Models with spatially unfavoured bosons

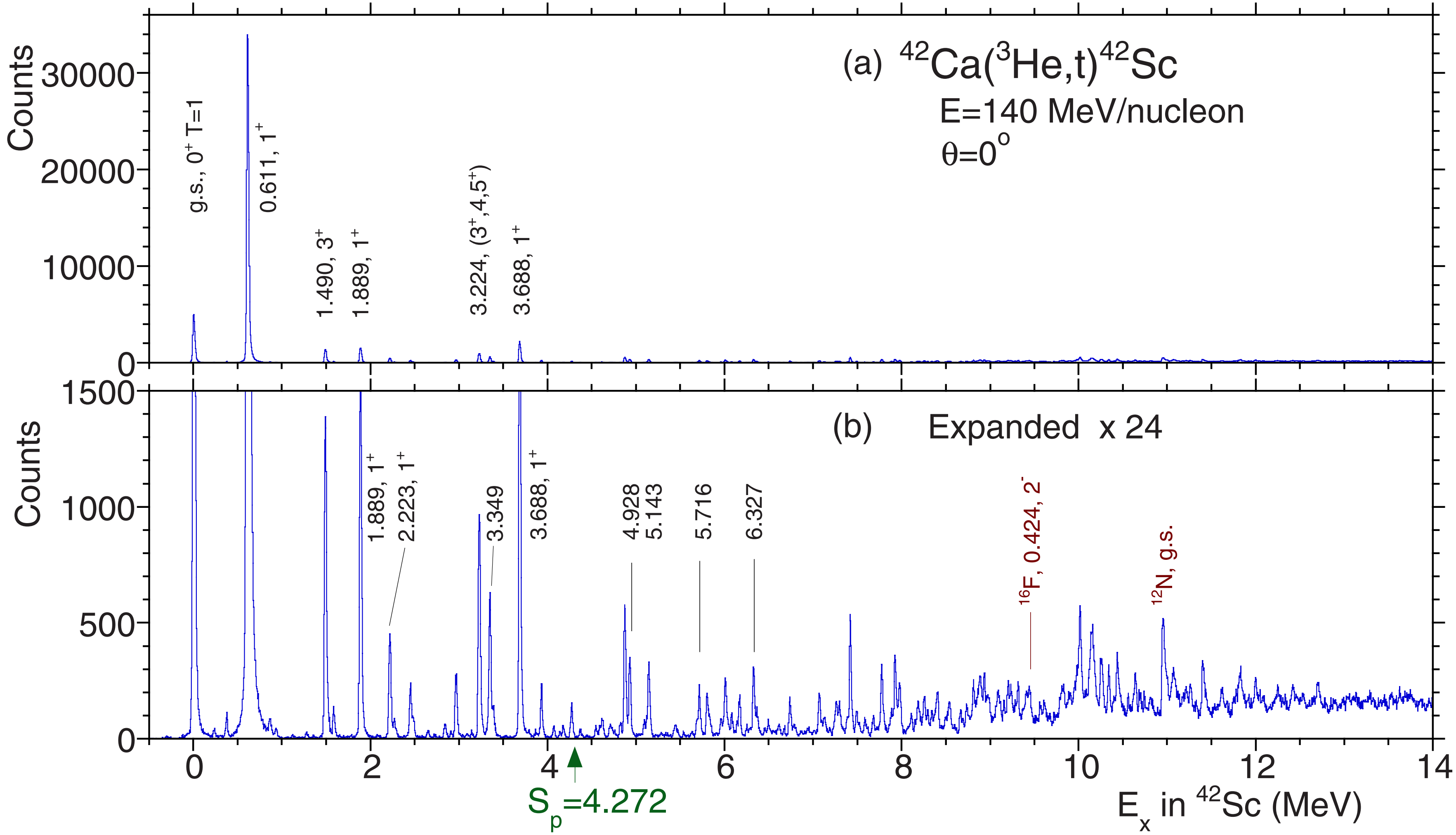
P. Van Isacker, GANIL, France

Charge exchange & deuteron transfer for $N=Z$

Two-shell fermionic model

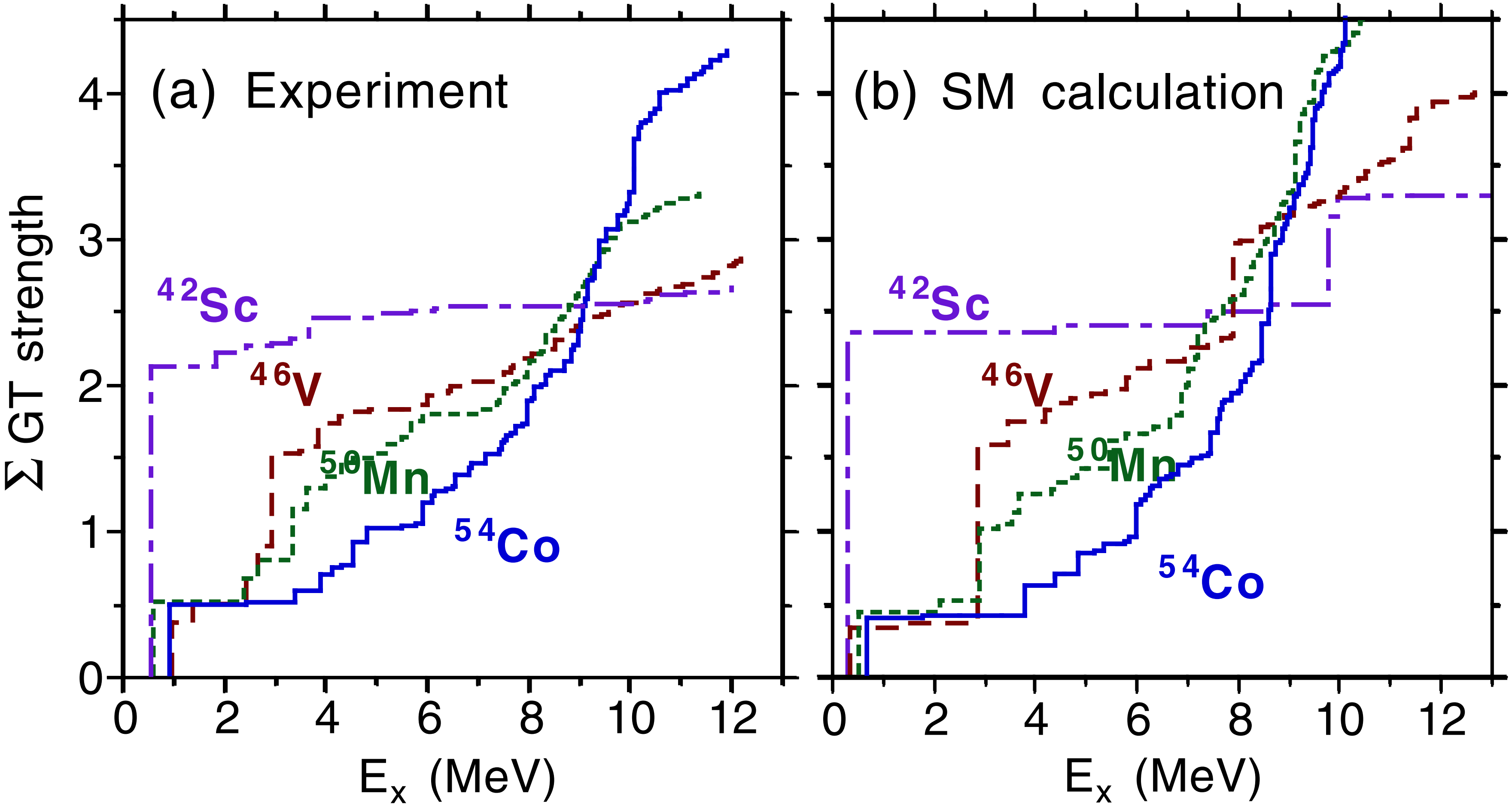
Models with spatially unfavoured bosons

Charge-exchange reactions



Y. Fujita *et al.*, Phys. Rev. Lett. **112** (2014) 112502

Charge-exchange reactions



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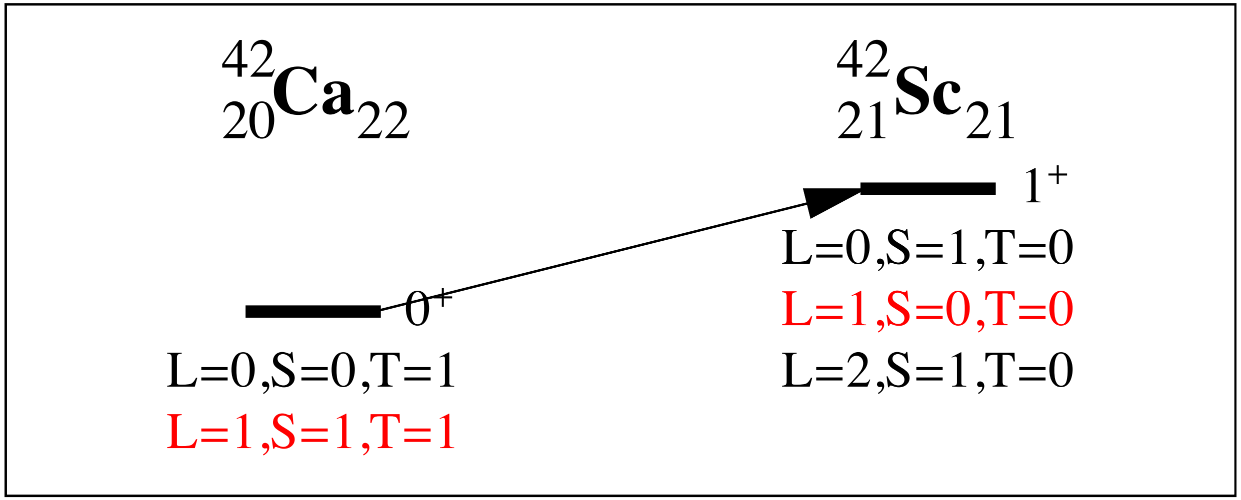
A two-shell model

Consider two *j* shells with $j=l\pm½$.

In *LS* coupling two-nucleon states $|l^2LSJT\rangle$ have $L+S+T$ = odd.

The initial and final states in ^{42}Ca and ^{42}Sc are admixtures:



A two-shell model

The spin-orbit interaction mixes favoured and unfavoured $|l^2LSJT\rangle$ states:

$$\hat{H}_{\text{so}} = \varepsilon_- \hat{n}_- + \varepsilon_+ \hat{n}_+ = \Delta\varepsilon \tfrac{1}{2}\left(\hat{n}_- - \hat{n}_+\right) + \bar{\varepsilon}\hat{n}$$

Energy matrices:

$$^{42}\text{Ca}\left(0^+\right): \quad 2\bar{\varepsilon} + \frac{\Delta\varepsilon}{2l+1}\begin{bmatrix} -1 & \sqrt{4l(l+1)} \\ \sqrt{4l(l+1)} & +1 \end{bmatrix}$$

$$^{42}\text{Sc}\left(1^+\right): \quad 2\bar{\varepsilon} + \frac{\Delta\varepsilon}{3(2l+1)}\begin{bmatrix} -3 & \sqrt{12l(l+1)} & 0 \\ \sqrt{12l(l+1)} & -3 & \sqrt{6(2l-1)(2l+3)} \\ 0 & \sqrt{6(2l-1)(2l+3)} & +6 \end{bmatrix}$$

A two-shell model

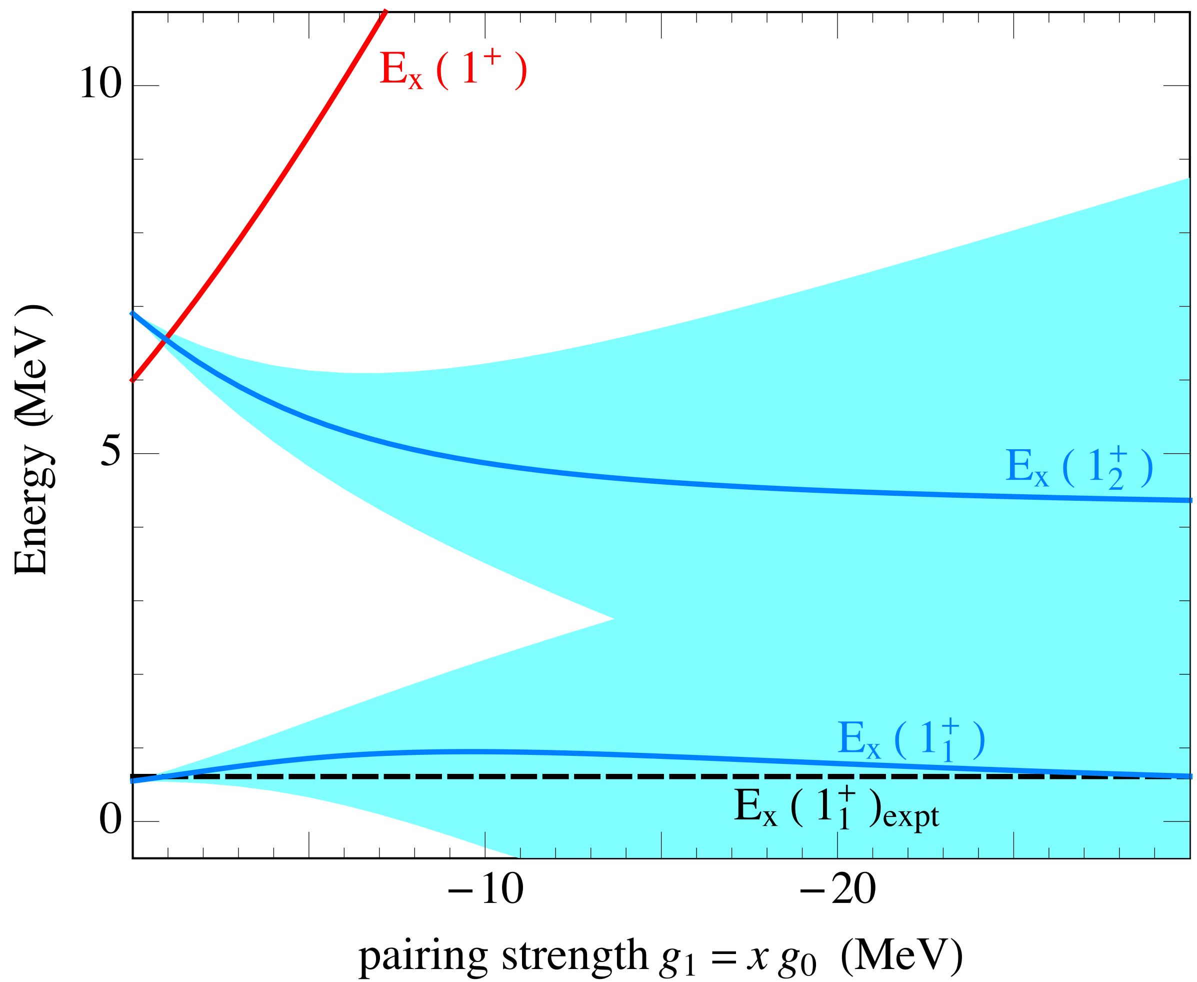
A schematic SM hamiltonian with a spin-orbit splitting, isoscalar and isovector pairing terms:

$$\hat{H} = \hat{H}_{\mathrm{so}} + g_0 P^{+}_{T=0} P_{T=0} + g_1 P^{+}_{T=1} P_{T=1} + \cdots$$

Can we describe observed energies, M1 transitions, GT strength and transfer cross sections in a consistent fashion?

$$\Delta^{(3)} = \frac{1}{2}\left[\mathrm{BE}\left({}^{42}\mathrm{Ca}\right) + \mathrm{BE}\left({}^{40}\mathrm{Ca}\right) - 2\mathrm{BE}\left({}^{41}\mathrm{Sc}\right)\right]$$

Energies



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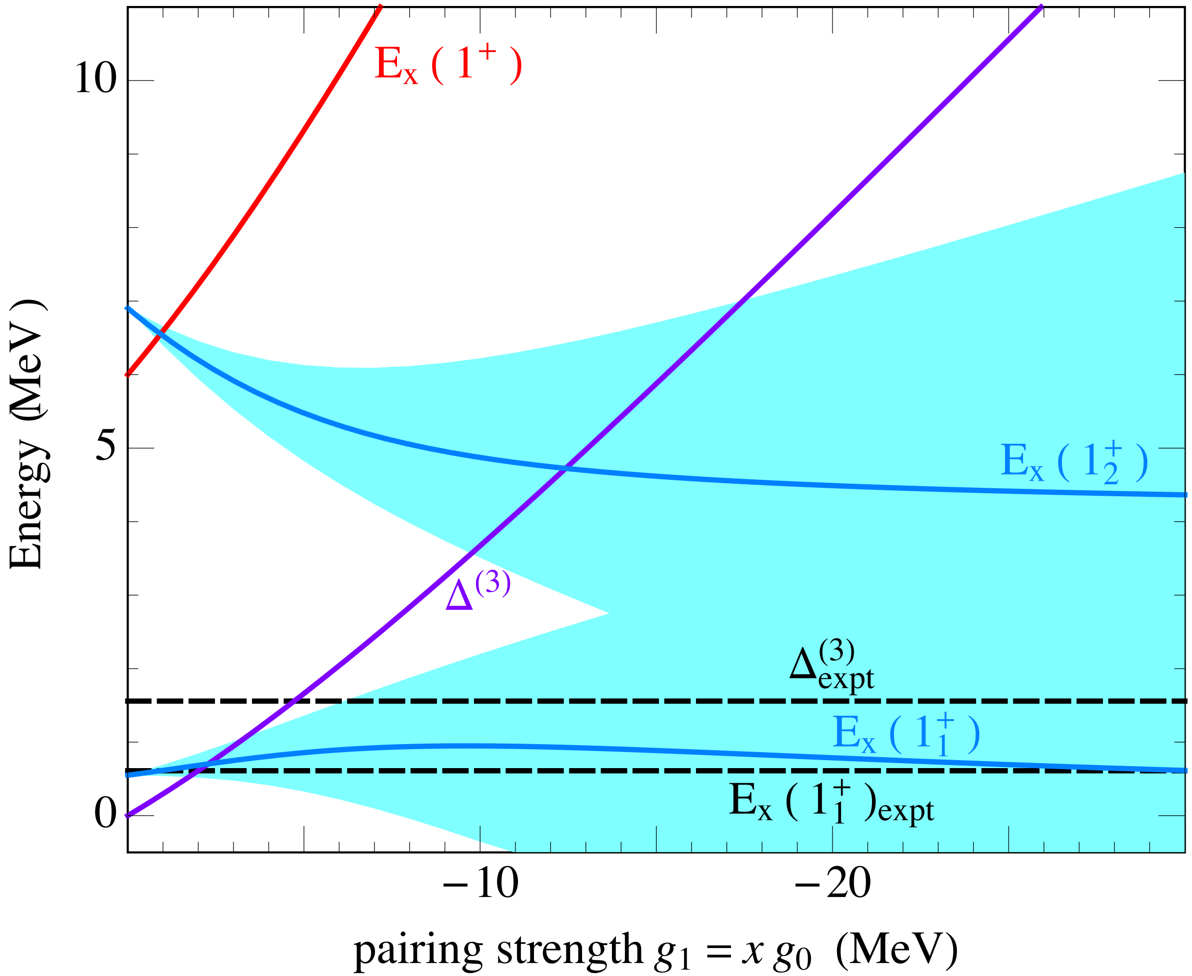

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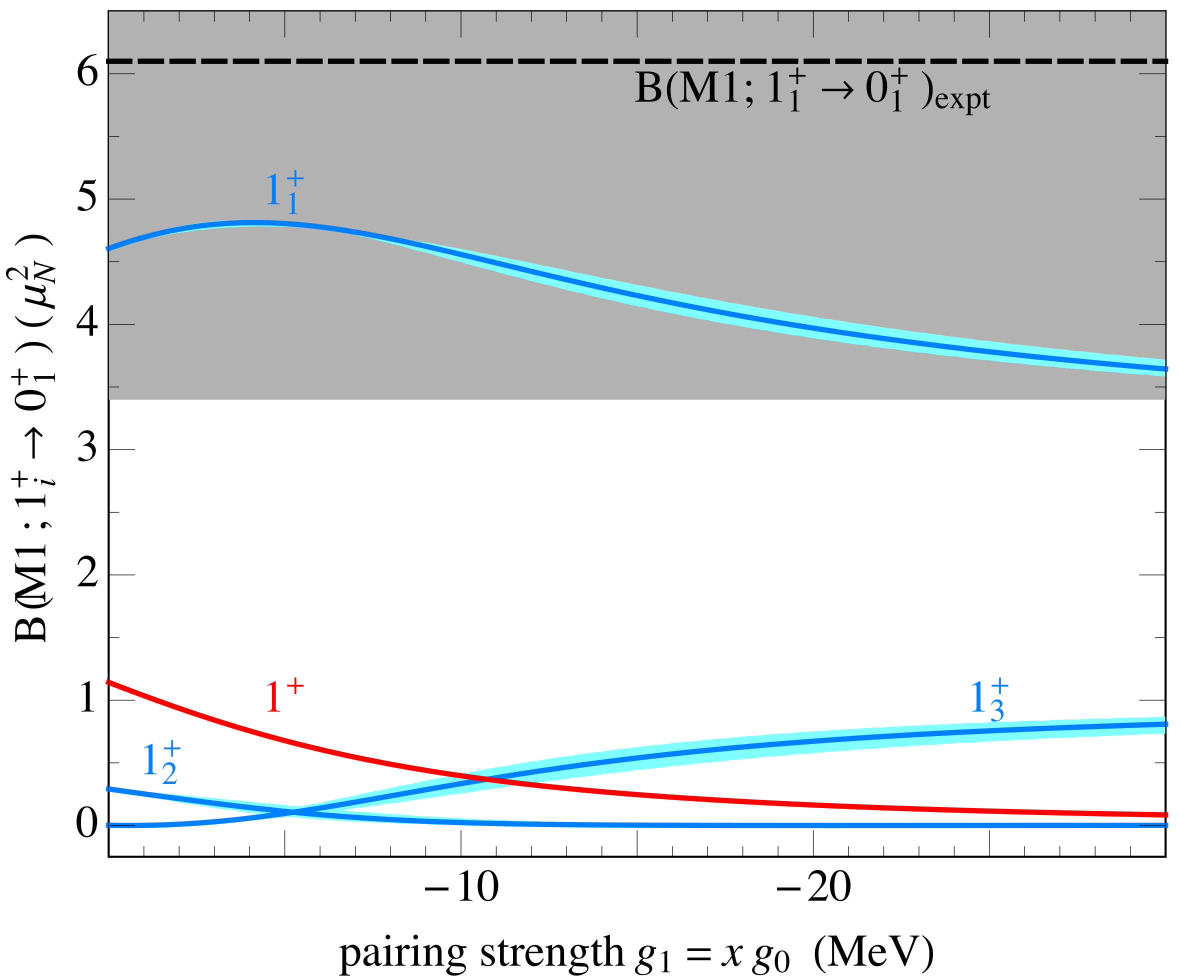
M1 transitions



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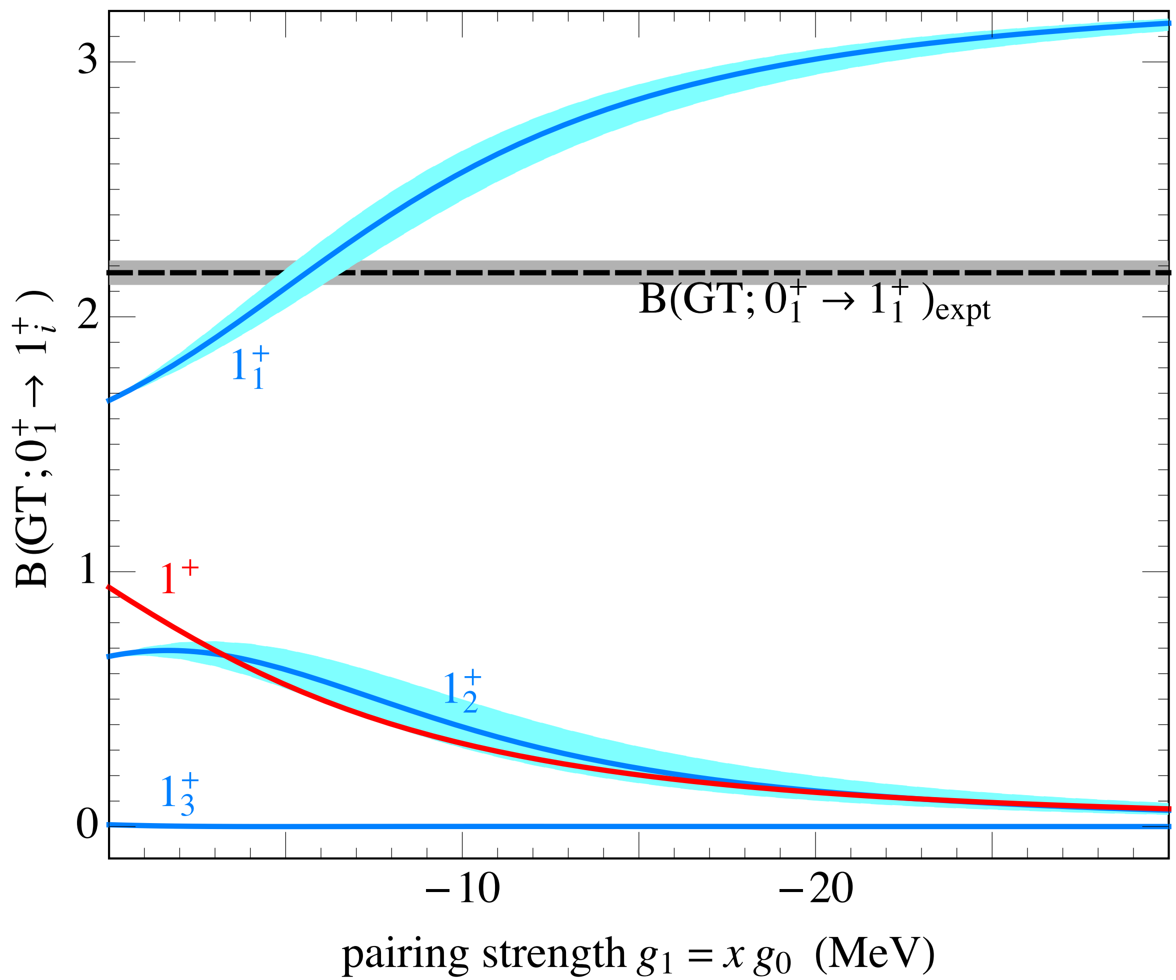
GT strength



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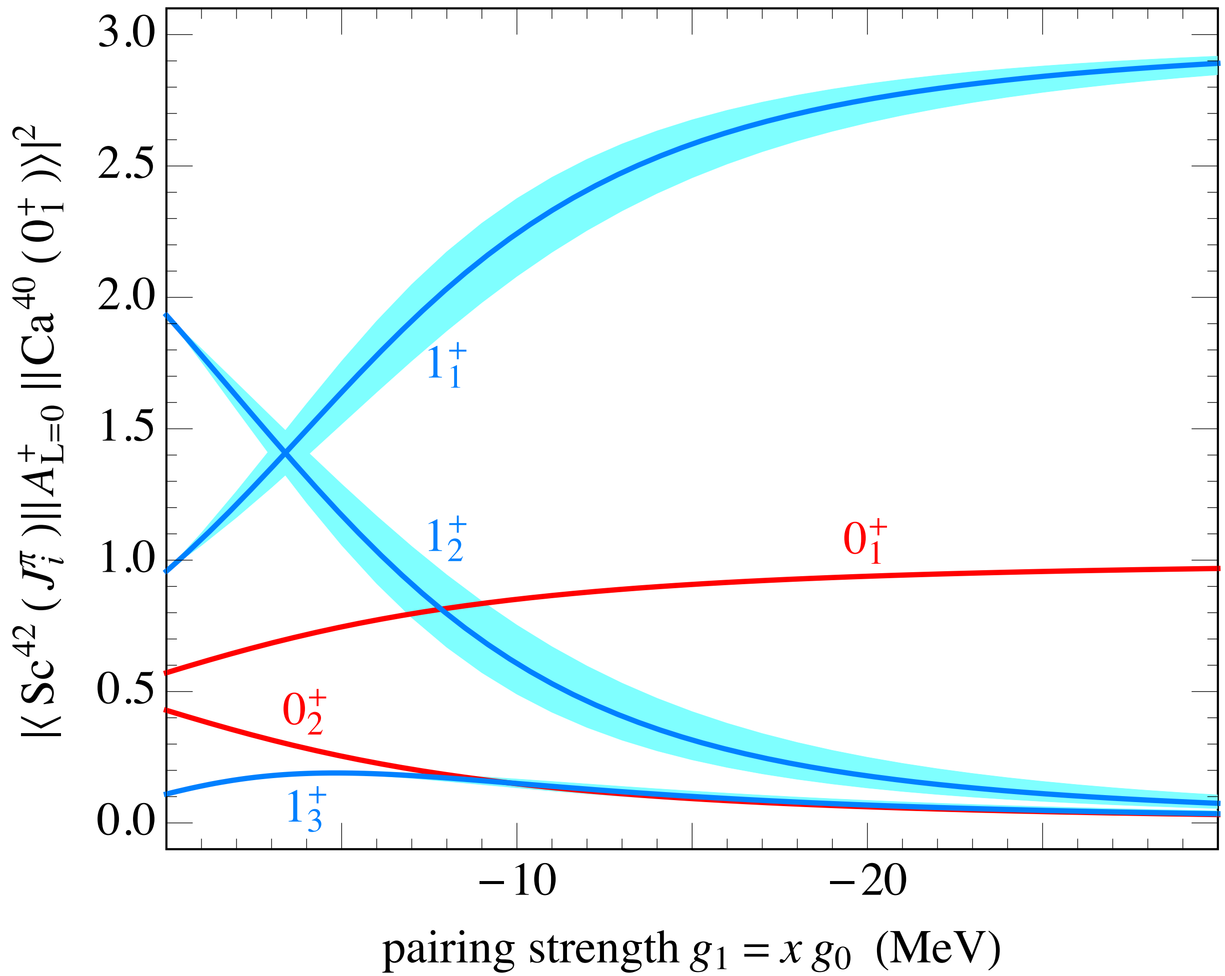
Transfer cross sections



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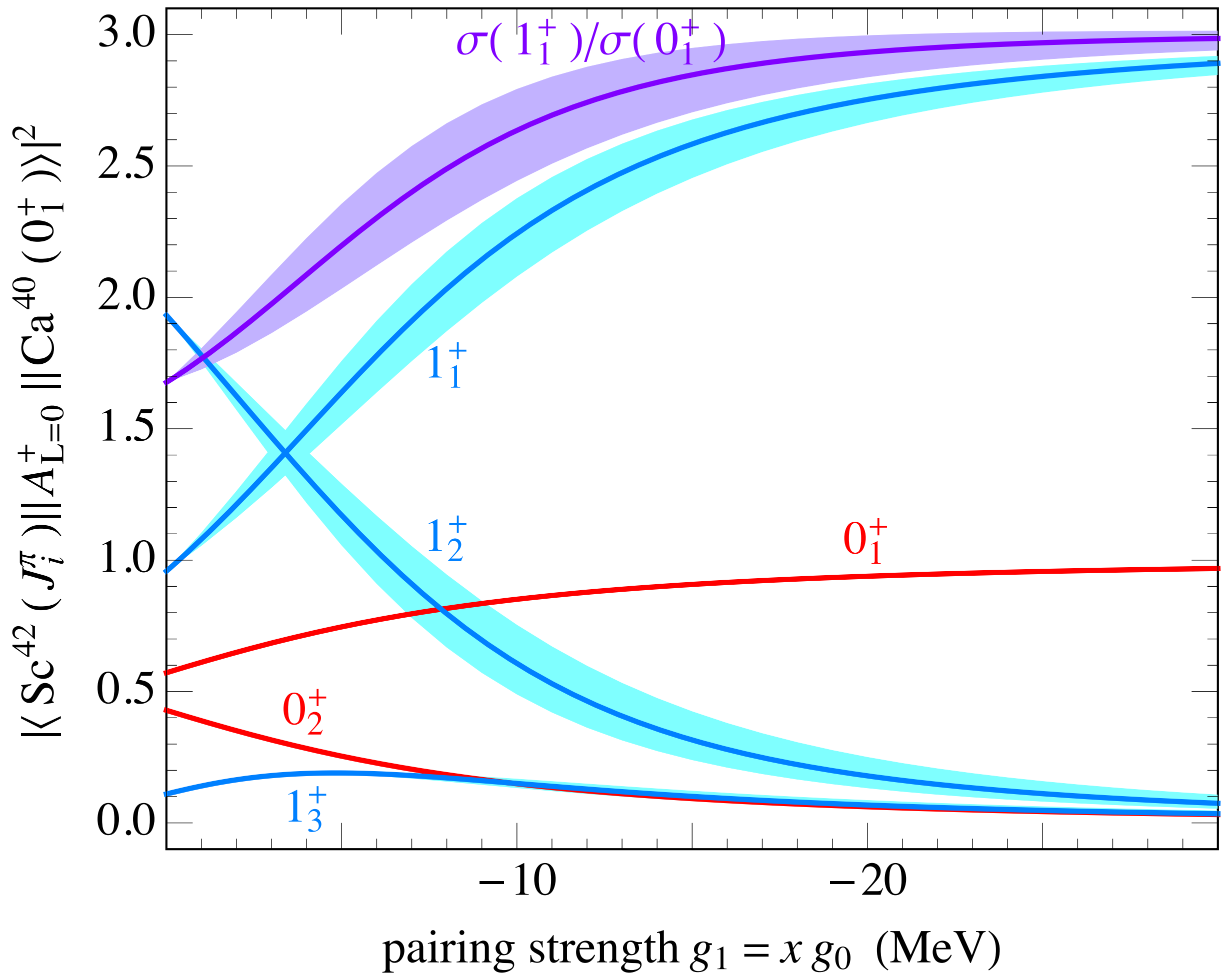

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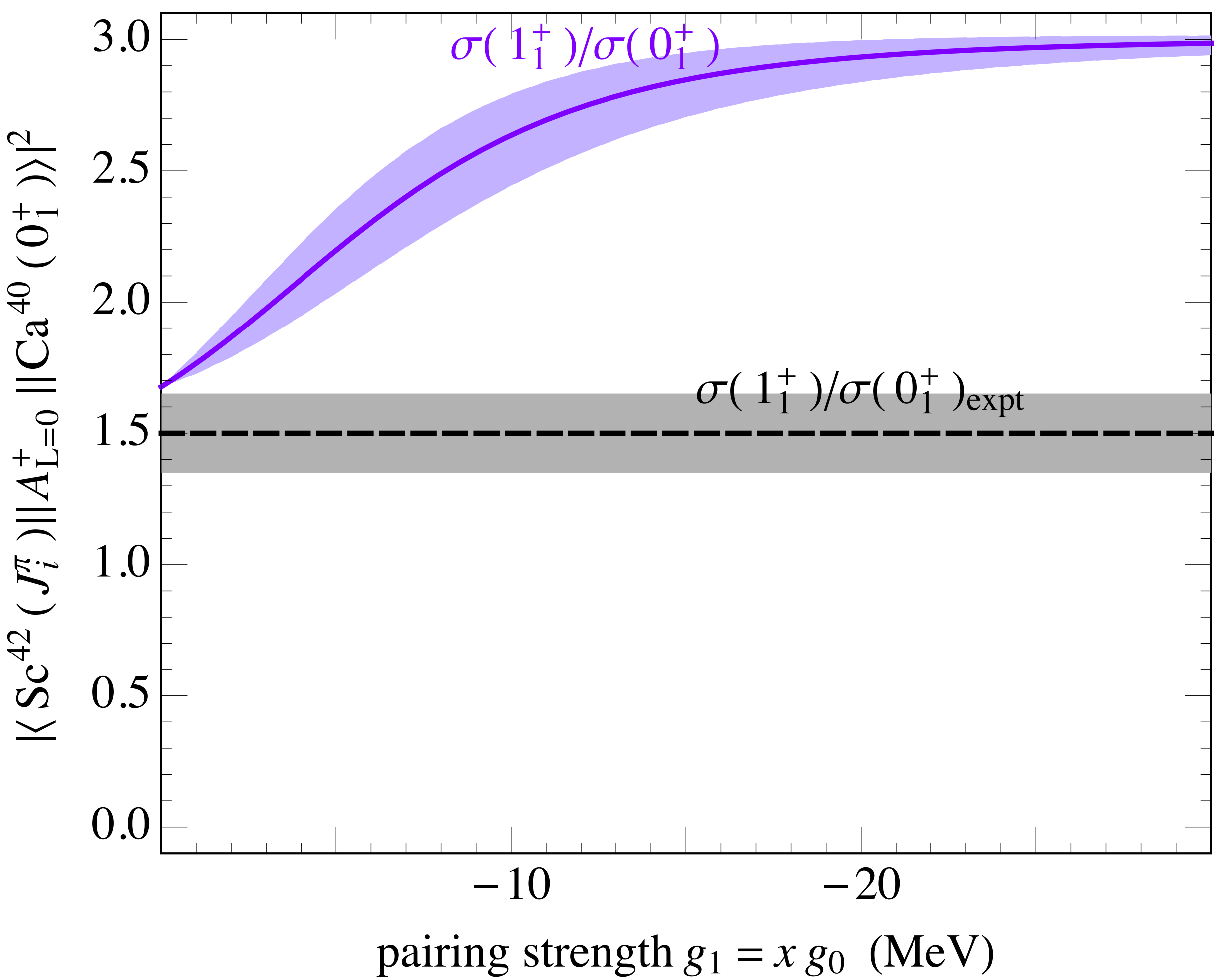

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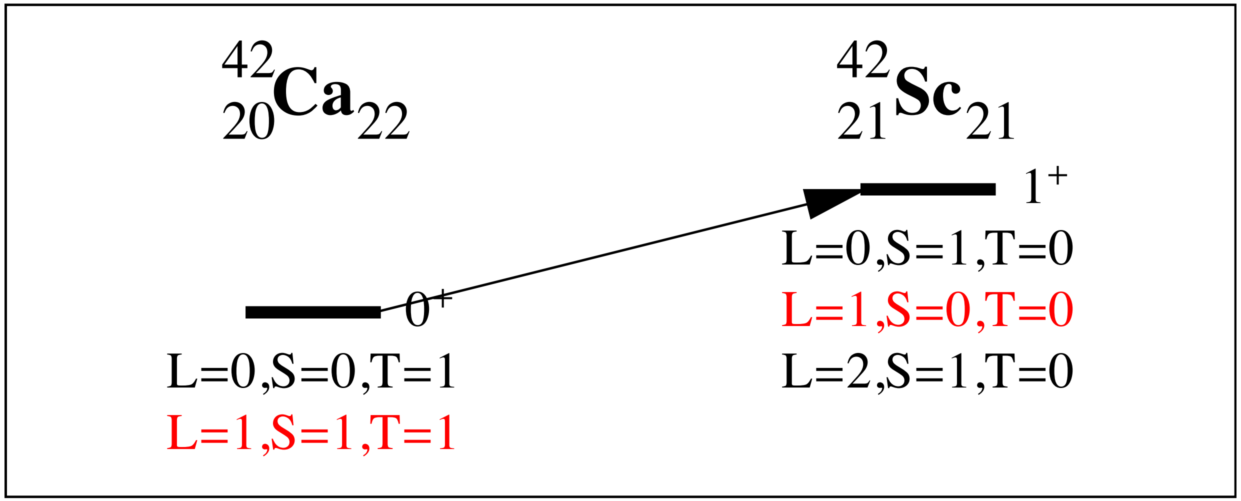

Boson mapping

Associate pairs of fermions with bosons:

$$\left(a^{+}_{l\frac{1}{2}\frac{1}{2}} \times a^{+}_{l\frac{1}{2}\frac{1}{2}}\right)^{(LST)}_{M_L M_S M_T} \mapsto b^{+}_{LM_L SM_S TM_T} \equiv b^{+}_{\ell m_\ell s m_s t m_t}$$

Remember: *L*+*S*+*T* must be odd!

In Elliott's IBM-4: only spatially favoured bosons with *L*=0,2.

Favoured-boson SU(4) algebra

L is even ➔ (S,T) = (0,1) and (1,0).

SU(4) algebra is generated by

$$\hat{S}^{\mathrm{e}}_{\mu} = \sqrt{2(2\ell+1)}\left(b^{+}_{\ell 10} \times \tilde{b}_{\ell 10}\right)^{(010)}_{0\mu 0}$$

$$\hat{T}^{\mathrm{e}}_{\nu} = \sqrt{2(2\ell+1)}\left(b^{+}_{\ell 01} \times \tilde{b}_{\ell 01}\right)^{(001)}_{00\nu}$$

$$\hat{Y}^{\mathrm{e}}_{\mu\nu} = \sqrt{2\ell+1}\left(\left(b^{+}_{\ell 10} \times \tilde{b}_{\ell 01}\right)^{(011)}_{0\mu\nu} \pm \left(b^{+}_{\ell 01} \times \tilde{b}_{\ell 10}\right)^{(011)}_{0\mu\nu}\right)$$

This algebra is at the basis of Elliott's IBM-4.

Favoured-boson classification

Many-boson states are classified by

$$\mathrm{U}\left[6(2\ell+1)\right] \supset \left(\mathrm{SU}_{ST}(6) \supset \mathrm{SU}_{ST}(4) \supset \mathrm{SU}_{S}(2) \otimes \mathrm{SU}_{T}(2)\right)$$
$$\otimes \left(\mathrm{U}_{L}(2\ell+1) \supset \cdots \supset \mathrm{SO}_{L}(3)\right)$$

Unfavoured-boson SU(4) algebra

L is odd ➔ (*S*,*T*) = (0,0) and (1,1).

SU(4) algebra is generated by

$$\hat{S}^{\mathrm{o}}_{\mu} = \sqrt{6(2\ell+1)}\left(b^{+}_{\ell 11} \times \tilde{b}_{\ell 11}\right)^{(010)}_{0\mu 0}$$

$$\hat{T}^{\mathrm{o}}_{\nu} = \sqrt{6(2\ell+1)}\left(b^{+}_{\ell 11} \times \tilde{b}_{\ell 11}\right)^{(001)}_{00\nu}$$

$$\hat{Y}^{\mathrm{o}}_{\mu\nu} = \sqrt{2\ell+1}\left(\left(b^{+}_{\ell 00} \times \tilde{b}_{\ell 11}\right)^{(011)}_{0\mu\nu} + \left(b^{+}_{\ell 11} \times \tilde{b}_{\ell 00}\right)^{(011)}_{0\mu\nu} \pm 2\left(b^{+}_{\ell 11} \times \tilde{b}_{\ell 11}\right)^{(011)}_{0\mu\nu}\right)$$

Unfavoured-boson classification

Many-boson states are classified by

$$\mathrm{U}\left[10(2\ell+1)\right] \supset \left(\mathrm{SU}_{ST}(10) \supset \mathrm{SU}_{ST}(4) \supset \mathrm{SU}_{S}(2) \otimes \mathrm{SU}_{T}(2)\right)$$
$$\otimes \left(\mathrm{U}_{L}(2\ell+1) \supset \cdots \supset \mathrm{SO}_{L}(3)\right)$$

Get your kicks with U(66)

Many-boson states are classified by

$$\mathrm{U}(6\Lambda_\mathrm{e}+10\Lambda_\mathrm{o})\supset \mathrm{U}(6\Lambda_\mathrm{e})\otimes \mathrm{U}(10\Lambda_\mathrm{o})\supset$$
$$\supset\left(\mathrm{SU}^\mathrm{e}_{ST}(6)\otimes \mathrm{SU}^\mathrm{o}_{ST}(10)\supset \mathrm{SU}_{ST}(4)\right)$$
$$\otimes\left(\mathrm{U}_L(\Lambda_\mathrm{e})\otimes \mathrm{U}_L(\Lambda_\mathrm{o})\supset\cdots\supset \mathrm{SO}_L(3)\right)$$

Bosons needed for states involved in the charge-exchange reaction:

L=0,2 and L=1 ➔ Λ_e=1+5=6 and Λ_o=3 ➔ U(66).

Mapping

Consider the schematic hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{so}} + g_0 P^+_{T=0} P_{T=0} + g_1 P^+_{T=1} P_{T=1} + \cdots$$

For the 0^+ and 1^+ two-nucleon states this hamiltonian can be mapped **exactly** onto *s*, *p* and *d* bosons.

We obtain an **exact** boson representation of the *A*=42 charge-exchange properties.

To do

Map realistic shell-model hamiltonian (KB3G or GXPF1J).

Use mapped hamiltonian to calculate charge-exchange strength for A=46, 50 and 54.

Study the relative importance of the different pairs.

Conclusions

The spin-orbit interaction mixes spatially favoured and unfavoured pairs.

Up to now only spatially favoured pairs have been mapped onto bosons (IBM-4).

A general formulation of the IBM, which includes unfavoured bosons, is possible.