 | -1 | 43 | -7.404 | 3 | 1 |
| 5 | -31.401 | 1 | 1 | 24 | **-16.586** | **1** | **1** | 44 | -7.058 | 1 | 1 |
| 6 | -30.202 | 3 | 1 | 25 | **-16.579** | **7** | **1** | 45 | -6.842 | 11 | -1 |
| 7 | -29.193 | 1 | 1 | 26 | **-14.649** | **1** | **1** | 46 | -6.603 | 3 | 1 |
| 8 | -28.322 | 5 | 1 | 27 | **-14.645** | **3** | **1** | 47 | -6.163 | 3 | -1 |
| 9 | -26.503 | 3 | 1 | 28 | **-14.435** | **9** | **1** | 48 | -5.765 | 5 | 1 |
| 10 | -25.983 | 1 | -1 | 29 | -13.723 | 1 | -1 | 49 | -5.367 | 5 | -1 |
| 11 | -25.804 | 1 | 1 | 30 | -13.179 | 3 | -1 | 50 | -4.815 | 1 | -1 |
| 12 | -25.100 | 3 | -1 | 31 | **-12.396** | **3** | **1** | 51 | -4.583 | 7 | 1 |
| 13 | -23.653 | 5 | -1 | 32 | -12.198 | 5 | -1 | 52 | -4.102 | 5 | -1 |
| 14 | -23.370 | 1 | -1 | 33 | **-12.073** | **5** | **1** | 53 | -3.076 | 9 | 1 |
| 15 | -21.625 | 7 | -1 | 34 | **-11.033** | **1** | **1** | 54 | -2.665 | 7 | -1 |
| 16 | -21.116 | 3 | -1 | 35 | -10.829 | 7 | -1 | 55 | -2.512 | 1 | 1 |
| 17 | -20.678 | 1 | -1 | 36 | **-9.785** | **5** | **1** | 56 | -2.483 | 1 | -1 |
| 18 | -20.056 | 1 | 1 | 37 | -9.308 | 1 | -1 | 57 | -1.901 | 3 | -1 |
| 19 | -19.379 | 3 | 1 | 38 | -9.072 | 9 | -1 | 58 | -1.772 | 7 | -1 |
| | | | | 39 | **-8.791** | **7** | **1** | 59 | -1.224 | 11 | 1 |
| | | | | | | | | 60 | -1.103 | 3 | 1 |
| | | | | | | | | 61 | -0.627 | 1 | 1 |
| | | | | | | | | 62 | -0.521 | 3 | -1 |
| | | | | | | | | 63 | -0.171 | 1 | -1 |
| | | | | | | | | 64 | 0.120 | 1 | -1 |

## $^{157}$Gd

**Unpaired valence neutron at level 47.**
**A=157, N=93, Z=64**

**$^{157}$Gd: $S_n$ = 6.36 MeV**

**$E_n$=0 … 1 MeV**

**n+$^{155}$Gd=**
**=$^{156}$Gd+$E_{ext}$**
**$^{157}$Gd-n=**

**$E_{ext}$=8.5 … 9.5 MeV**

**$^{156}$Gd: $S_n$ = 8.536 MeV**

**$BE_n$ ~ -14.8987 MeV single particle levels relevant for the surrogate reaction**

# Excited states in $^{156}$Gd

## *model: neutron hole in the $^{157}$Gd core*

$$|^{156}Gd, \Omega = |K \pm \nu| > \; = \; \psi^\dagger_{\pm\nu} |^{157}Gd, K = 3/2^- >$$

$$E(J^\pi; \Omega = |K \pm \nu|) \; = \; \epsilon_0 - \epsilon_\nu + \frac{\hbar^2}{2\mathcal{I}}(J(J+1) + \delta_\pm)$$

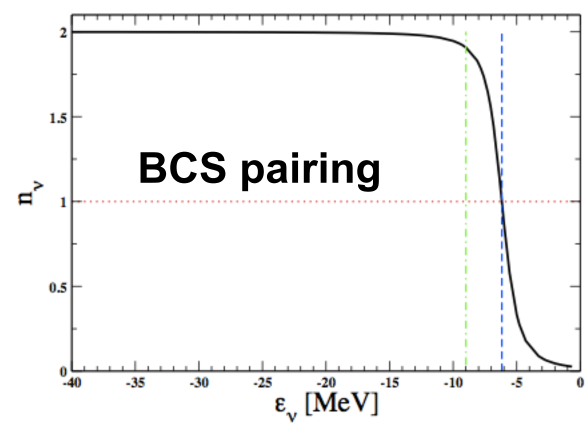
| $J^\pi_n$ | $E_{exp}$ [MeV] | $E_{th}$ [MeV] | $J^\pi_n$ | $E_{exp}$ [MeV] | $E_{th}$ [MeV] |
|---|---|---|---|---|---|
| $0^+_{gs}$ | 0 | 0.004 | $3^+_1$ | 1.248 | 1.255 |
| $2^+_1$ | 0.089 | 0.085 | $4^+_3$ | 1.355 | 1.363 |
| $4^+_1$ | 0.288 | 0.275 | $5^+_1$ | 1.507 | 1.499 |
| $6^+_1$ | 0.585 | 0.574 | $6^+_3$ | 1.644 | 1.662 |
| $8^+_1$ | 0.965 | 0.982 | $7^+_1$ | 1.85 | 1.852 |

TABLE III: Comparison of the experimentally-observed low-energy states in $^{156}$Gd [14] and excitation energies calculated considering a neutron hole $\nu = 3/2^-$ at $\epsilon_{47} = -6.361$ coupled to the core system $^{157}$Gd with $K = 3/2^-$. The first three columns represent the $\Omega = 0^+$ ground state band; the next three columns represent the $\Omega = 3^+$ band. The theoretical ground state is not exactly zero because $\epsilon_{47}$ is not exactly equal the neutron separation energy $S_n$ in $^{157}$Gd.

**Coriolis coupling**

$$H_C = -\frac{\hbar^2}{2\mathcal{I}}(I_+ j_- + I_- j_+).$$

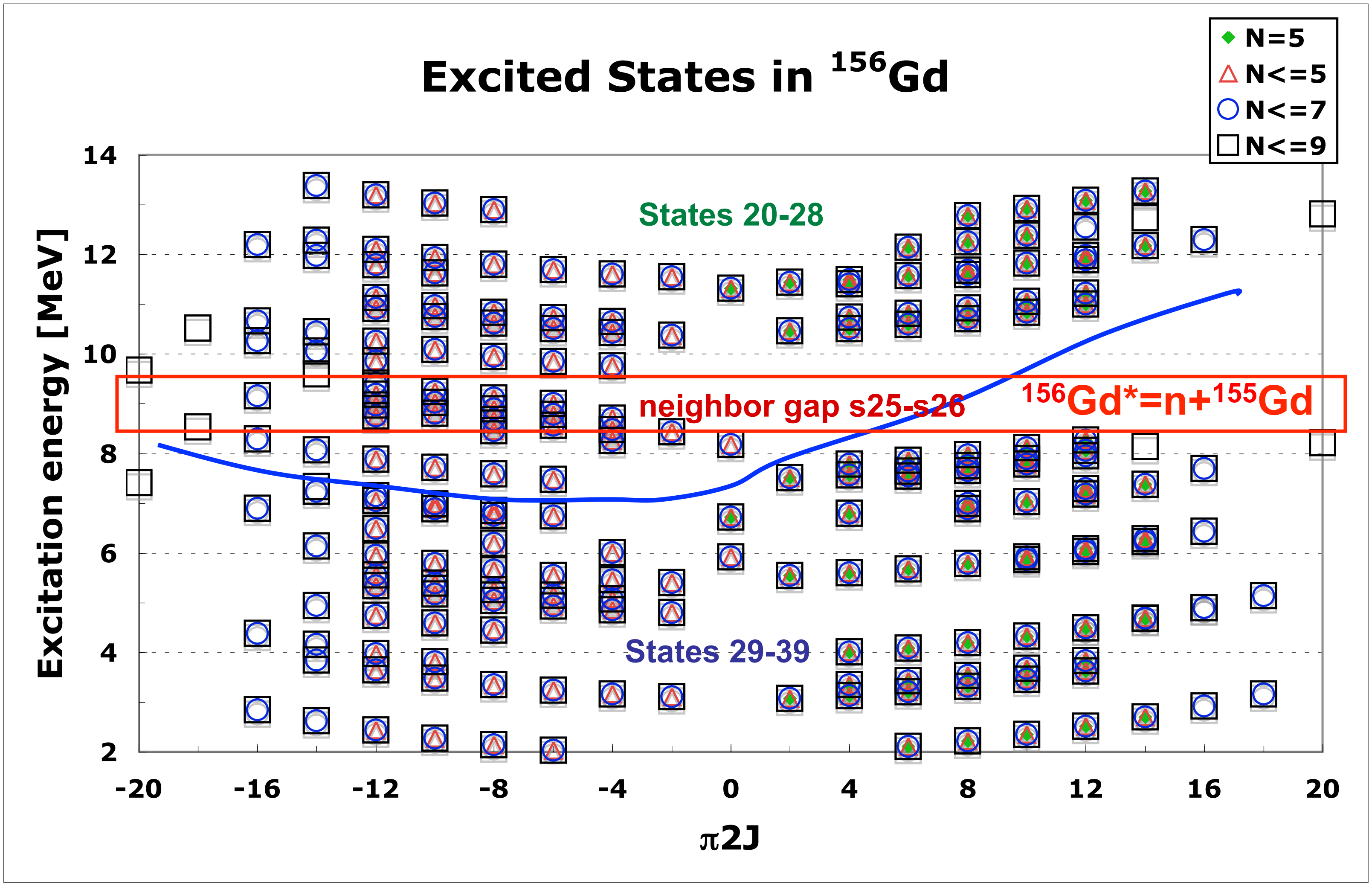
$$\Delta E(\Omega_\pm = |K \pm \nu|) = \pm c \frac{\hbar^2}{2\mathcal{I}} K\nu.$$




$$n_\nu = 1 - \frac{\epsilon_\nu - \mu}{\sqrt{(\epsilon_\nu - \mu)^2 + \Delta^2}}$$

# Excited States in $^{156}$Gd

$$E_{\nu}^{J,K=\Omega_{\nu}\pm\Omega_{0}} = E_{F} - BE_{\nu} + \frac{\hbar^{2}}{2\wp_{1,2}}J(J+1) \pm \frac{\Omega_{\nu}\Omega_{0}}{\wp_{3}}$$

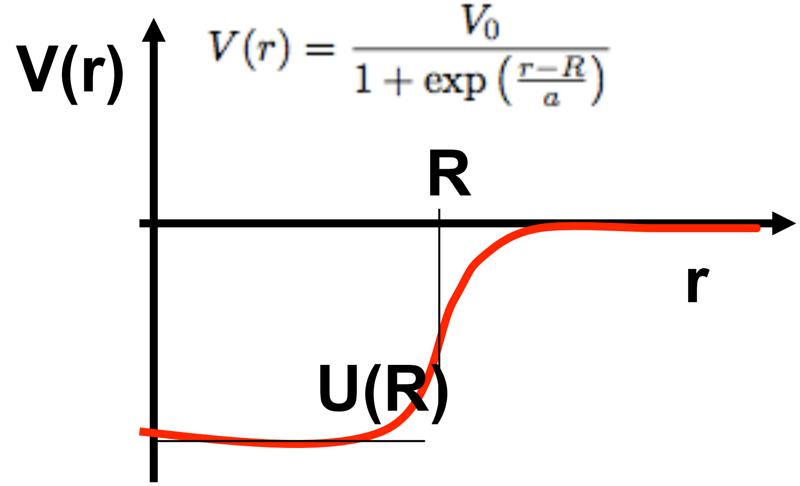


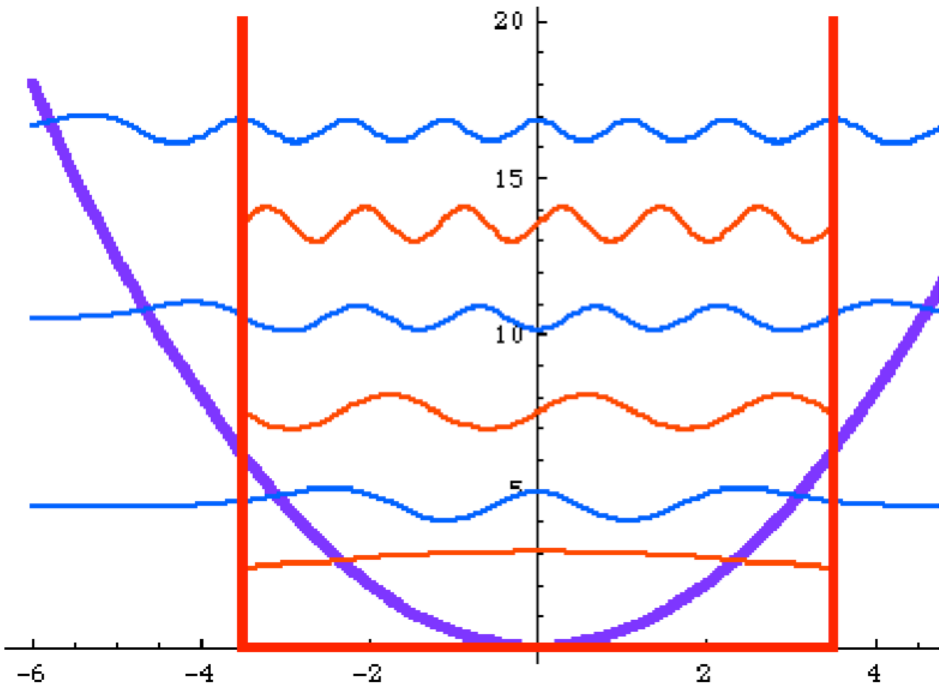
# Wave Function Spreading

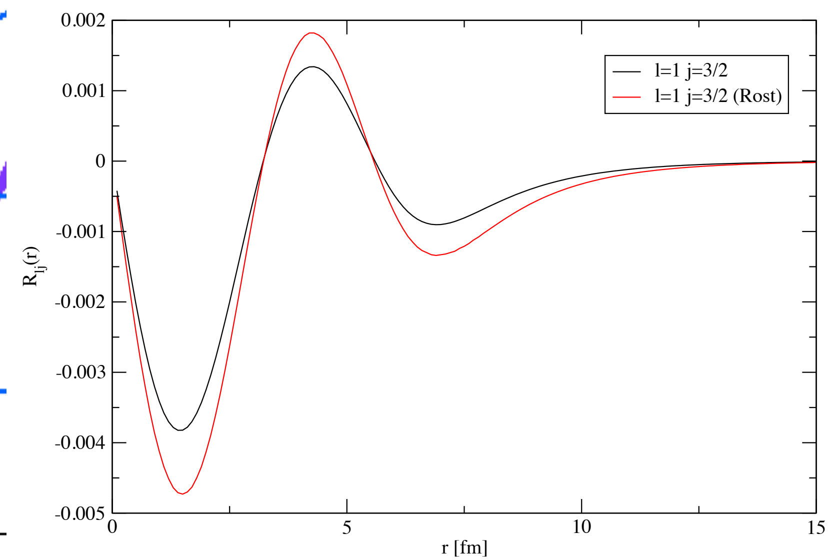
$$H_1 = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}q^2.$$

$$\Psi_n(q) = \sqrt{\frac{1}{bn!\,2^n\sqrt{\pi}}}\,H_n\!\left(\frac{q}{b}\right)\exp\!\left(-\frac{1}{2}\frac{q^2}{b^2}\right),$$

$$E_n^{\mathrm{HO}} = \hbar\,\omega\left(n+\frac{1}{2}\right),\quad b=\sqrt{\frac{\hbar}{m\omega}},$$



# Model space convergence in Sturmian basis

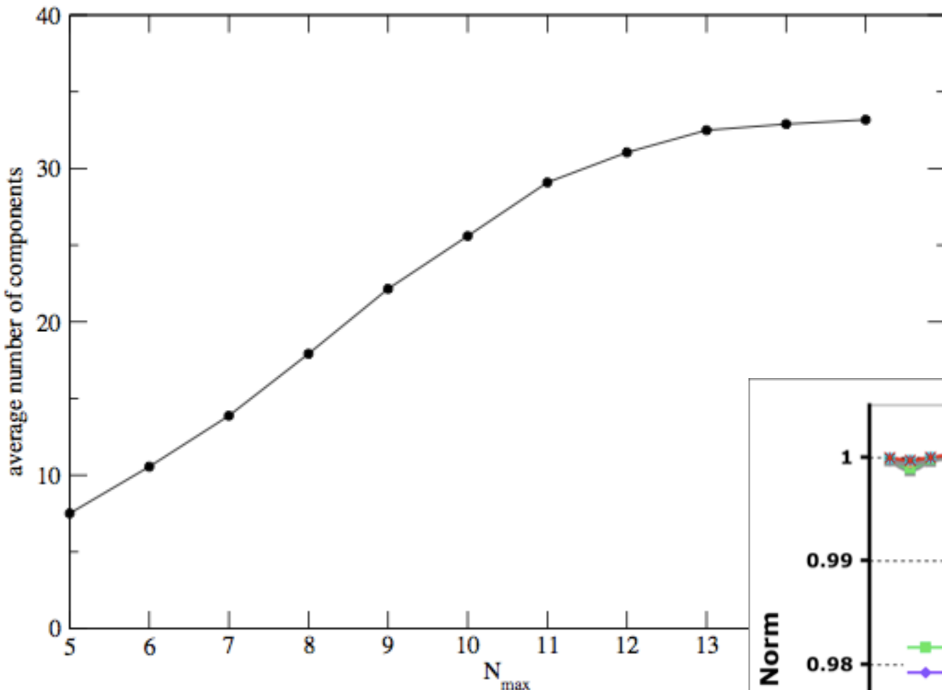
B. L. Andersen, B. B. Back, and J. M. Bang, Nucl. Phys. A 147, 33 (1970).

$$\psi_{\varepsilon m} = \sum_{nlj} c^{nlj} \phi_{\varepsilon nljm}$$

$$H_{\beta \neq 0} \psi_{\varepsilon \Omega} = \varepsilon \psi_{\varepsilon \Omega}$$

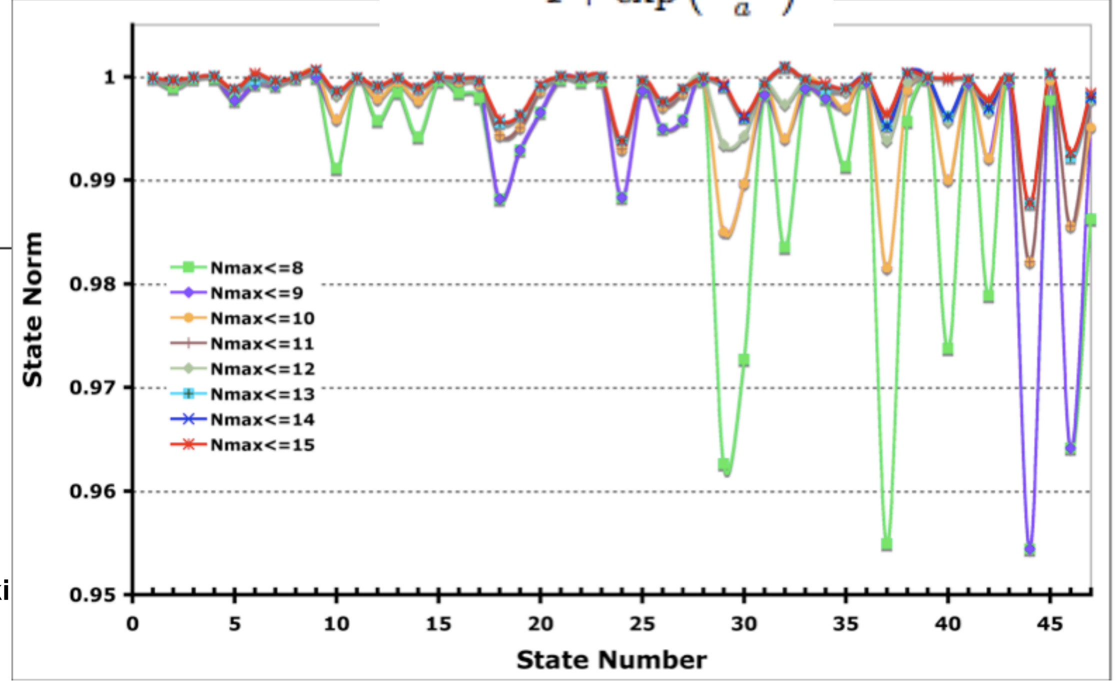
$$\boxed{H_{\beta \equiv 0}(V_0)\phi_{V_0;\varepsilon nljm} = \varepsilon \phi_{V_0;\varepsilon nljm}}$$

$$V(r) = \frac{V_0}{1 + \exp\left(\frac{r-R}{a}\right)}$$



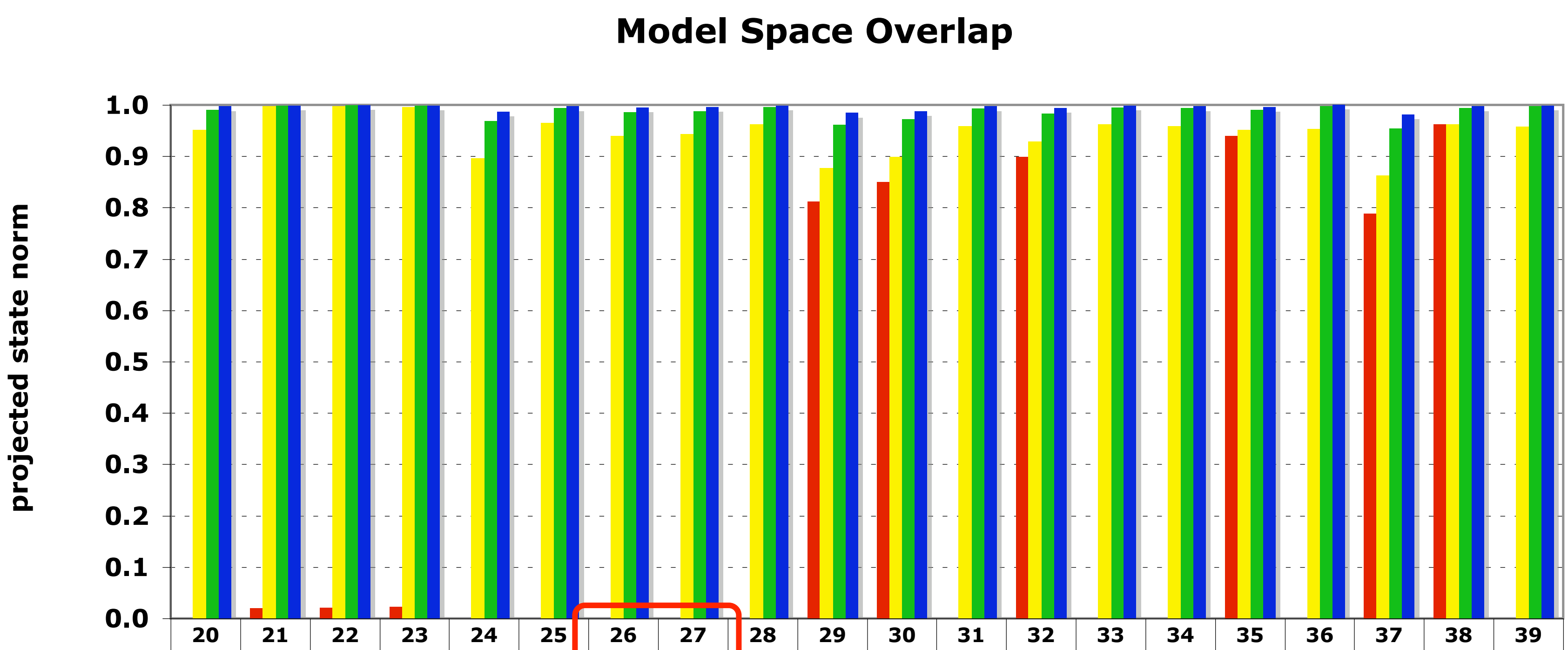

$$\phi_{\varepsilon nljm} = \sum_{NN_z\Lambda} D_{nljm}^{NN_z\Lambda} |NN_z\Lambda m\rangle$$

$$\psi_{\varepsilon \Omega} = \sum_{NN_z\Lambda} B_{\Omega}^{NN_z\Lambda} |NN_z\Lambda \Omega\rangle$$




WSBETA: S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski and T. Werner, Comp. Phys. Comm. 46, 379 (1987).

# s.p.s. in Sturmian basis



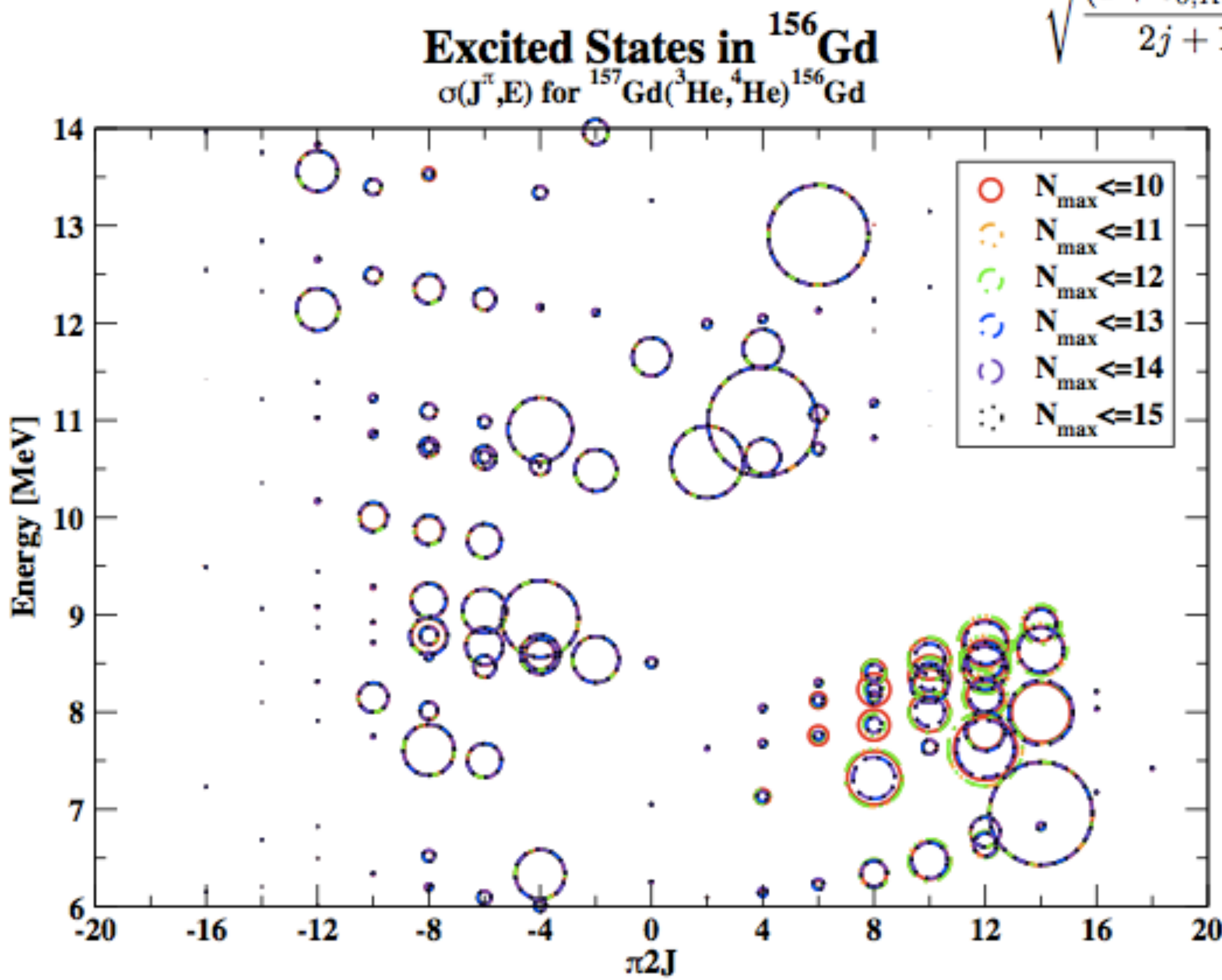

| | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nmax=5 | | 0.020 | 0.021 | 0.023 | | | | | | 0.812 | 0.850 | | 0.900 | | | 0.939 | | 0.789 | 0.963 | |
| Nmax<=5 | 0.95 | 1 | 1 | 1 | 0.9 | 0.96 | 0.94 | 0.94 | 0.96 | 0.88 | 0.9 | 0.96 | 0.93 | 0.96 | 0.96 | 0.95 | 0.95 | 0.86 | 0.96 | 0.96 |
| Nmax<=7 | 0.991 | 0.999 | 1.000 | 0.999 | 0.969 | 0.995 | 0.986 | 0.988 | 0.997 | 0.962 | 0.972 | 0.993 | 0.984 | 0.995 | 0.994 | 0.990 | 0.998 | 0.955 | 0.995 | 0.998 |
| Nmax<=9 | 0.997 | 0.999 | 1.000 | 0.999 | 0.988 | 0.998 | 0.995 | 0.996 | 0.999 | 0.985 | 0.989 | 0.998 | 0.994 | 0.999 | 0.998 | 0.996 | 1.001 | 0.982 | 0.998 | 0.999 |

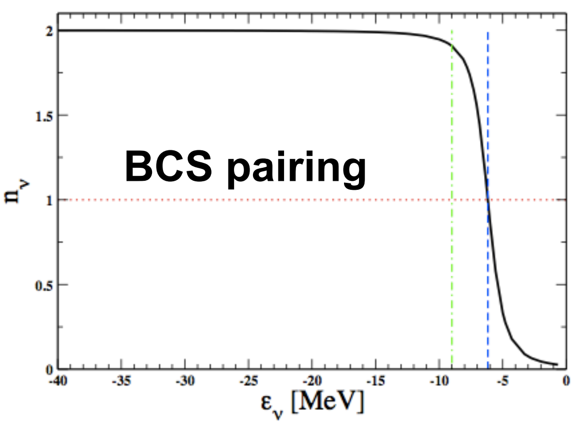
deformed state

# Individual x-sections

$$d\sigma(J_iK_i \to J_fK_f;\nu) = \sum_{lj}\sum_{n}(a_\nu v_\nu c_\nu^{nlj})^2 d\sigma_{nlj}^{DW}$$

$$\sqrt{\frac{(1+\delta_{0,K_iK_f})}{2j+1}}D_0 \times c_{nlj}^{\nu} \times (J_fK_f|jm,J_i,K_i).$$



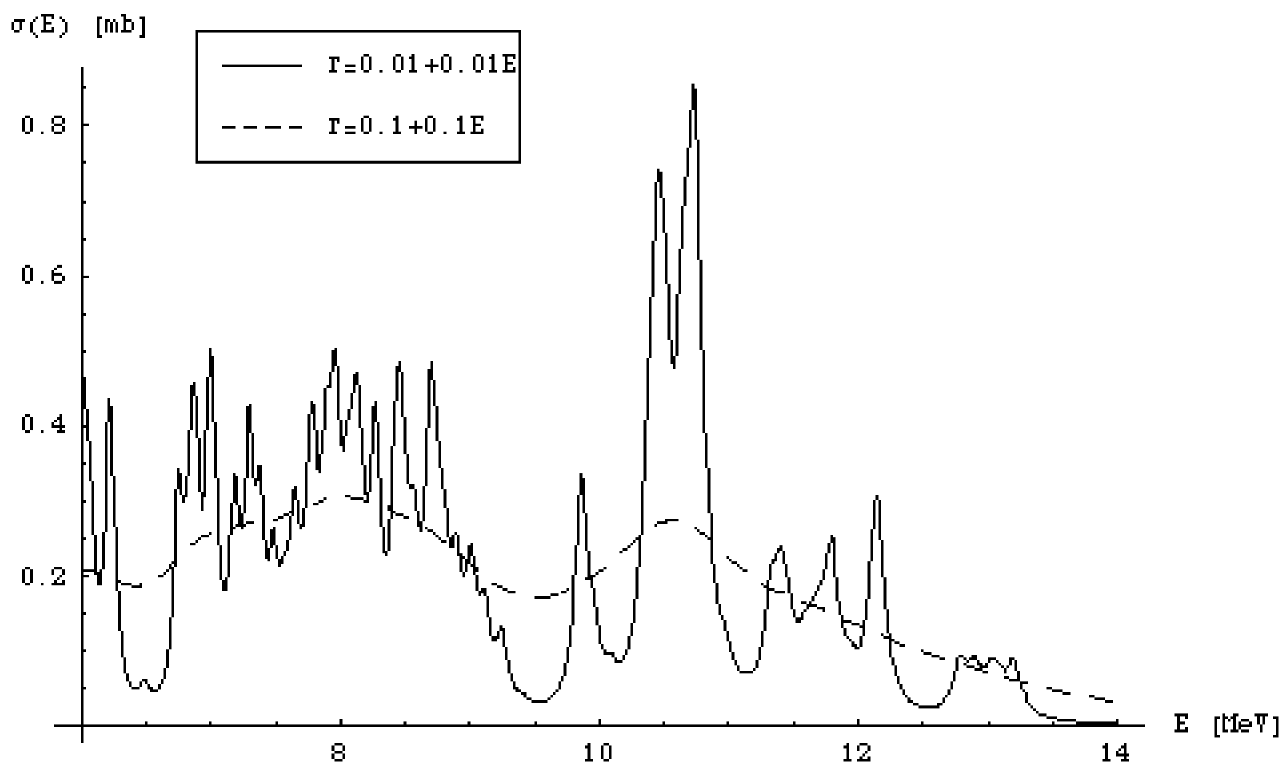




$$n_\nu = 1 - \frac{\epsilon_\nu - \mu}{\sqrt{(\epsilon_\nu-\mu)^2+\Delta^2}}$$

# Total cross section σ(E)

*smearing function* $\rho(E,\Gamma)$ *dependence*

$$\sigma(E)=\sum_{\nu}\rho_{\nu}(E)\sigma_{\nu},\ \ \rho_{\nu}(E)=\frac{1}{2\pi}\frac{4\Gamma}{4(E-E_{\nu})^{2}-\Gamma^{2}},\ \ \Gamma=a+bE$$

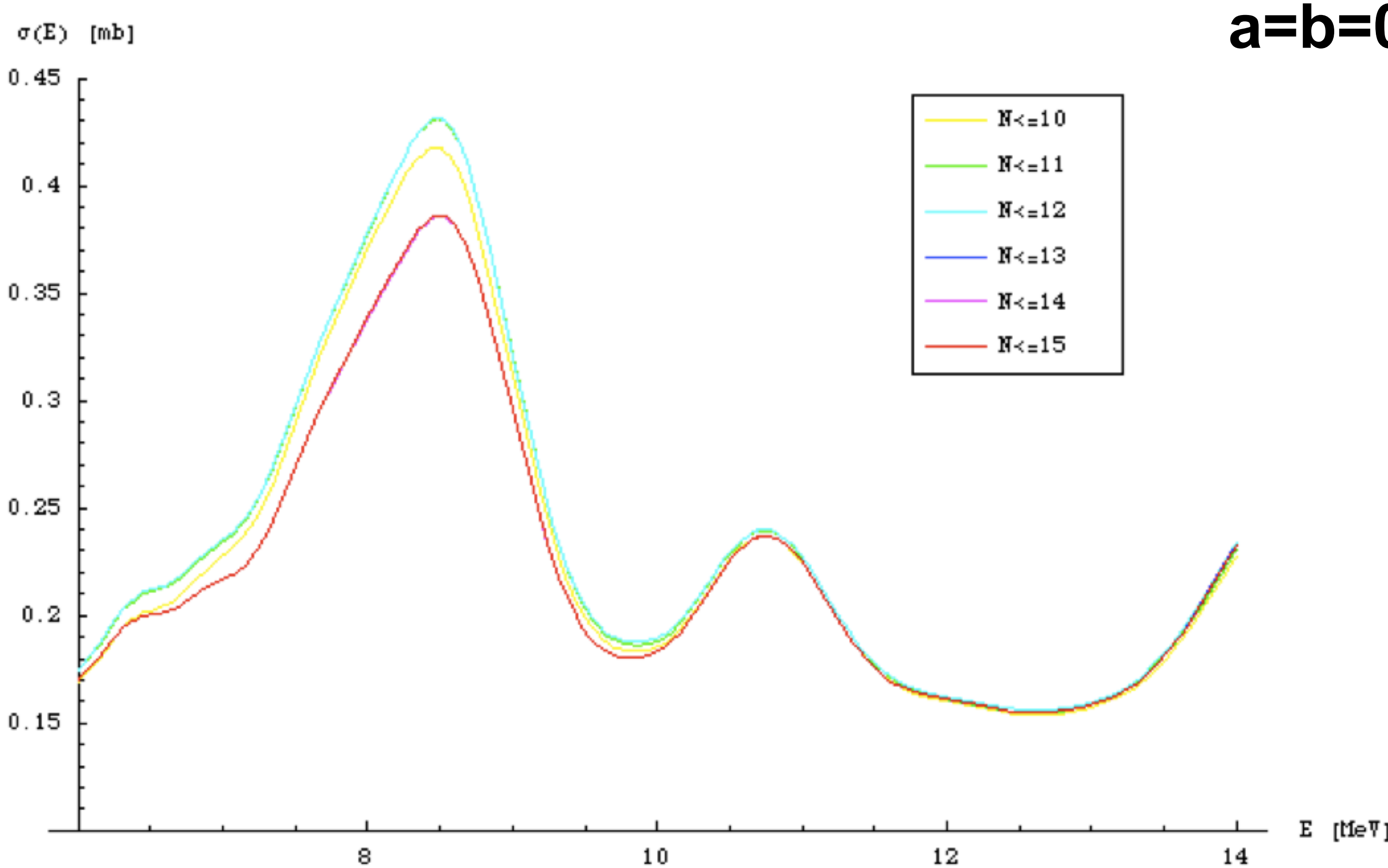


# Total cross section σ(E)

*model space N-dependence*

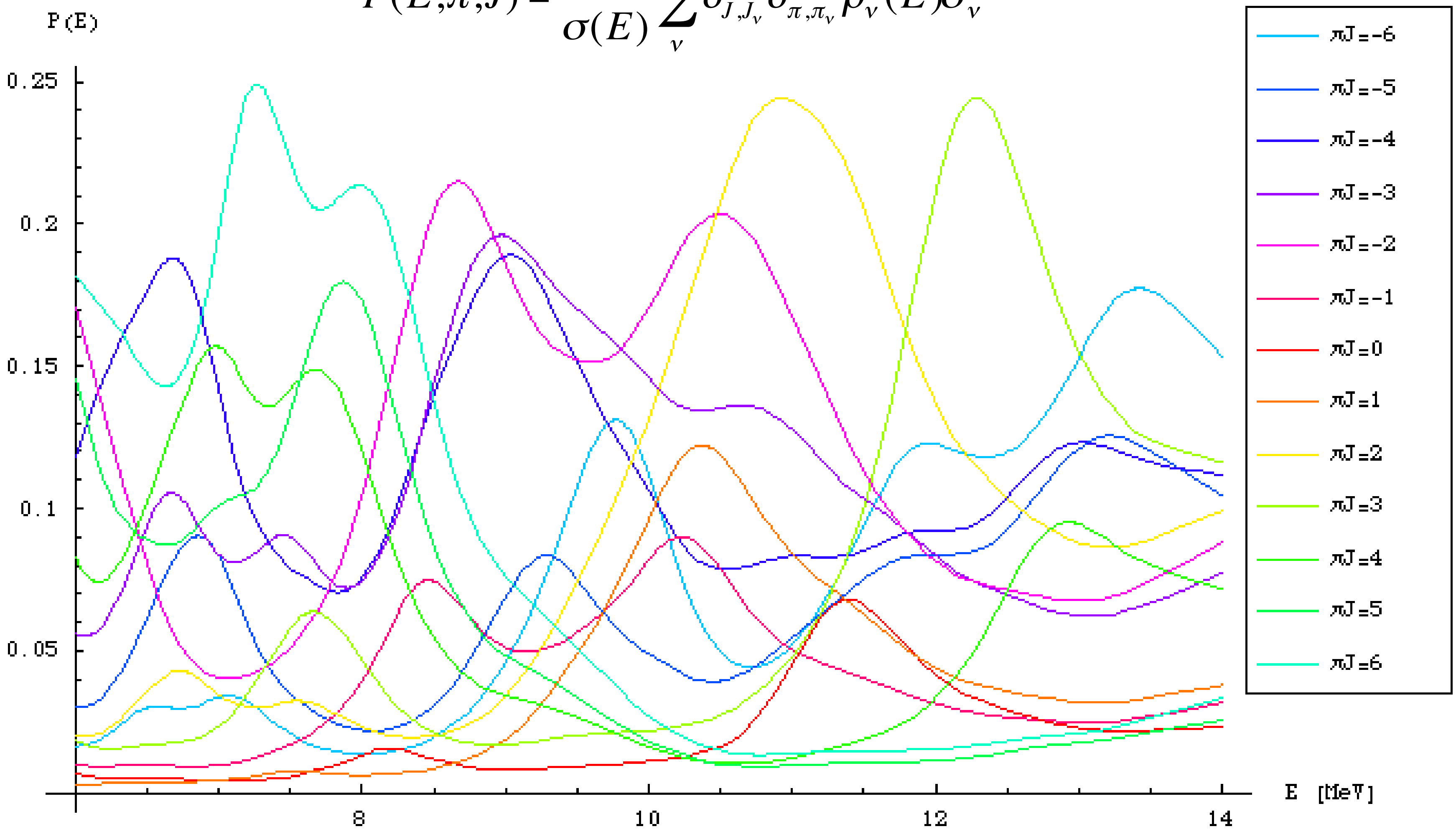
$$\sigma(E) = \sum_{\nu} \rho_{\nu}(E)\sigma_{\nu}, \;\; \rho_{\nu}(E) = \frac{1}{2\pi}\frac{4\Gamma}{4(E-E_{\nu})^2 - \Gamma^2}, \;\; \Gamma = a + bE$$

**a=b=0.1**

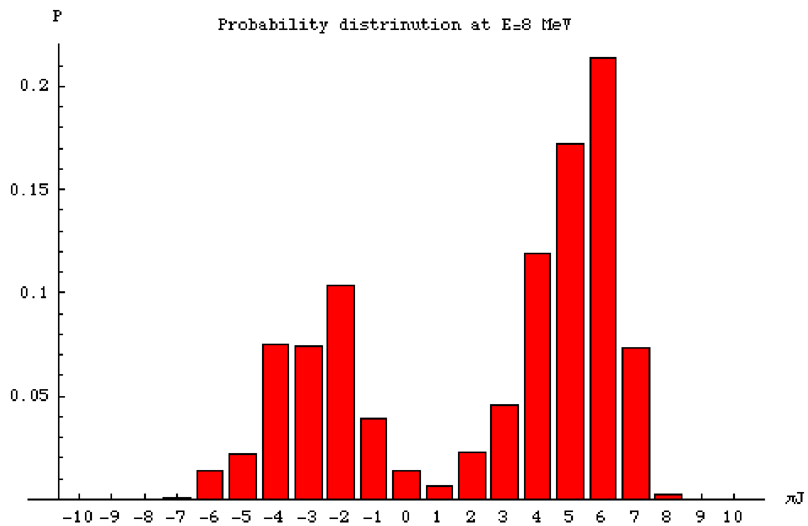


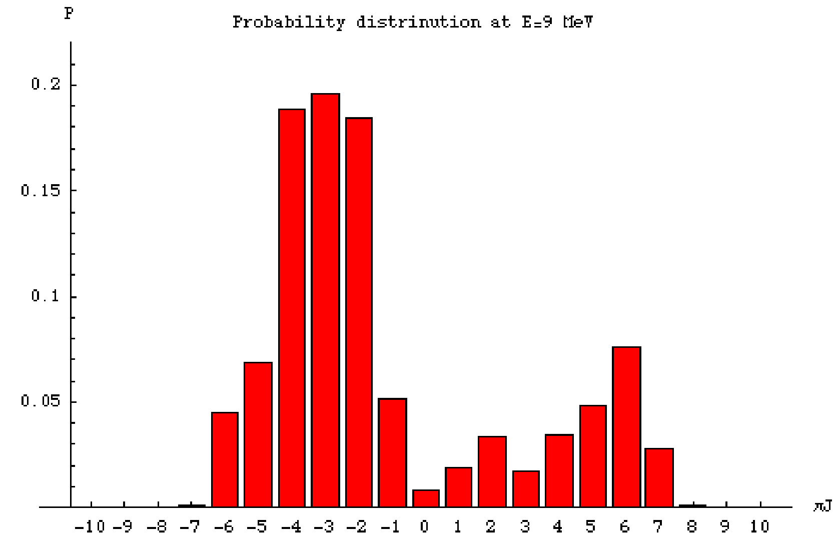
# $P(J^{\pi}, E)$ Distribution

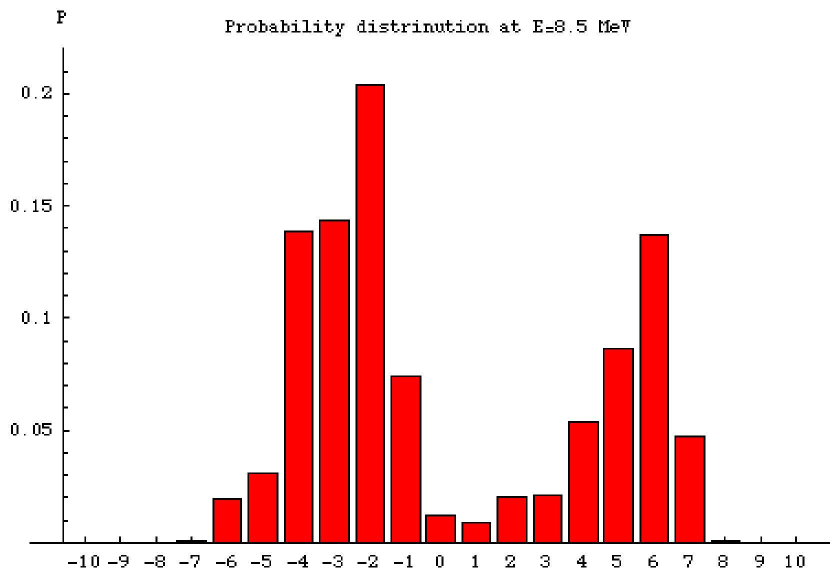
$$P(E,\pi,J) = \frac{1}{\sigma(E)} \sum_{\nu} \delta_{J,J_\nu} \delta_{\pi,\pi_\nu} \rho_\nu(E) \sigma_\nu$$

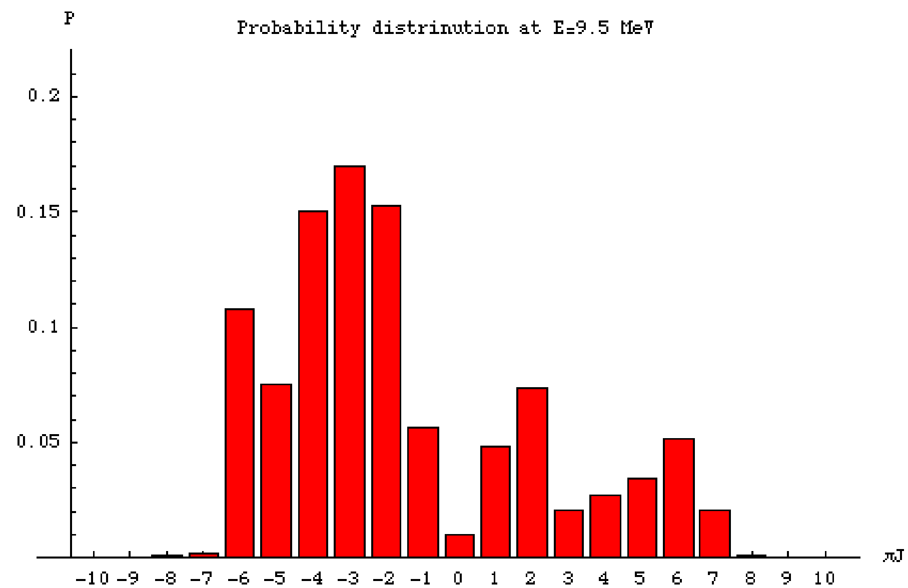


# $P(J^{\pi})$ Distributions E= 8, 8.5, 9, 9.5 MeV



# Outlook

- Sturmian basis:
  - Guarantees the correctness of the wave-function tail;
  - Provides **fast convergence**!
  - Consistency checks:
    - Calculate basis overlap matrix for DWUCK w.f.
    - Compare the basis overlap matrices

- Need better understanding of:
  - the norm issue for few of the states;
  - E-deviations for large model spaces;
  - Smearing function for comparison to experiments!
  - When are the Pairing and Coriolis mixing important?

- Compare to other methods, codes, and experiments …
- **Study the x-sections for the desired reaction n+$^{155}$Gd!**