Nuclear structure effects involving pear-shape deformation

Nikolay Minkov

Institute of Nuclear Research and Nuclear Energy
Bulgarian Academy of Sciences, Sofia, Bulgaria
Research Group on Complex Deformed Atomic Nuclei

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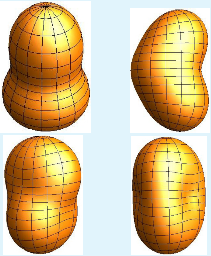
Shell model origin of octupole deformation



Coupling of orbitals with different parity and $(\Delta l = 3)$ near the Fermi level $\rightarrow (N, l, j) \otimes (N-1, l-3, j-3)$

$\Rightarrow$ Particle numbers favouring strong octupole correlations:

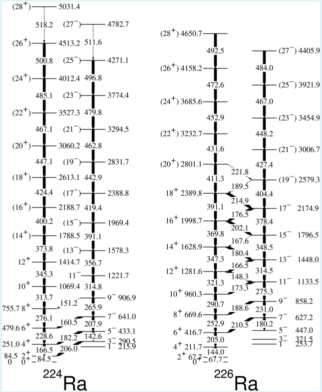
$$\mathbf{34}\ (g_{\frac{9}{2}} \otimes p_{\frac{3}{2}})\ \mathbf{56}\ (h_{\frac{11}{2}} \otimes d_{\frac{5}{2}})\ \mathbf{88}\ (i_{\frac{13}{2}} \otimes f_{\frac{7}{2}})\ \mathbf{134}\ (j_{\frac{15}{2}} \otimes g_{\frac{9}{2}})$$

$\Rightarrow$ regions of pronounced octupole deformations/collectivity:

^{144}Ba $(Z = 56,\ N = 88) \Rightarrow$ Xe (Z=54) – Ba (Z=56) – Ce (Z=58)

^{222}Ra $(Z = 88,\ N = 134) \Rightarrow$ Rn (Z=86) – Ra (Z=88) – Th (Z=90)

Alternating-parity bands (APBs) at stable octupole mode



J. F. C. Cocks et al

PRL **78**, 2920 (1997)

Recent confirmations of GS octupole deformation:

^{224}Ra – L. Gaffney et al, Nature **497**, 199 (2013); P. Butler, JPG **43**, 073002 (2016)

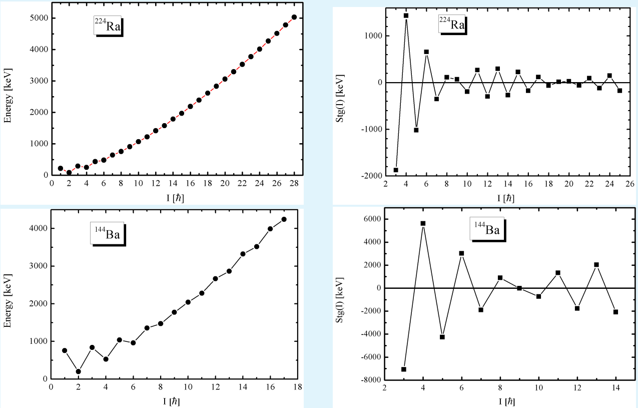
^{144,146}Ba – B. Bucher et al, PRL **116**, 112503 (2016); PRL **118**, 152504 (2017)

New data expected:

^{142}Ba, ^{222,224,226}Rn and ^{222,228}Ra – P. Butler, L. Gaffney et al – HIE-ISOLDE

^{140}Ba – T. Mertzimekis et al – IFIN-HH

Fine staggering effects in octupole bands



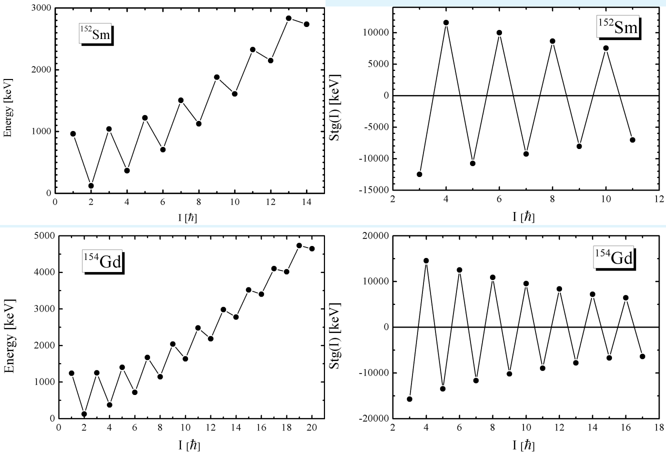
$$\mathrm{Stg(I)} = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2)$$

$$\Delta E(I) = E(I+1) - E(I)$$

$I < 10 - 12 \rightarrow$ octupole vibration and collective rotation

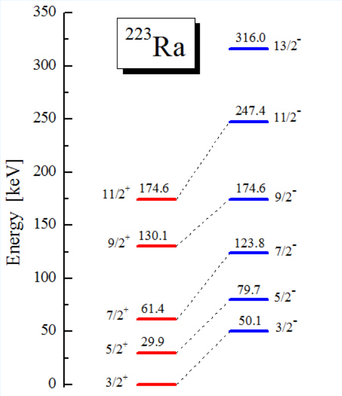
$I > 12 \rightarrow$ rotation of a stable quadrupole-octupole shape

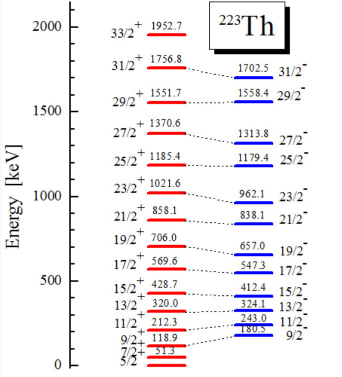
Alternating-parity bands in ^{152}Sm and ^{154}Gd



soft quadrupole-octupole mode

Split parity-doublet bands in ^{223}Ra and ^{223}Th (data from ENSDF)

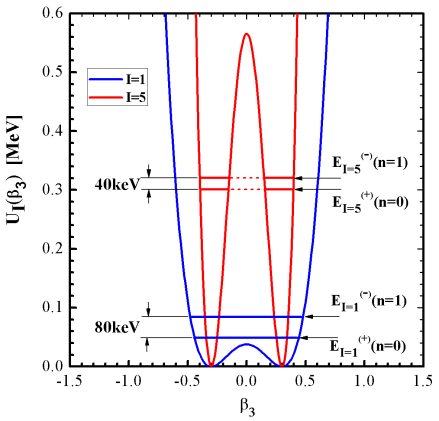


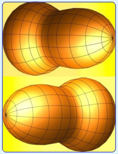


Model of octupole vibrations and quadrupole-octupole rotations

[N. Minkov, S. Drenska, P. Yotov and W. Scheid, JPG 32, 497(2006)]

$$U(\beta_3, I) = \frac{1}{2}C(I)\beta_3{}^2 + \frac{I(I+1)}{2(d_2 + d_3\beta_3^2)} \qquad C(I) = \frac{2X(I)d_3}{(d_2 + d_3\beta_{3\,\mathrm{min}}^2)^2}$$



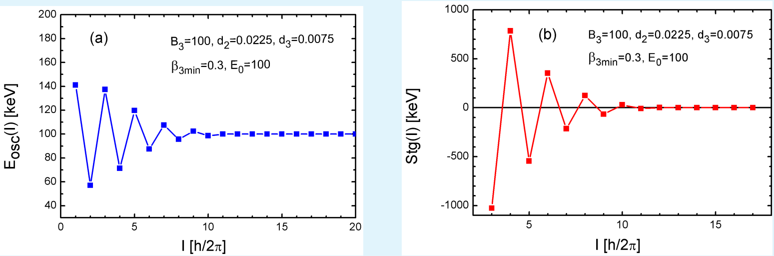


I=odd

I=even

$$E_{\mathrm{vib}}^{\mathrm{oct}} = E_0 - (-1)^I \frac{[E_I^{(-)} - E_I^{(+)}]}{2}$$

Energy and parity shift from double-well potential



⇒ **Low-spin parity shift effect**

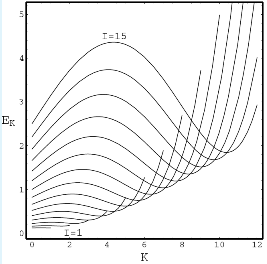
Quadrupole-octupole rotation Hamiltonian. Octahedron symmetry

$$\hat{H}_{\text{qorm}} = \hat{H}_{\text{quad}} + \hat{H}_{\text{oct}} + \hat{H}_{\text{qoc}}$$

$$\hat{H}_{\text{oct}} = \hat{H}_{A_2} + \sum_{r=1}^{2} \sum_{i=1}^{3} \hat{H}_{F_r(i)}$$

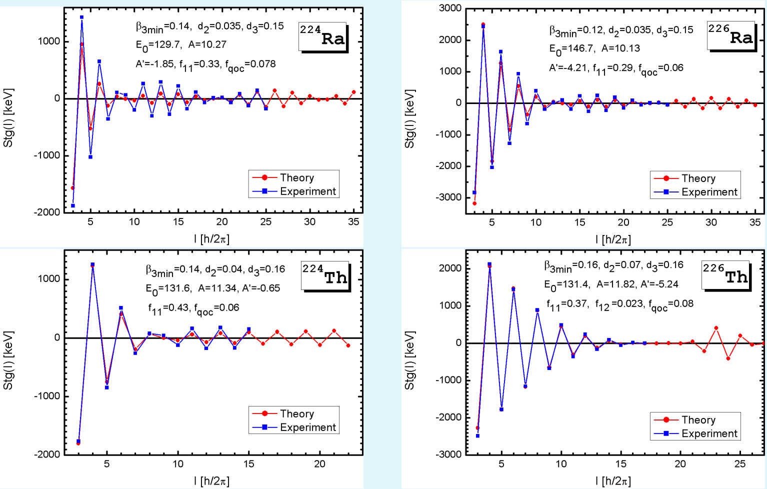
Point-symmetry contents: $\hat{H}_{F_1(1)} \rightarrow Y_{30} \rightarrow \mathrm{D}_{\infty}$; $\hat{H}_{F_1(2)} \rightarrow (Y_{3\,1} - Y_{3\,-1}) \rightarrow \mathrm{C}_{2v}$; $\hat{H}_{F_2(1)} \rightarrow (Y_{3\,2} + Y_{3\,-2}) \rightarrow \mathrm{T}_d$; $\hat{H}_{F_2(2)} \rightarrow (Y_{3\,3} - Y_{3\,-3}) \rightarrow \mathrm{D}_{3h}$ [Octahedron irreps]

$$\begin{aligned} E_K^{\text{qorm}}(I) &= AI(I+1) + A'K^2 + (1/2)f_{11}\left[5K^3 - 3KI(I+1)\right] \\ &+ f_{\text{qoc}}(1/I^2)\left[15K^5 - 14K^3 I(I+1) + 3KI^2(I+1)^2\right] \end{aligned}$$



$K_{\text{min}} \rightarrow$ high-spin "beat" staggering effect
[N. M. et al, PRC **63**, 044305 (2001)]

$E_{\mathrm{coll}} = E_{\mathrm{vib}}^{\mathrm{oct}} + E_{\mathrm{qorm}}$. "Beat" staggering patterns in Ra and Th nuclei.



[N. Minkov, P. Yotov, S. Drenska and W. Scheid, JPG **32**, 497 (2006)]

Model of Coherent quadrupole-octupole motion (CQOM)

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I)$$

$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{1}{2}\frac{d_0 + I(I+1)}{d_2\beta_2^2 + d_3\beta_3^2}$$

$$\beta_2 = \sqrt{d/d_2}\eta\cos\phi\ , \quad \beta_3 = \sqrt{d/d_3}\eta\sin\phi\ , \quad d = (d_2+d_3)/2$$

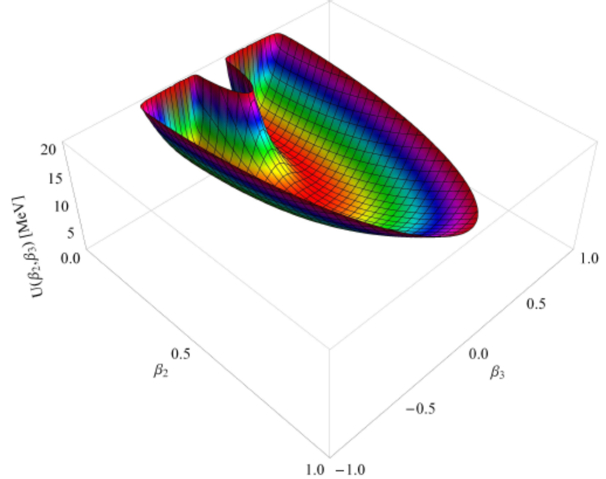
coherent quad-oct mode: $\omega = \sqrt{C_2/B_2} = \sqrt{C_3/B_3} \equiv \sqrt{C/B}$

$$E_{n,k}(I,\pi) = \hbar\omega\left[2n + 1 + \sqrt{k^2 + b[d_0 + I(I+1)]}\right]\ , \quad b = \frac{2B}{\hbar^2 d}$$

n=0,1,2,...; k=1,2,3,...

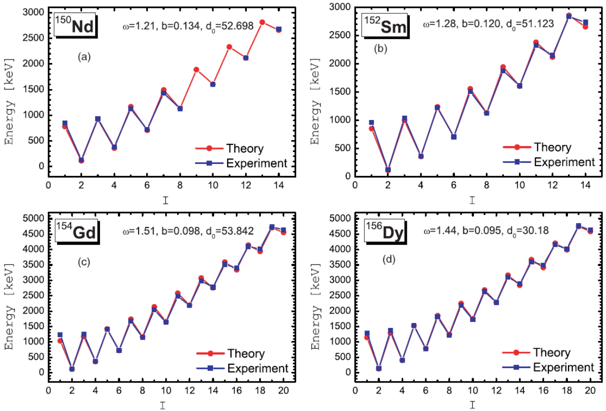
[N. M. et al, Phys. Rev. C **73**, 044315 (2006); **85** 034306 (2012)]

Quad.-oct. potential of a coherent mode $\omega = \sqrt{C_2/B_2} = \sqrt{C_3/B_3}$



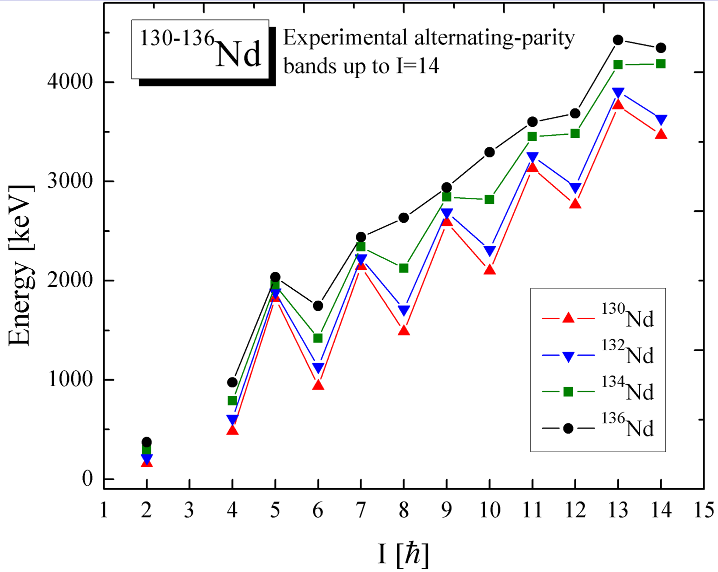
[Phys. Rev. C **73**, 044315 (2006)]

Theoretical and experimental alternating-parity bands in ^{150}Nd, ^{152}Sm, ^{154}Gd and ^{156}Dy

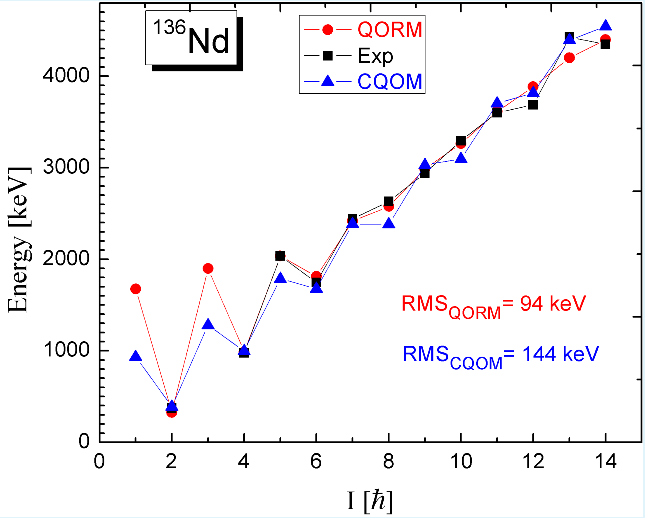


[Phys. Rev. C **73**, 044315 (2006)]

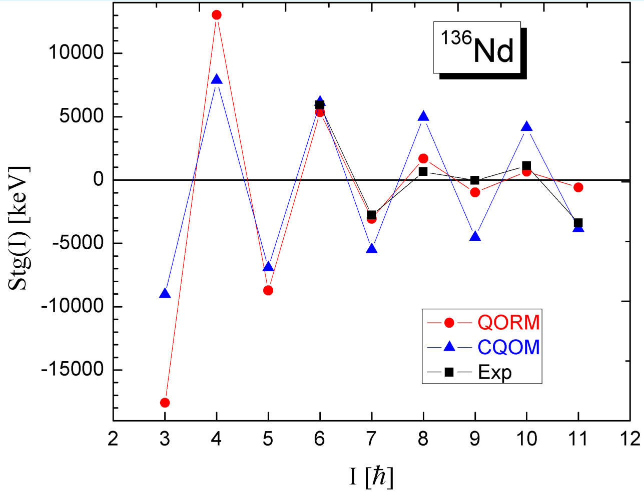
$^{130-136}$Nd: Experimental alternating parity bands (APBs) up to $I = 14$. Data from ENSDF.



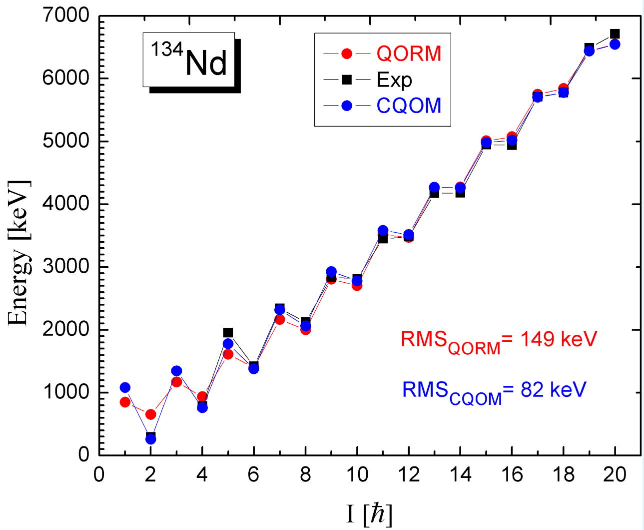
QORM and CQOM descriptions of ^{136}Nd APB. Data from ENSDF. Communication C. Petrache.



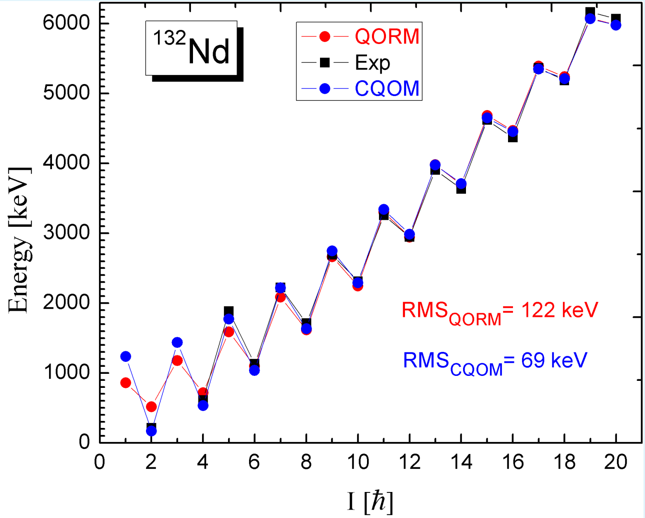
Experimental and theoretical staggering patterns for ^{136}Nd APB



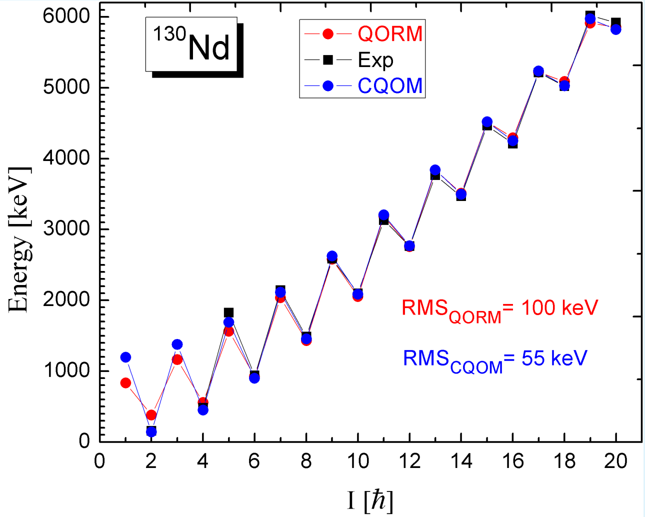
QORM and CQOM descriptions of ^{134}Nd APB. Data from ENSDF.



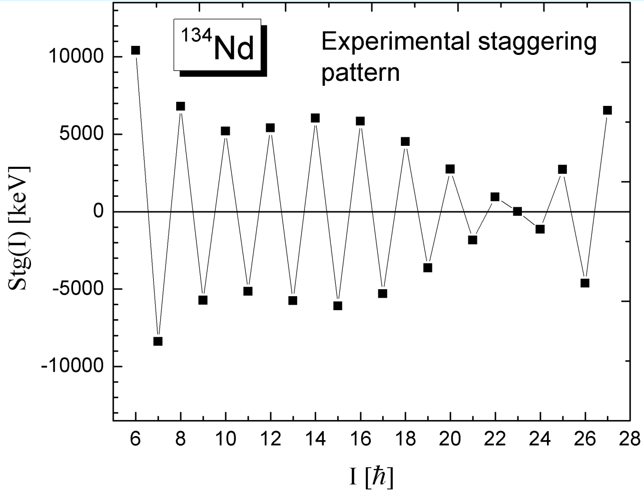
QORM and CQOM descriptions of ^{132}Nd APB. Data from ENSDF.



QORM and CQOM descriptions of ^{130}Nd APB. Data from ENSDF.



Experimental staggering pattern for ^{134}Nd APB up to I=30



Model interpretation of $^{130-136}$Nd APB data

^{136}Nd $\rightarrow$ possible stabilization of the pear-shape deformation, quadrupole-octupole (QO) rotation mode

^{130,132}Nd $\rightarrow$ soft QO rotation-vibration mode

^{134}Nd $\rightarrow$ possible transition between stable QO rotations and soft rotation-vibration motions

Nd isotopes $\rightarrow$ **Z=60** $>$ $(+4)$ **Z=56** octupole "magic number"

$^{130-136}$Nd $\rightarrow$ **N= 70,...,76** $<$ $(-18, ..., -12)$ **N=88** "mag"

$\Rightarrow$ **Riddle:** If our analysis is correct, what is the reason for the enhanced octupole deformation mode around Z=60, N=76?

$\rightarrow$ Needs for a deep microscopic analysis of the underlying octupole correlations in a deformed shell framework.

Quadrupole-octupole core plus particle Hamiltonian

$$H = H_{\mathrm{qo}} + H_{\mathrm{s.p.}} + H_{\mathrm{pair}} + H_{\mathrm{Coriol}}$$

$$H_{\mathrm{qo}} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I)$$

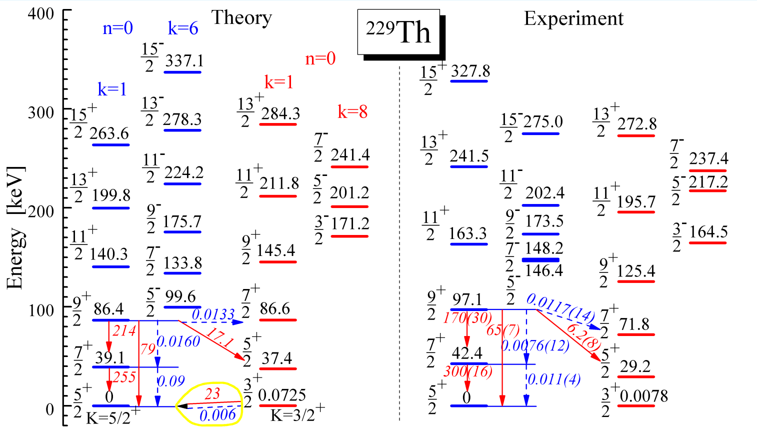
$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{d_0 + \hat{I}^2 - {\hat{I}_z}^{\,2}}{2\mathcal{J}(\beta_2,\beta_3)}$$

$$H_{\mathrm{Coriol}} = -\frac{(\hat{I}_+\hat{j}_- + \hat{I}_-\hat{j}_+)}{2\mathcal{J}(\beta_2,\beta_3)}, \qquad \mathcal{J}(\beta_2,\beta_3) = (d_2\beta_2^2 + d_3\beta_3^2)$$

$$H_{\mathrm{sp}} = T + V_{\mathrm{ws}}(\beta_2,\beta_3,...) + V_{\mathrm{s.o.}} + V_{\mathrm{c}}$$

$$H_{qp} \equiv H_{\mathrm{s.p.}} + H_{\mathrm{pair}} \rightarrow \epsilon_{\mathrm{qp}}^{K} = \sqrt{(E_{\mathrm{sp}}^{K} - \lambda)^2 + \Delta^2}$$

Theoretical and experimental quasi parity-doublet spectrum of ^{229}Th



DSM: s.p. orbitals GS(5/2[633]), IS(3/2[631]); $\beta_2 = 0.240$, $\beta_3 = 0.115$

BCS: $g_0 = 18.8$, $g_1 = 7.4$

CQOM: $\omega = 0.06$ MeV$/\hbar$, $b = 4.5\ \hbar^{-2}$, $d_0 = 45\ \hbar^2$, $c = 320$, $p = 1$

Coriol: $A = 0.144$ keV

Theoretical B(E2) and B(M1) transition values for ^{229}Th at different parameter sets

ω	b	d_0	c	p	A	$k_{yr}^{(-)}$	$k_{ex}^{(-)}$	rms$_{yr}$	rms$_{ex}$	rms$_{tot}$	$E_{ex}(\frac{3}{2}^+)$	$B(E2)$	$B(M1)$
0.2039	0.28	18	79	1.0	0.158	2	2	39.9	26.0	34	0.4263	27.04	0.0076
0.2361	0.28	33	89	1.0	0.141	2	2	41.2	26.4	35	0.0078	23.05	0.0061
0.0912	2.39	49	245	1.0	0.152	4	6	37.6	15.8	29	0.3556	25.80	0.0071
0.0635	4.51	45	321	1.0	0.144	6	8	36.4	12.4	28	0.0725	22.86	0.0063

$\Rightarrow$ transition probabilities for the $3/2^+$-isomer decay in ^{229}Th expected in the limits:

$B(E2)$=20–30 W.u. $\Rightarrow \tau \sim 10^{13} s$
$B(M1)$=0.006–0.008 W.u. $\Rightarrow \tau \sim 10^{4} s = 2.8h$

N. M. and A. Pálffy, Phys. Rev. Lett. **118**, 212501 (2017)
P. Bilous, N.M. and A. Pálffy, PRC **97**, 044320 (2018) [predictions for M1 and E2 internal conversion rates]

Magnetic moment in a s.p./q.p. state with $K = K_{bh}$

$$\hat{M}1 = \sqrt{\frac{3}{4\pi}}\mu_N[g_R(\hat{I} - \hat{j}) + g_s\,\hat{s} + g_l\,\hat{l}], \quad \hat{j} = \hat{l} + \hat{s}, \quad \mu_N = \frac{e\hbar}{2mc}$$

$$\mu = \sqrt{\frac{4\pi}{3}}\langle\widetilde{\Psi}_{IIK_b}|\hat{M}1_z|\widetilde{\Psi}_{IIK_b}\rangle$$

$g_s = q_s \cdot g_s^{free}$

spin gyromagnetic quenching (core polarization effect): $q_s = 0.6$

$g_R = q_R \cdot Z/A$

collective gyromagnetic quenching (pairing effect):

q_R=1.0, **0.8** (from exp. M1/E2), **0.7** (Nilsson), **0.6** (HF+BCS)

Magnetic moments (in μ_N) in the $3/2^+_{\mathrm{IS}}$ and $5/2^+_{\mathrm{GS}}$ states of ^{229}Th

N. Minkov and A. Pálffy, Phys. Rev. Lett., **122**, 162502 (2019).

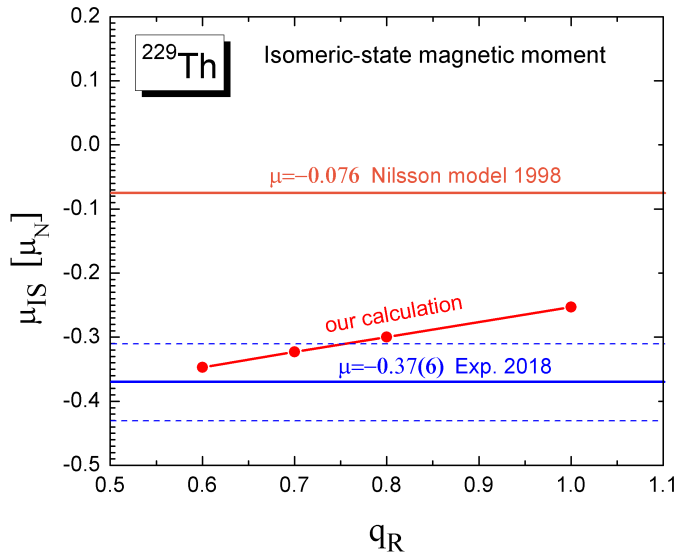
μ	q_R (our work)				other theories		laser spectroscopy			
	1.0	0.8	0.7	0.6	Th77	Th98	Exp74	Exp13	Exp18a	Exp18b
μ_{GS}	0.654	0.591	0.559	0.528	0.54	–	0.46(4)	0.360(7)	–	–
μ_{IS}	−0.253	−0.300	−0.323	−0.347	–	−0.076	–	–	(−0.3)-(−0.4)	−0.37(6)

Th77: Modified Woods-Saxon Model,
R. Chasman et al, Rev. Mod. Phys. **49**, 833 (1977)

Th98: Nilsson Model,
A. Dykhne and E. Tkalya, JETP Lett. **67**, 251 (1998)

Exp74: S. Gerstenkorn et al., J. Phys. (Paris) **35**, 483 (1974)
Exp13: M. Safronova et al, Phys. Rev. A **88**, 060501(R) (2013)
Exp18a: R. Müller et al., Phys. Rev. A **98**, 020503(R) (2018)
Exp18b: J. Thielking,..., P.Thirolf, E.Peik, Nature **556**, 321 (2018)

Magnetic moment (in μ_N) in the $3/2^+_{\mathrm{IS}}$ state of ^{229}Th

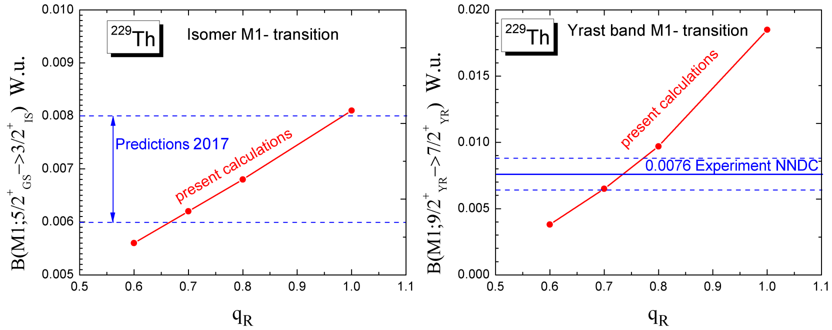


Predicted $B(M1)$ values (in W.u.) for ^{229}Th in dependence of q_R

Decay	q_R 1.0	q_R 0.8	q_R 0.7	q_R 0.6	Experiment
$\frac{3}{2}^{+}_{\text{ex}} \rightarrow \frac{5}{2}^{+}_{\text{yr}}$	0.0081	0.0068	0.0062	0.0056	–
$\frac{7}{2}^{+}_{\text{yr}} \rightarrow \frac{5}{2}^{+}_{\text{yr}}$	0.0096	0.0043	0.0025	0.0011	0.0110 (40)
$\frac{9}{2}^{+}_{\text{yr}} \rightarrow \frac{7}{2}^{+}_{\text{yr}}$	0.0185	0.0097	0.0065	0.0038	0.0076 (12)
$\frac{9}{2}^{+}_{\text{yr}} \rightarrow \frac{7}{2}^{+}_{\text{ex}}$	0.0144	0.0147	0.0149	0.0151	0.0117 (14)

N. Minkov and A. Pálffy, Phys. Rev. Lett. **122**, 162502 (2019)

Predicted $B(M1)$ values (in W.u.) for ^{229}Th in dependence of q_R



$\Rightarrow \mu_{GS}, \mu_{IS}$ constraints in the determination of isomer decay rates

Concluding remarks

- Model: CQOM+DSM+BCS with Coriolis mixing – description of K-suppressed EM transitions at axial symmetry
- ^{229}Th spectrum: $\rightarrow$ quadrupole-octupole-shape driven quasi parity-doublet structure built on 5/2[633], 3/2[631] qp states
- 7.8 eV $3/2^{+}$ isomer interpretation: a bandhead of an excited quasi parity-doublet, built on 3/2[631] q.p. state coupled to a collective quadrupole-octupole vibration mode and rotation motion – very fine interplay between all these modes
- Predictions for: B(E2) and B(M1) isomer decay probabilities; Magnetic dipole moment $\Rightarrow$ surprisingly good reproduction of $\mu_{\rm IS}^{\rm exp}$, further constraint on model conditions $[\mu_{\rm GS}^{\rm exp}]$
- Perspective: Possibility for a highly precise determination of ^{229m}Th decay rates and life-time $\Rightarrow$ input into the efforts for achieving Nuclear Clock frequency standard.