INTRINSIC VORTICAL MODE AND THE ROTATIONAL BANDS OF WELL-DEFORMED NUCLEI

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SDANCA 17, Sofia, Bulgaria 5-7 October 2017

INTRODUCTORY REMARKS

Beyond the rigid rotor approach of nuclear rotational states

$$E(I) = \frac{\hbar^2}{2J} I(I+1) \qquad J(\Omega) = cst$$

There are two dominating perturbating effects

Rotational stretching / anti-stretching

- collective due to the centrifugal forces

$\Omega \uparrow \quad \beta \uparrow \quad J \uparrow$

- « quantal » due to HF field changes

Weakening of pair correlations by pair alignment

- gradual, collective

$\Omega \uparrow \quad \Delta \downarrow \quad J \uparrow$

- sudden, individual

backbending, a particular case of band crossing

Other effects

as e.g. coupling with vibrations (as octupole modes in SD bands)

Therefore the moment of inertia J is varying with I

$$\text{with} \quad \bar{I} = \sqrt{I(I+1)} \quad \text{and thus} \quad E(I) = \frac{\hbar^2}{2J(I)}\bar{I}^2$$

One gets due to the I-dependence of J(I)

$$\frac{1}{\bar{I}}\frac{dE(I)}{d\bar{I}} \neq \frac{\hbar^2}{J(I)} \neq \frac{d^2E(I)}{d\bar{I}^2}$$

One then defines two moments of inertia

$$\text{Kinematical} \quad \frac{J^{(1)}(I)}{\hbar^2} = \bar{I}\ [\frac{dE(I)}{d\bar{I}}]^{-1}$$

$$\text{Dynamical} \quad \frac{J^{(2)}(I)}{\hbar^2} = [\frac{d^2E(I)}{d\bar{I}^2}]^{-1}$$

With the Lagrange parameter Ω of the Routhian approach, thus such that

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$$\Omega(I) \equiv \Omega = \frac{dE(I)}{\hbar\, d\bar{I}} \quad \text{and thus one has} \quad \frac{J^{(1)}(I)}{\hbar^2} \approx \frac{\bar{I}}{\hbar\Omega} \quad \text{and} \quad \frac{J^{(2)}(I)}{\hbar^2} \approx \frac{d\bar{I}}{d\hbar\Omega}$$

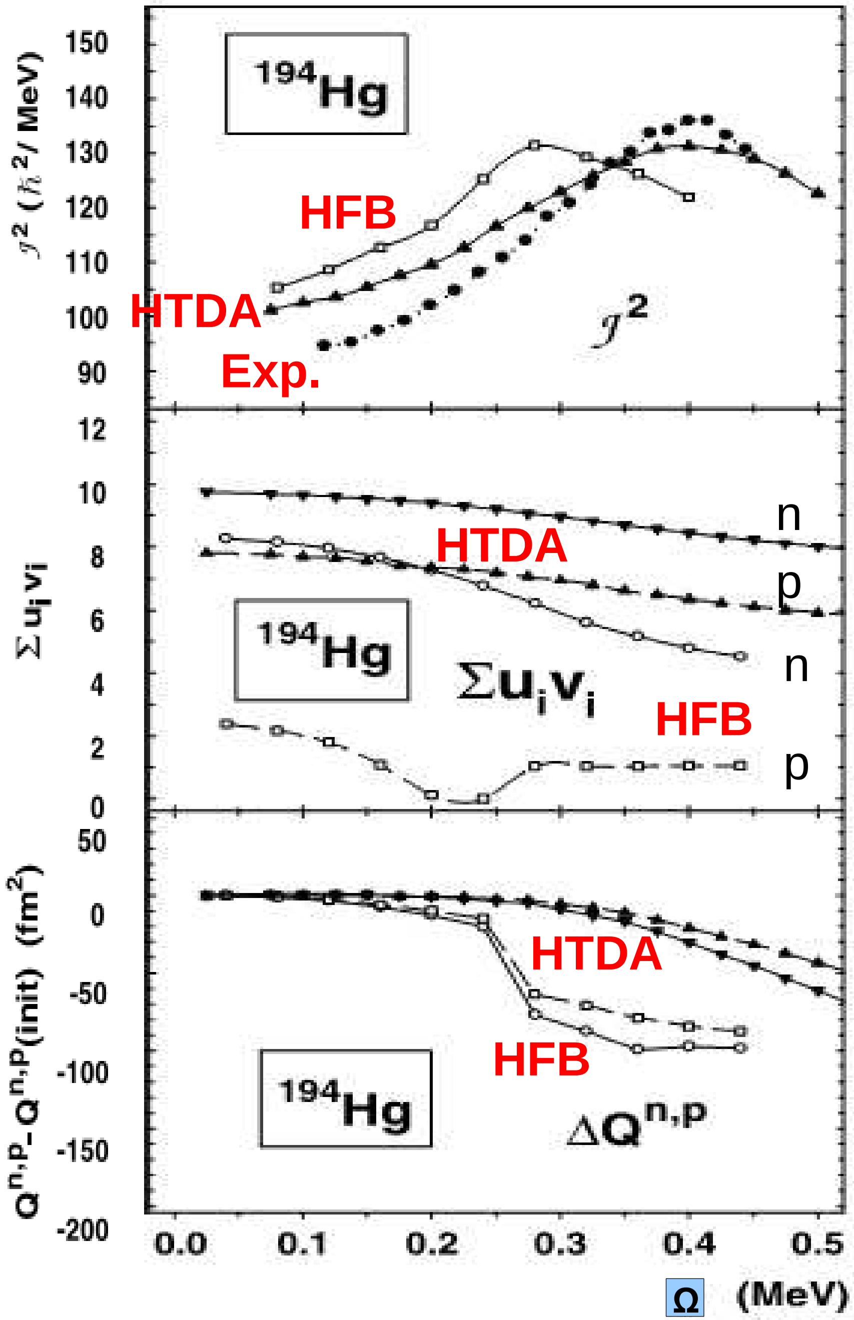
Through some approximations (eg finite differences $I \rightarrow I \pm 2$ and $\bar{I} \approx I$)

one connects usually these local quantities with transition energy data

$$E_\gamma^+(I) = E(I+2) - E(I) \quad \text{and} \quad E_\gamma^-(I) = E(I) - E(I-2) \quad \text{so that}$$

$$\hbar\Omega(I) \approx \frac{E_\gamma^+(I) + E_\gamma^-(I)}{4} \quad , \quad \frac{J^{(1)}(I)}{\hbar^2} \approx \frac{2\mathrm{I}-1}{E_\gamma^-(I)} \quad , \quad \frac{J^{(2)}(I)}{\hbar^2} \approx \frac{4}{E_\gamma^+(I) - E_\gamma^-(I)}$$

An example, among many, of the I-dependence of J



H. Laftchiev, J. Libert, P. Quentin, Ha Thuy Long, Nucl. Phys. A845 (2010)33

Routhian HTDA calculations

Yrast SD rotational band of ^{194}Hg

- Small rotational deformation effect (anti-stretching) at low Ω

-influence on J of a proton pairing gap spuriously vanishing (HFB) or merely diminishing (HTDA)

$$\sum_i u_i v_i = Tr\,\{\hat{\rho}^{1/2}(1-\hat{\rho})^{1/2}\}$$

In this talk we will consider only one of the collective effects namely the pairing quenching effects

As well-known in the adiabatic limit (low Ω),
for the same canonical basis
the Inglis-Belyaev cranking formula yields moment of inertia
is reduced from its normal phase value (as in the Inglis formula)
in the presence of pairing correlations

This has been first analyzed by B.R. Mottelson and P.G. Valatin
and dubbed as the Coriolis Anti Pairing (CAP) effect

WHAT ARE WE PROPOSING HERE ?

A polynomial expression (3rd order in Ω^2)
for the energies within a rotational band of well-deformed nuclei

Different from mere parametrizations
- original VMI approach (from G. Scharff-Goldhaber et al.,1976 on)
- quadratic expression in $\overline{I}^2$ (see eg R. Casten's textbook)
- variational approach with an ad hoc dependence of J on the pairing gap
  (K. Pomorski et al.)

Here the polynomial mathematical expression is fully determined
by the assumed dynamical behaviour

Two adjustments only are made for each nucleus from data by inserting
- the equilibrium quadrupole deformation
- the energy of the 2^+ state

We will consider a model involving intrinsic vortical currents
as proposed in

P. Quentin, H. Laftchiev, D. Samsoen and I.N. Mikhailov, Phys. Rev. C69, 054315 (2004)

in which one couples pairing generated intrinsic vortical currents
(i.e. in the inertial frame)
with those responsible for the global rotation
(of the inertial frame with respect to the lab frame)

We describe this coupling using the linear divergence-free velocity field
within the so-called
Chandrasekhar S-ellipsoid's theoretical framework

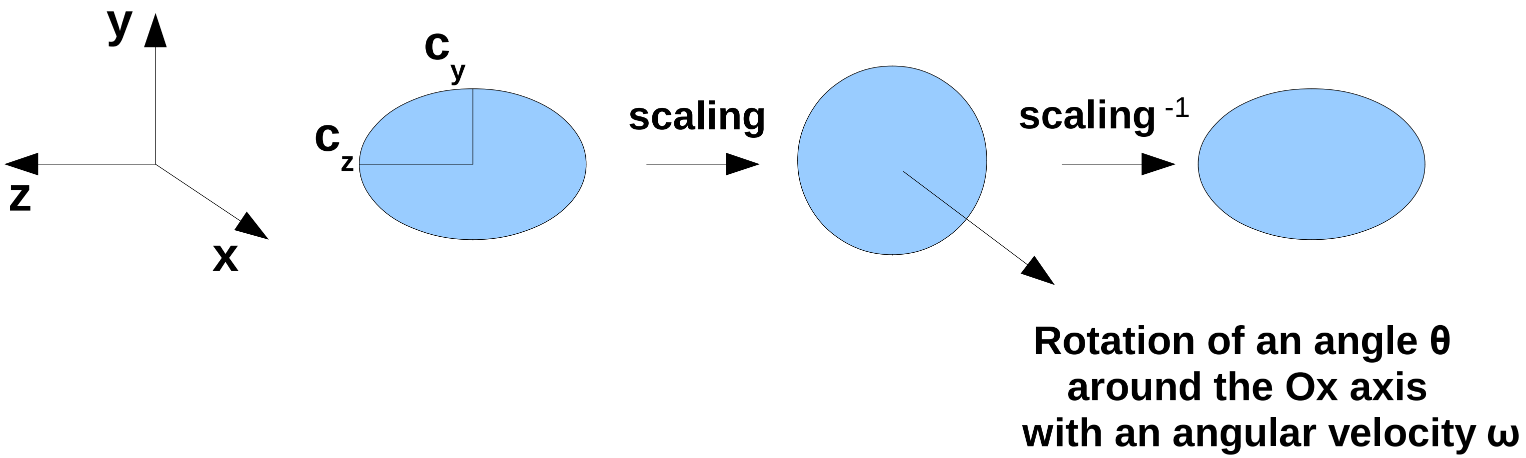
INTRINSIC VORTICAL CURRENTS
AND THE ROTATIONAL PAIRING QUENCHING

In what follows we couple two rotational modes : global and intrinsic

From Routhian HFB calculations, one may quantify the intrinsic vortical currents generated by the global rotation in terms of the expectation value of the so-called
Kelvin circulation operator $\hat{\vec{K}}$ defined as follows.
Consider the case of the so-called Chandrasekar's S type ellipsoids further assumed to be axially symmetrical (with Oz as the symmetry axis) and where the vorticities of the global and intrinsic modes of an enclosed fluid are aligned or anti-aligned (e.g. on the x-axis)
The operator–form of the Kelvin circulation (acting on wavefunctions) is

$$K_y = K_z = 0 \qquad K_x = \frac{\hbar}{i}\left\{ y\, q \frac{\partial}{\partial z} - \frac{z}{q}\frac{\partial}{\partial y}\right\} \quad \text{with} \quad q = c_z / c_y \qquad \text{double stretching in } \vec{r} \text{ and } \vec{p}$$

where q is the semi-axis ratio of the considered ellipsoid



K is canonically conjugated with θ

$$\omega = \frac{d\theta}{dt}$$

This product of transformations yields a rather general field linear in $\vec{r}$
divergence-free (i.e. tangential and thus non-deforming)
of a fluid bounded by an ellipsoidal container
in the container's body-fixed (i.e. intrinsic) frame

It generates a wide class of modes
when combined
with the global rotation

$$a_z > a_x = a_y$$

$$\xi = \frac{1 + \frac{a_z}{a_y}\frac{\omega}{\Omega}}{1 + \frac{a_y}{a_z}\frac{\omega}{\Omega}}$$

(lab frame)

I.N. Mikahailov, D. Samsoen, P. Quentin, Nucl. Phys. A 627 (1997) 259

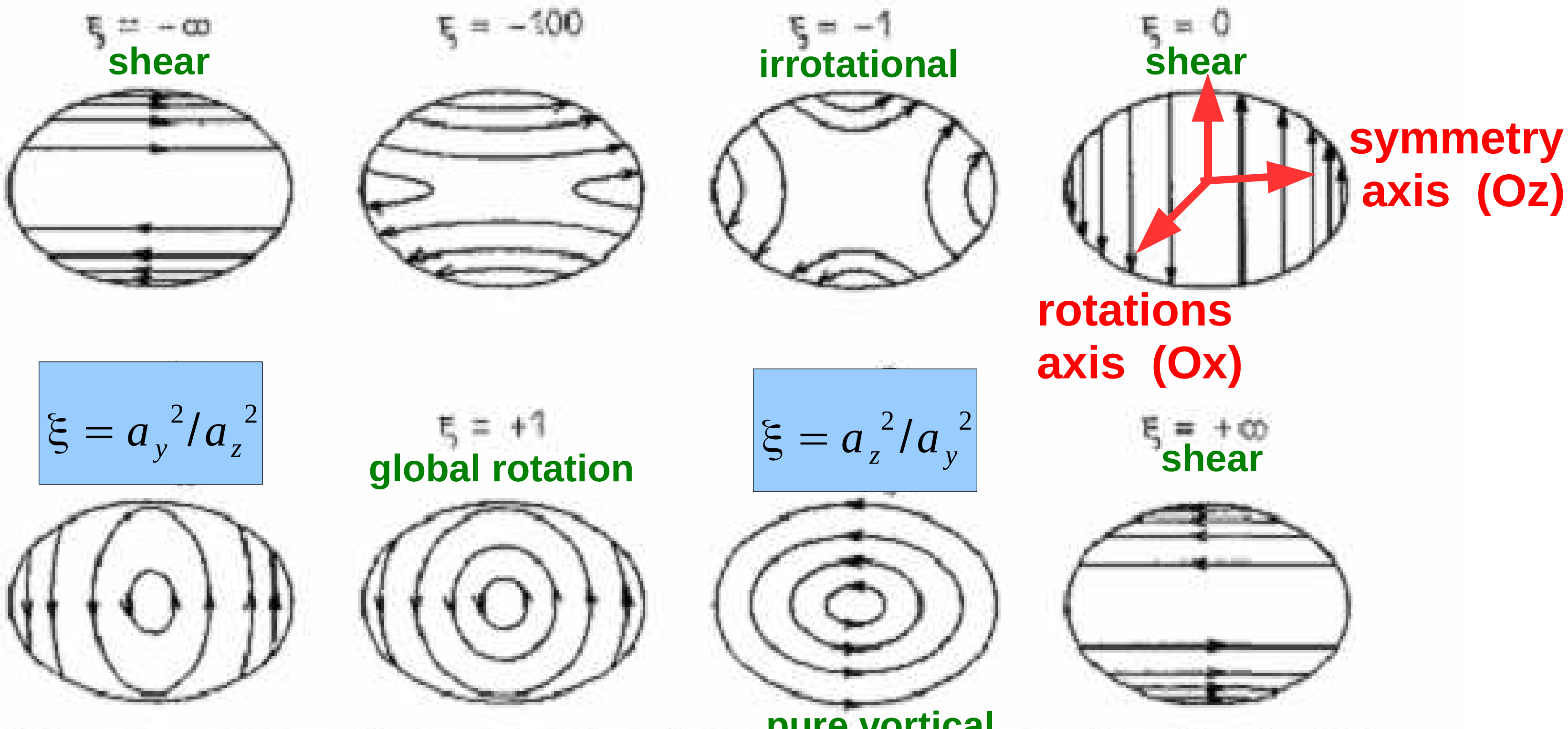


Fig. 1. Flow patterns corresponding to various values of the dimensionless parameter ξ describing the relative importance of global rotation versus intrinsic vortical motion, whose angular velocities are respectively Ω and ω, for a given prolate ellipsoidal shape. In this figure located in the (x, y) plane, Ω and ω are aligned on the z-axis. The quantities a_x and a_y are semi-axis values on the x-axis and y-axis respectively, with $a_x > a_y$.

It has been shown for both normal or super deformed nuclear states that

Routhian HFB calculations $\delta\{\hat{H} - \Omega \hat{I}_x\} = 0$

Constrained Routhian HF calculations $\delta\{\hat{H} - \Omega \hat{I}_x - \omega \hat{K}_x\} = 0$

yielded astonishingly similar solutions when constraining the RHF solutions to have the same expectation value of $\hat{K}_x$ as in RHFB

H. Laftchiev, P. Quentin, D. Samsoen and I .N. Mikhailov, Phys. Rev. C 67 (2003) 014307.

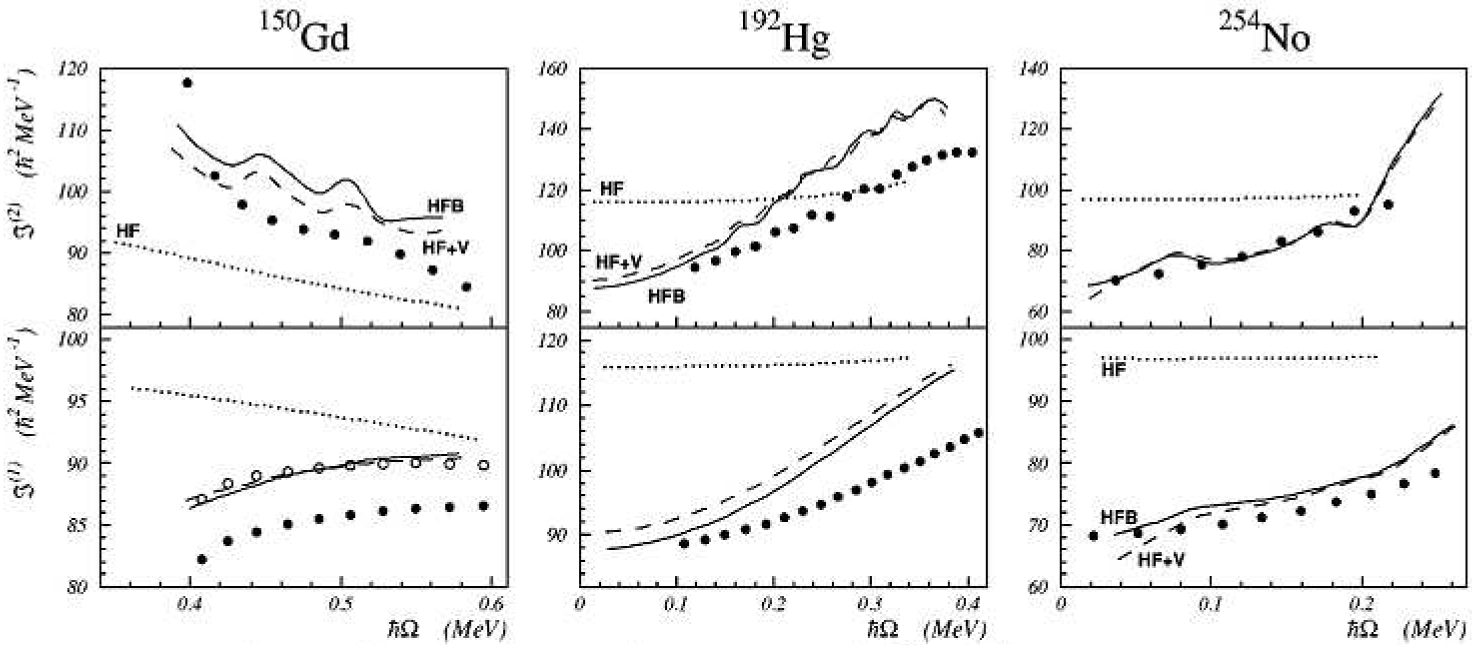


FIG. 1. Dynamical (upper panel) and kinetic (lower panel) moments of inertia (in $\hbar^2$ MeV^{-1}) for the three considered nuclei as functions of the angular velocity Ω in MeV. The conventions in use are the following: HF value (dotted line), HFB value (solid line), and HF+V value (dashed line). Experimental data are represented as solid circles, except for the ^{150}Gd kinetic moment of inertia with the assumption $I_{\rm init} = 34\hbar$ as opened circles (lower left panel). The Lipkin-Nogami correction has been applied for ^{150}Gd and ^{192}Hg.

Conclusions

The S-ellipsoids type of intrinsic vortical currents are relevant here

But, using this for constrained Routhian HF calculations is not useful in practice, since one has first to perform Routhian HFB calculations

Thus one has built up a simple model relating the two relevant angular velocities ω and Ω as follows

The frequency ω is described as a function of Ω such that

- **the product Ωω is negative (a counter rotation)**
- **|ω| is proportional (all other things being kept constant) to Ω (the faster the rotation, the stronger the counter effect)**
- **ω is also proportional to a "pair condensation energy" (all other things being kept constant) E_{cond} defined as the expectation value of the residual interaction (thus a positive quantity) which is a decreasing function of Ω and approximated as**

$$E_{cond}(\Omega) = E_0 \, \{1 - (\frac{\Omega}{\Omega_c})^2\}$$

where E_0 is the pair condensation energy at zero spin

Thus $$\omega(\Omega) = -k\,\Omega\,\{1 - (\frac{\Omega}{\Omega_c})^2\}$$

The critical frequency Ω_c (vanishing pairing) is defined (somewhat as in superconductors) by equating the rotational energy at zero pairing with twice E_0 (pair condensation energy) at zero spin

$$\Omega_c^2 = \frac{4|E_0|}{J_{rigid}}$$ using J_{rigid} since at zero pairing, the rigid moment of inertia is a valid approximation

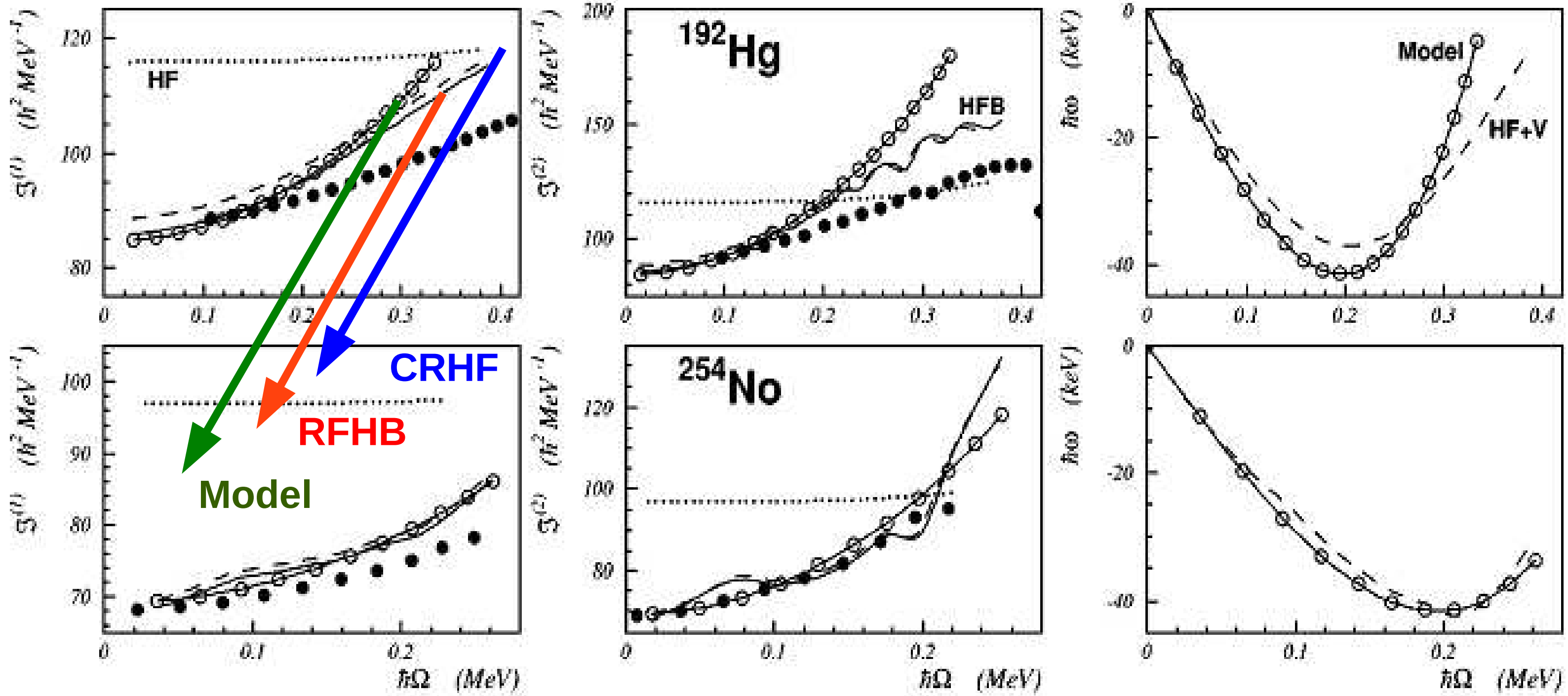
The model parameters E_0 ,k and J_{rigid}

are **determined from nuclear properties at zero angular velocity**

Then, performing Constrained RHF calculations with the model ω(Ω)

$$\delta\{\hat{H} - \Omega\hat{I}_x - \omega(\Omega)\hat{K}_x\} = 0$$

one gets results in **excellent agreement with those of RHFB calculations**
as long as merely the collective pairing quenching mode is at work



P. Quentin,
H. Lafchiev,
D. Samsoen
and
I.N. Mikhailov,
Phys. Rev. C69,
054315 (2004)

FIG. 5. Results of our calculations for ^{192}Hg (upper panels) and ^{254}No (lower panels). From left to right we show (as functions of the rotation angular velocity Ω in MeV) for each nuclei the kinetic moment of inertia and the dynamical moment of inertia (in $\hbar^2$ MeV^{-1} units) as well as the Kelvin circulation velocity ω (in keV). The results obtained within various formalisms are represented as follows: dotted line for the (pure) HF formalism, full line (dashed line, respectively) for the HFB (HF+V, respectively) formalisms and full line with opened circles for our model. Experimental data for the moments of inertia are represented as filled circles. The Lipkin-Nogami correction has been applied for the Hg isotope.

From that, build a polynomial expression of the energy as a function of Ω

Take a quadratic approximation for the excitation energy as a function of the angular velocities is postulated (a relatively low velocity ansatz) as

$$E(\Omega,\omega) = \frac{1}{2}\left(A\omega^2 + 2B\omega\Omega + C\Omega^2\right)$$

introducing the generalized moments of inertia A,B,C almost equal to the rigid body moment of inertia J_R (see below)

The function ω(Ω) could be approximated at low velocity Ω as

$$\omega(\Omega) \approx -k\Omega$$

thus introducing the experimental first 2^+ energy

$$E_2^{\exp} = \frac{6\hbar^2}{2J(2)} = \frac{1}{2}J(2)\Omega_2{}^2 \quad \text{and thus} \quad J(2) = \frac{3\hbar^2}{E_2^{\exp}} \quad \text{and} \quad \Omega_2{}^2 = \frac{2E_2^{\exp}}{J(2)}$$

one gets a second order equation for k as

$$J(2) = Ak^2(1-y_2{}^2)^2 - 2Bk(1-y_2{}^2) + C \quad \text{with} \quad y_2 = \Omega_2/\Omega_c$$

with 0 < k < 1 (the counter-rotation never equates the global rotation)

$$k = \frac{B}{A(1-y_2{}^2)}\left[1 - \sqrt{1 - \frac{A(C - J(2))}{B^2}}\right] \quad \text{with} \quad y = \Omega/\Omega_c$$

defining thus fully the polynomial expression of the energy

$$E(\Omega) = \frac{\Omega^2}{2}\left[C - 2Bk(1-y^2) + Ak^2(1-y^2)^2\right]$$

From the semiclassical ETF calculations (up to the second order in $\hbar$) of

I.N. Mikhailov, P. Quentin and D. Samsoen, Nucl. Phys. A627 (1997) 259

one gets for an axial ellipsoid defined by an axis ratio q, q > 1 when prolate

$$A = \eta\Theta\{1 - X\ \Theta\} \qquad B = \eta\ \{1 - X\ \Theta\} \qquad C = \eta\Theta\ \{1 - \frac{X}{\Theta}\}$$

$$\text{with} \qquad \Theta = \frac{1}{2}\left(q + \frac{1}{q}\right) \qquad \text{and} \qquad \eta = J_{rigid}^{sphere}\ q^{1/3}$$

$$X \approx 5\left(\frac{8}{9\pi}\right)^{2/3} \left(\frac{m^*}{m}\right)_{NM}^{-1} A^{-2/3}$$

(to which a proper spin contribution - known for C- should be added)
from which one easily gets the hierarchy B < A < C

Such an ordering of A, B and C has been actually confirmed
in the adiabatic microscopic calculations of
P. Quentin, H. Lafchiev, D. Samsoen and I.N. Mikhailov, Phys. Rev. C69 (2004) 054315
where typically (C – B) / A ~ 10% (ND states), 15% (SD states)

For all nuclei we have taken as a rough evaluation
of the zero spin « pair condensation » energy
E(0) = - 4.6 MeV approximated as twice the correlation energy
as given in S.G. Nilsson et al., Nucl. Phys. A131 (1969) 1.

Apart from the 2^+ energies,
the only piece of data introduced for each nucleus are
the rigid body J_{rigid} values at equilibrium deformation

For that we use a rough estimate of their A-dependence at sphericity
from the semiclassical self-consistent ETF calculations of
K. Bencheikh, J. Bartel and P. Quentin, Nucl. Phys. A571 (1994) 518

$$J^{0}_{rigid} = \frac{1}{68.4} A^{5/3}\ MeV^{-1}\hbar^{2}$$

The deformation dependence is given as in a usual LDM
for an axial quadrupole deformation at lowest orders

$$J(\alpha) = J^{0}_{rigid}\{1 + \frac{1}{2}\alpha + \frac{9}{7}\alpha^{2}\} \qquad \text{with} \qquad \alpha = \sqrt{5/4\pi}\,\beta$$

The β parameter values are taken from the B(E2) data compilation of
S. Raman et al., Atomic Data Nucl. Data Tables 78 (2001) 1

and when not available from the theoeretical systematic study of
www-phynu.cea.fr/HFB-5DCH-table.htm
J.-P. Delaroche et al., Phys. Rev. C81 (2010) 014303

The last theoretical task is to determine the value of the velocity Ω corresponding to a given spin I theoretically Ω(I)
It is performed through the Lagrange multiplier relation

$$\frac{dE\left(\Omega,\omega\left(\Omega\right)\right)}{\left(d\,\bar{I}\right)} = \hbar\,\Omega \quad \text{from which} \quad I\left(\Omega_1\right) = I\left(\Omega_0\right) + \frac{1}{\hbar}\int_{\Omega_0}^{\Omega_1} \frac{d\,\Omega}{\Omega}\,\frac{d\,E\left(\Omega,\omega\left(\Omega\right)\right)}{d\,\Omega}$$

Then one obtains with

$$\frac{dE}{\left(d\,\Omega\right)} = \{[C - 2\,Bk + A\,k^2] + 4[B\,k - A\,k^2]\,y^2 + 3\,A\,k^2\,y^4\}\,\Omega \quad \text{with}\; y = \frac{\Omega}{\Omega_c}$$

$$I\left(\Omega\right) = \{[C - 2\,Bk + A\,k^2] + \frac{4}{3}[B\,k - A\,k^2]\,y^2 + \frac{3}{5}A\,k^2\;y^4\}\,\Omega$$

This expression of I as a function of Ω is monotonically increasing and can thus be safely inverted to get Ω for each desired value of I

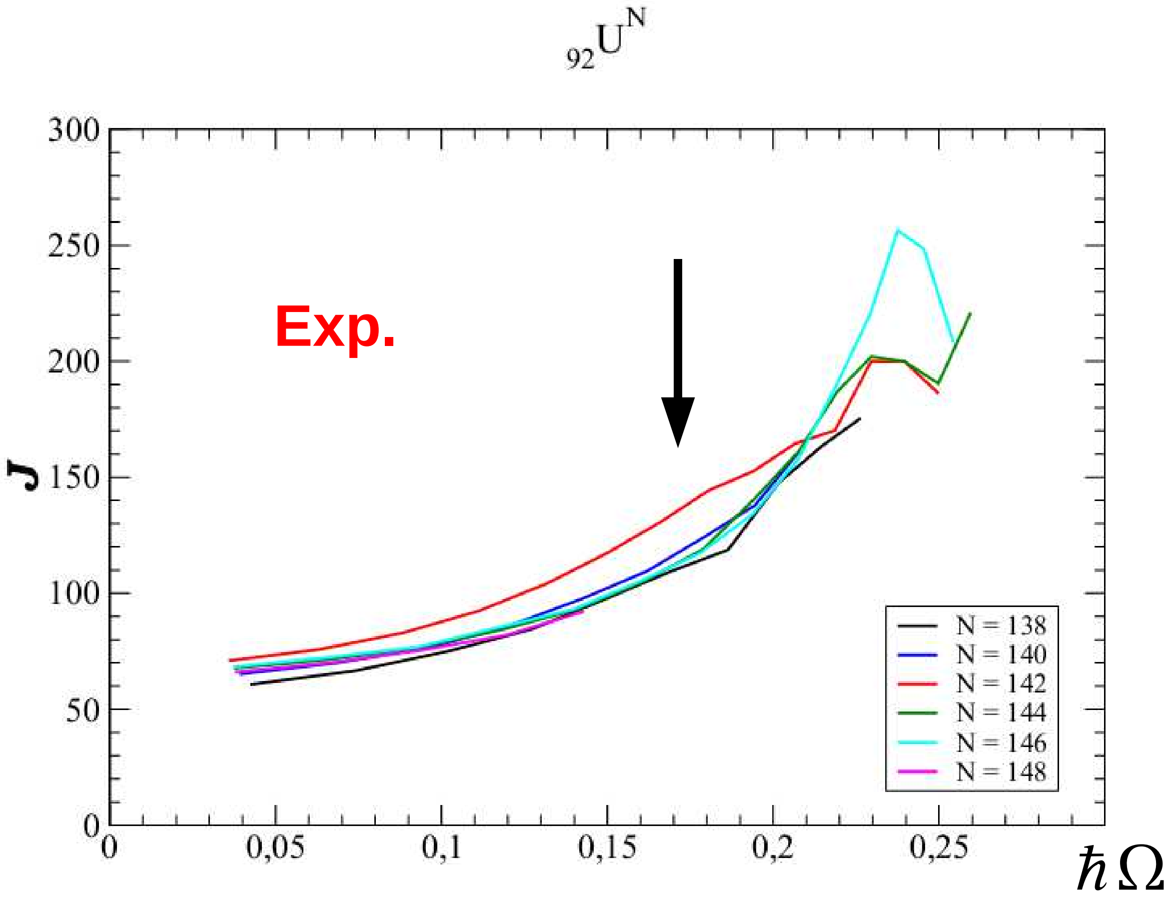
Calculations have been performed for all nuclei in and around the rare-earth (52 nuclei) and actinide (31 nuclei) regions

for which the ratio $R = \dfrac{E^{\exp}\left(4^+\right)}{E^{\exp}\left(2^+\right)}$ **is larger than 3**

LIMITS OF THE APPROACH

As already mentioned it takes only into account CAP effects.
Other effects as eg centrifugal stretching are not described.

As an example we show the data on $J^{(2)}$
for even-even Uranium isotopes (from ^{230}U to ^{240}U).
A reasonable agreement is *a priori* only expected for $\hbar\Omega \leqslant\sim 0.17\,\mathrm{MeV}$
corresponding for these isotopes to band states with E_{excit} ~ 1.5 MeV



INITIAL TEST OF OUR INTRINSIC VORTICAL MODEL

Calculations labelled (1) : Experimental angular velocities
Calculations labelled (2) : Model-derived angular velocities

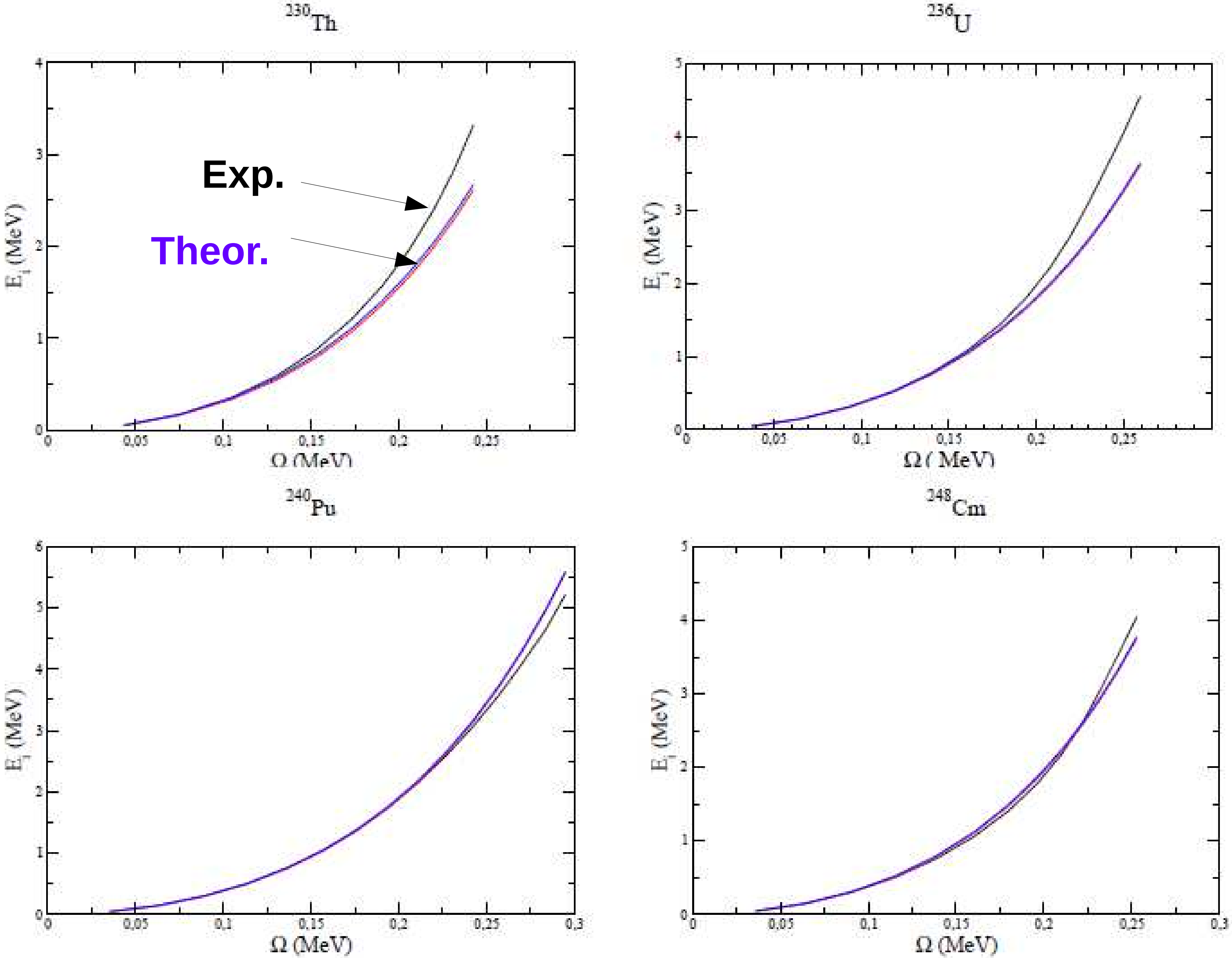
^{232}U

I (ħ)	$E_{calc}(2)$	E_{exp} (keV)	$E_{calc}(1)$
2	48	48	48
4	159	157	153
6	328	323	320
8	553	541	544
10	829	806	817
12	1151	1112	1135
14	1515	1454	1491
16	1918	1828	1879
18	2356	2232	2284

^{238}U

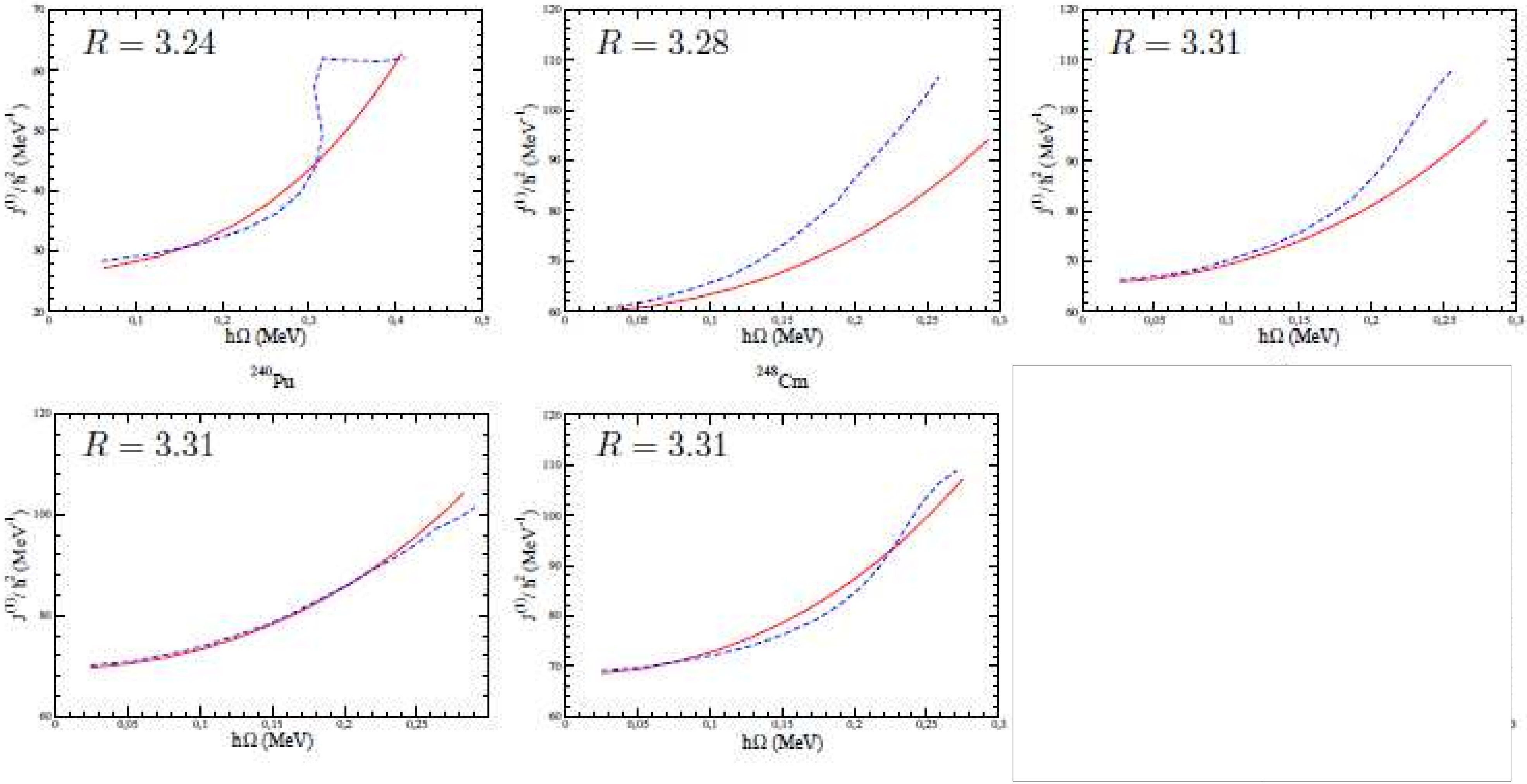
I (ħ)	$E_{calc}(2)$	E_{exp} (keV)	$E_{calc}(1)$
2	45	45	45
4	150	148	147
6	310	307	312
8	523	518	542
10	785	776	837
12	1091	1077	1192
14	1437	1416	1601
16	1821	1788	2052
18	2239	2191	2284
20	2689	2619	2978
22	3167	3068	3421
24	3673	3535	3846
26	4204	4018	4285
28	4759	4517	4823

Even though calc. (2) are less phenomenological than calc. (1), they lead to results comparable and sometimes better e.g. in ^{238}U yielding some confidence in the model



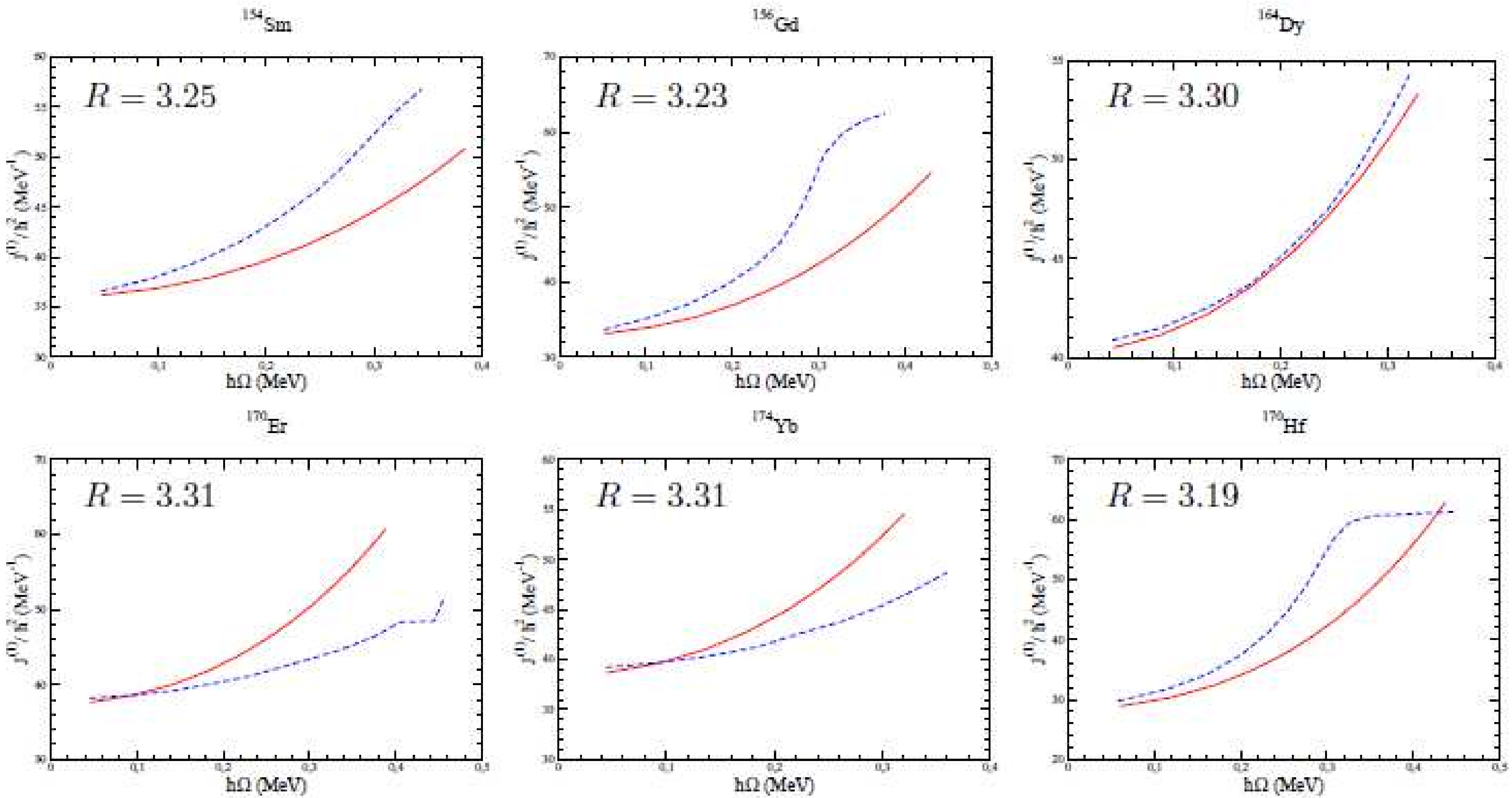
230Th
Exp.
Theor.
E_i (MeV)
Ω (MeV)
236U
E_i (MeV)
Ω (MeV)
240Pu
E_i (MeV)
Ω (MeV)
248Cm
E_i (MeV)
Ω (MeV)

A more demanding comparison : kinematical moments of inertia
Some actinide nuclei

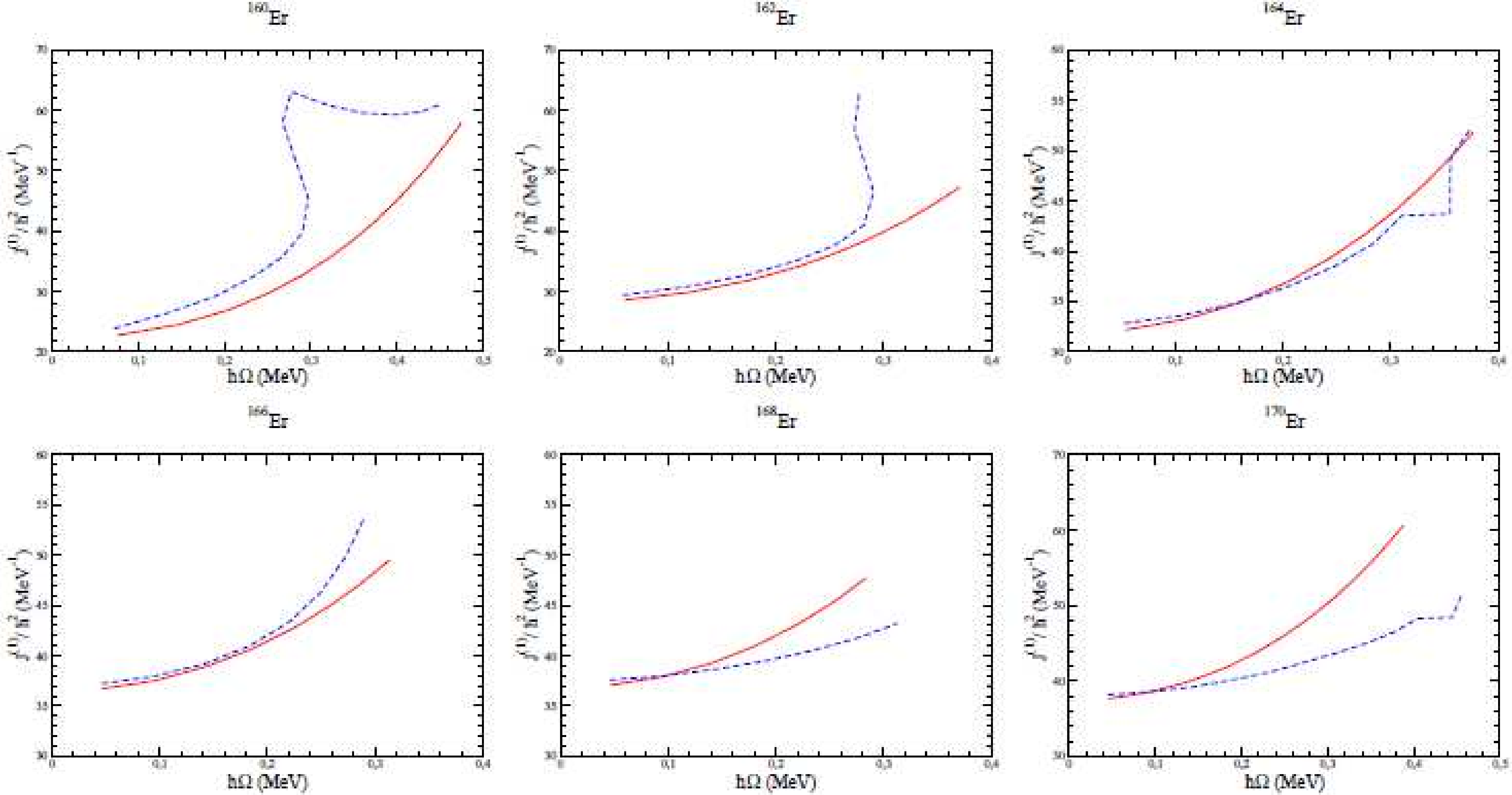


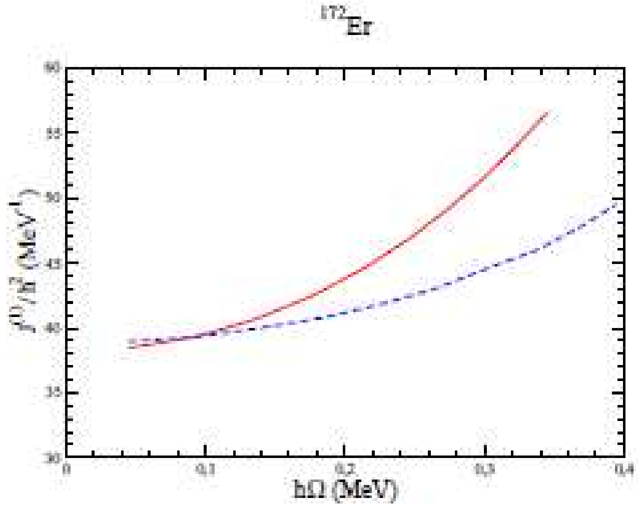
Appearance in some cases of
backbending effects and/or coupling to other modes

In and around the rare earth region



Rare-earth nuclei are in general less prone to show a pure CAP behaviour (see eg Z. Szymanski et al.)





Erbium isotopic series (from A =160 to A = 172)

CONCLUSIONS

Our Variable Moment of Inertia model
stemming from the coupling of two collective currents
à la Chandrasekhar (S-ellipsoids)
yields a simple yet efficient sixth order polynomial formula for E(Ω)

What is it good for ?

- confirms the already established relevance of these intrinsic vortical currents to describe the CAP

- may indicate from data
when and where (in I) the CAP is the only mechanism at work
when and where it couples with other dynamical effects

- may serve to correct with an energy uncertainty of less than ~ 100 keV theoretical results at moderately high spins relying on adiabatic moments of inertia (as eg microscopically based Bohr Hamiltonian calculations)