Collective coupling between pairing correlations and global rotation modes revisited

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INTRODUCTION

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Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. Bohr, B. R. Mottelson, and D. Pines*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

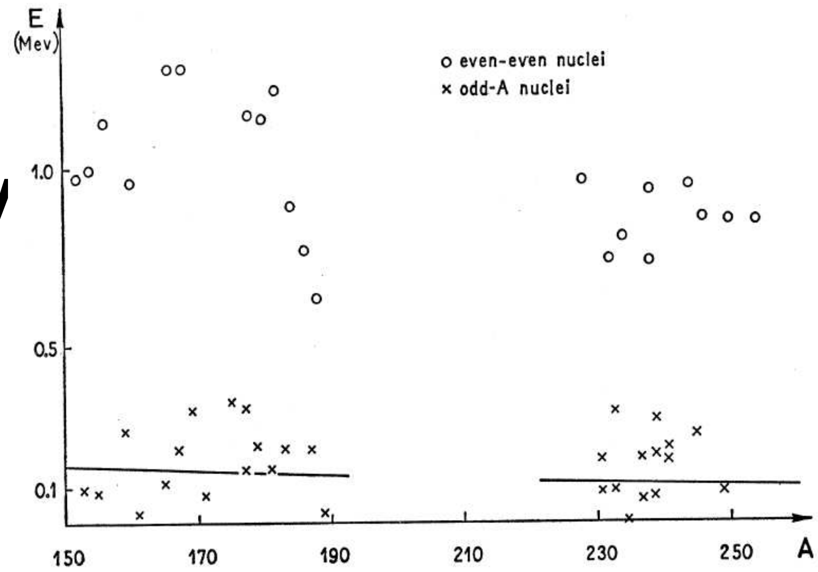
(Received January 7, 1958)

Evidence for an energy gap in the intrinsic excitation spectrum of nuclei

a) Comparison of first excited states in odd-odd and even-A nuclei

b) Moments of inertia appreciably smaller than rigid body values

c) Odd-even mass differences



Here we investigate the point b) :
Effect of pairing correlations on moments of inertia (M.o.I.) in two directions

1) as a collective coupling between pairing correlations and the global rotation mode

2) in connexion with the point c) related to odd-even-even mass differences

Both can be currently tackled quantitatively within a systematic study in particular within HFB or HF+BCS using phenomelogical effective N-N interactions

Limiting to well-deformed nuclei: obviously for M.o.I. and also to avoid large range collective fluctuations as well as to minimize energy effects of short range collective correlations (both not taken into account here)

1) COLLECTIVE COUPLING BETWEEN PAIRING AND GLOBAL ROTATION MODES

(J. Bartel, P.Q.)

After Bohr, Mottelson and Pines (1958), Migdal (1959) Griffin and Rich (1960) and Belyaev (1961) have made theoretical investigations of the effect of pairing on M.o.I. Mottelson and Valatin (1960) have made a fruitful parallel

VOLUME 5, NUMBER 11 PHYSICAL REVIEW LETTERS DECEMBER 1, 1960

EFFECT OF NUCLEAR ROTATION ON THE PAIRING CORRELATION

Ben R. Mottelson and J. G. Valatin*
NORDITA and The Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark
(Received November 8, 1960)

Analogy : Lorentz force and Coriolis pseudo-force

$$\overrightarrow{F_L} = q\,(\vec{v}\ X\ \vec{B}) \quad \text{and} \quad \overrightarrow{F_C} = 2m\,(\vec{v}\ X\ \vec{\Omega})$$

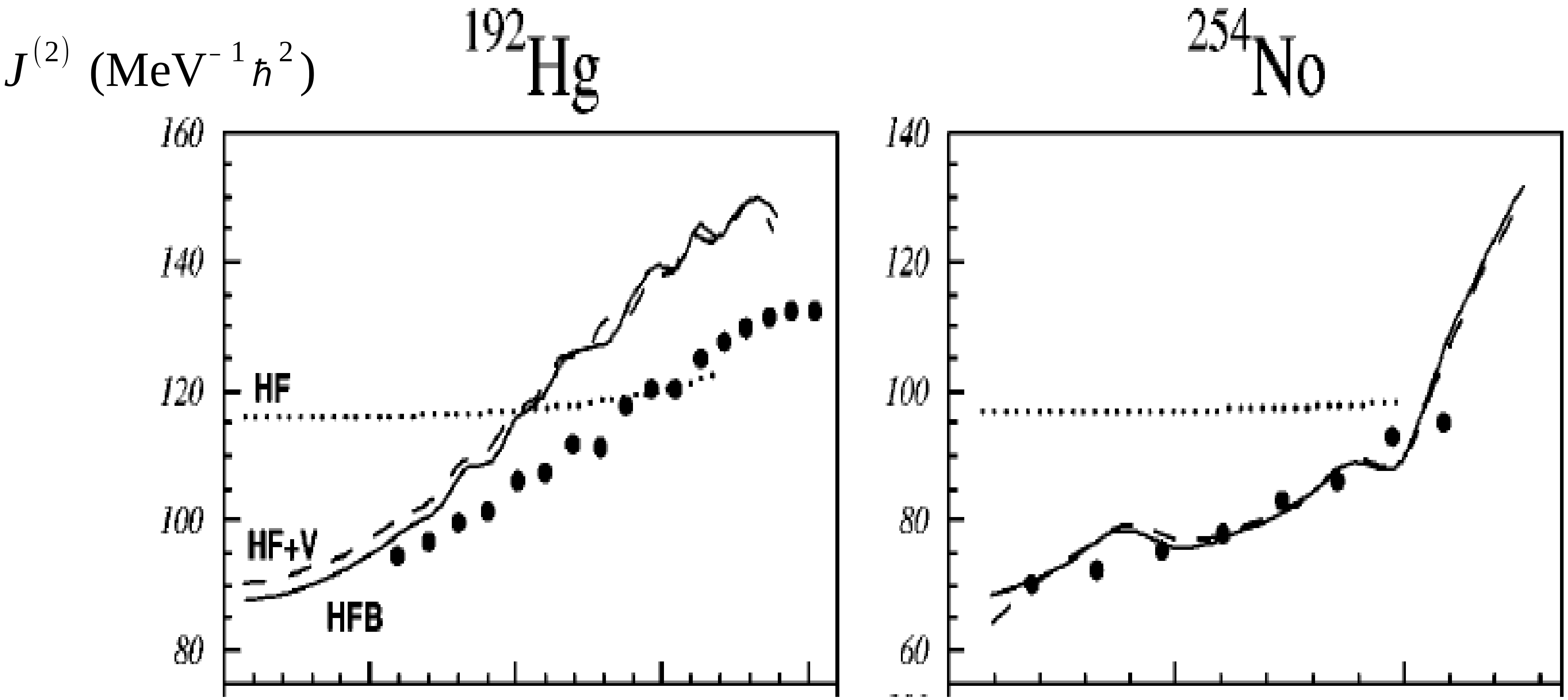
Coriolis anti-pairing (CAP) effect (~ type I superconductor)
<u>collective</u> gradual alignment within Cooper pairs

Other pairing-rotation couplings occur but are not of a collective nature as :
single pair breaking $\rightarrow$ qp alignment $\rightarrow$ backbending
Here we focus on the CAP phenomenon for even-even nuclei

It has been shown that the effect of pairing correlations on global rotation is well described in terms of a coupling à la Chandrasekhar (type-S ellipsoids) between rotational currents and those issued from a linear divergence-free intrinsic vortical current field (counter-rotating with respect to the global rotation)

See H.Lafchiev et al., Phys. Rev. C 67, 054315 (2003)
P. Quentin et al. Phys. Rev. C 69, 014307 (2004)

Equivalence of rotational properties between Routhian HFB and Routhian HF under constraint on the Kelvin circulation operator $\widehat{K}$



0.2 $\hbar\Omega$ (MeV) **0.1** $\langle K_x \rangle$ **constrained to its HFB value**

H.Lafchiev et al., Phys. Rev. C 67, 054315 (2003)

The Lagrange parameter (intrinsic angular velocity) ω **associated with** $\widehat{K}$ **in the constrained HF Routhian**

$$\delta(\widehat{H} - \Omega \widehat{J}_x - \omega \widehat{K}_x) = 0$$

has been phenomenologically linked to Ω **as follows**

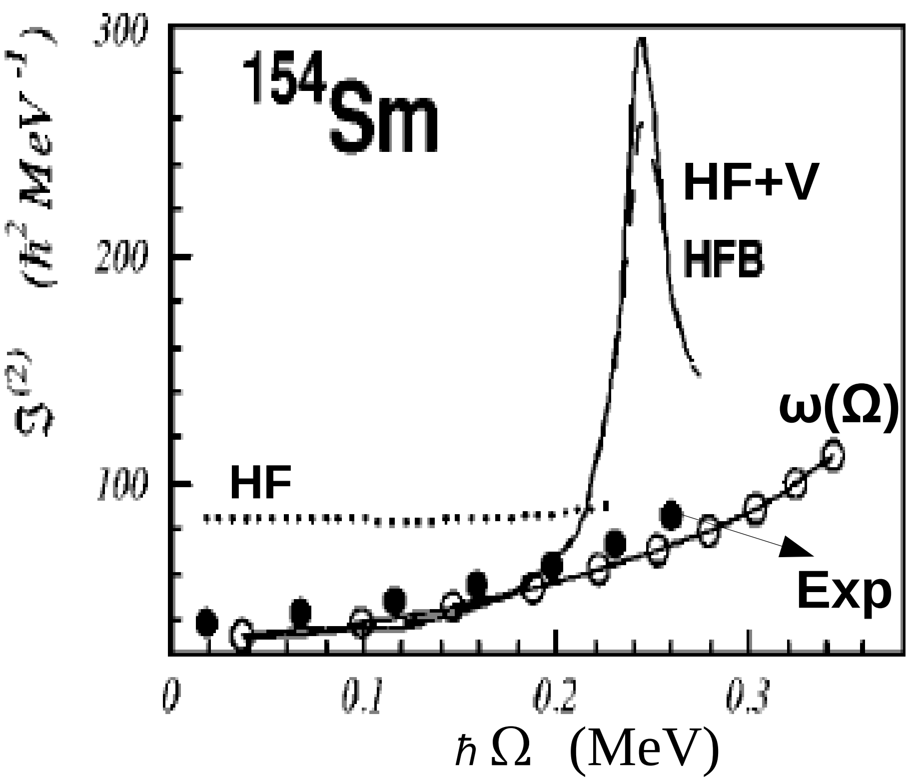
$$\omega = - \, k \, \Omega [1 - (\frac{\Omega}{\Omega_c})^2] \quad \text{with} \;\; k > 0 \;\; (\omega, \Omega > 0)$$

$\Omega \uparrow \quad \rightarrow \quad$ increasing counter rotation $\quad \rightarrow \quad (-\omega) \uparrow$

But $\Omega \uparrow \quad \rightarrow \quad$ decreasing pairing $\quad \rightarrow \quad (-\omega) \downarrow$

Consequently $|\omega|$ reaches a maximum and decreases to 0 at the critical angular velocity for pairing disappearance Ω_c

The critical angular velocity Ω_c **corresponds to the disappearance of pairing correlations**



(It depends a priori on the ground state intensity of pairing correlations)

The coupling strength k **has to be determined**

Aim: Find an analytical form for a Harris type M.o.I. up to third order Ω^2 terms in $E(\Omega)$ from

$$E(\Omega) = 1/2\,[A\,\omega(\Omega)^2 + 2B\,\omega(\Omega)\,\Omega + C\,\Omega^2] \text{ thus}$$

$$E(\Omega) = 1/2\,\Omega^2\,[C - 2Bk(1-\xi^2) + Ak^2(1-\xi^2)^2] \text{ with } \xi = \Omega/\Omega_c$$

The inertia parameters (A, B, C) have been calculated semiclassically within a Wigner-Kirkwood approximation up to $\hbar^2$ terms with $B \ll A \ll C$

They depend on the intrinsic deformation β (assuming an equivalent ellipsoidal shape)

Three quantities have to be introduced per nuclei to define

Ω_c, k, β

They are not fitted for each nucleus, they depend on zero or low E* experimental data

Question: how well these low E* inputs are able to reproduce higher spin energies whenever the CAP is valid

The critical value Ω_c **is determined by equating the zero pairing rotational energy with some estimate of a relevant pairing energy at zero spin**

$$\frac{1}{2} J_R \Omega_c^2 = E^{pair} \text{ with } E^{pair} = E_0 \frac{Q_0^{pair}}{Q_{pair}}$$

$$Q^{pair} = \sqrt{\Delta_n(N,Z)^2 + \Delta_p(N,Z)^2}$$

$$\text{where } \Delta_n(N,Z) = \frac{1}{2}\left(\delta_n^{(3)}(N-1,Z) + \delta_n^{(3)}(N+1,Z)\right) \quad (N \text{ even})$$

$$\text{with } \delta_n^{(3)}(N,Z) = \frac{(-1)^N}{2} \left[S_n(N,Z) - S_n(N+1,Z)\right]$$

and similarly for $\Delta_p(N,Z)$

The strength parameter k **is obtained in the lowest** Ω **limit by linearizing** $\omega(\Omega)$ **(i.e. from the first** 2^+ **energy)**

The deformation β **is obtained from** $B(E2;0^+ \rightarrow 2^+)$ **data**

The angular velocity Ω **associated to a spin value** I **is determined from the Lagrange parameter equation**

$$\hbar\,\Omega = \frac{dE(\Omega)}{d\,\tilde{I}} \text{ with } \tilde{I} = \sqrt{I(I+1)}$$

That is inverting the monotonic function

$$\tilde{I}(\Omega) = \int_0^{\Omega} \frac{1}{\hbar\,\Omega'} \frac{dE(\Omega')}{d\,\Omega'} d\,\Omega'$$

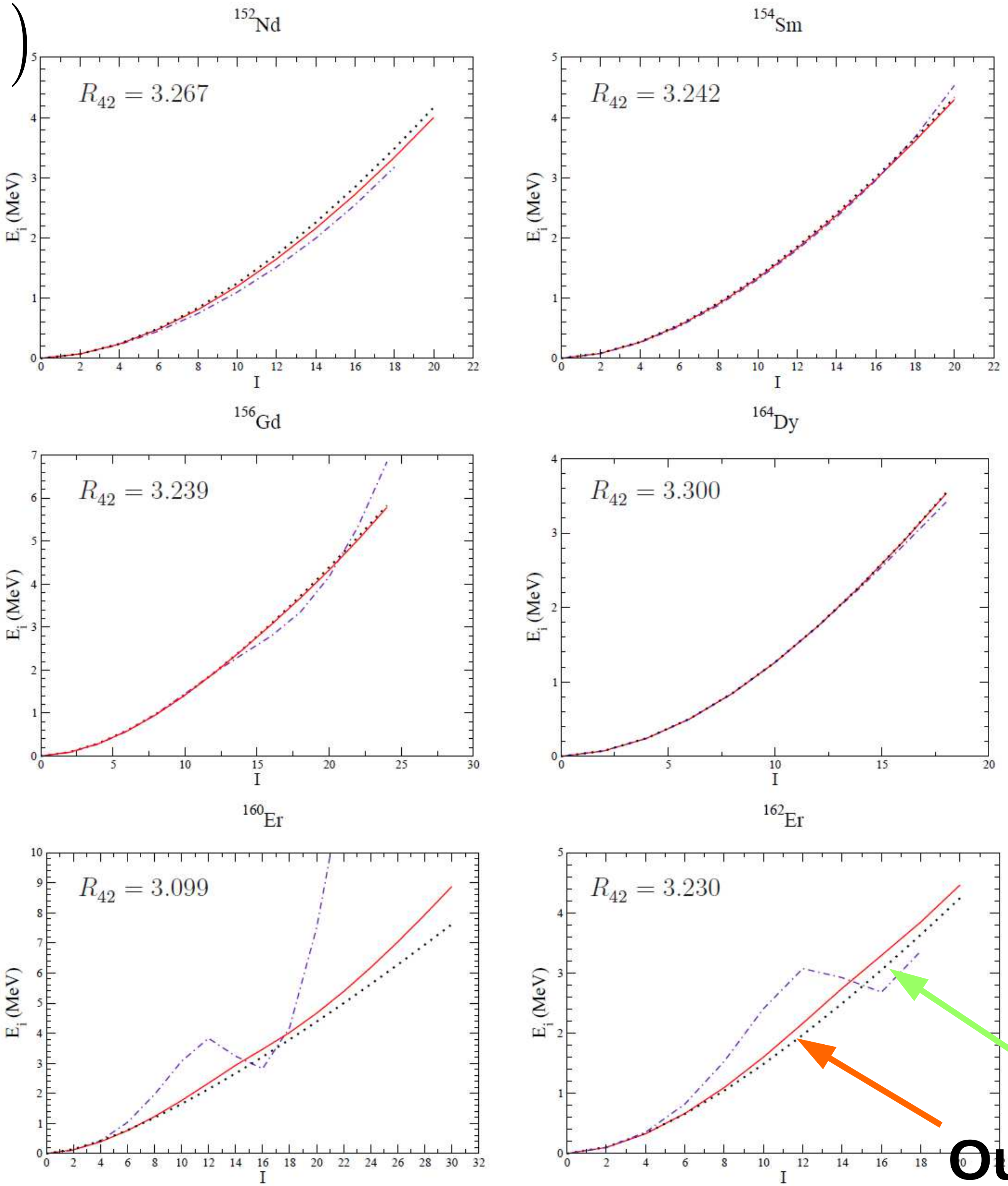
We compare for a sample of well-deformed even-even and actinide well-deformed nuclei experimental data and model results and shall present

- Ground band state energies E **as functions of** I

- Kinematical M.o.I. $J^{(1)}$ **as functions of** $(\hbar\,\Omega)^2$

The rotational indicator $R_{42} = \dfrac{E(4^+)}{E(2^+)}$ **is also reported**

$E(I)$



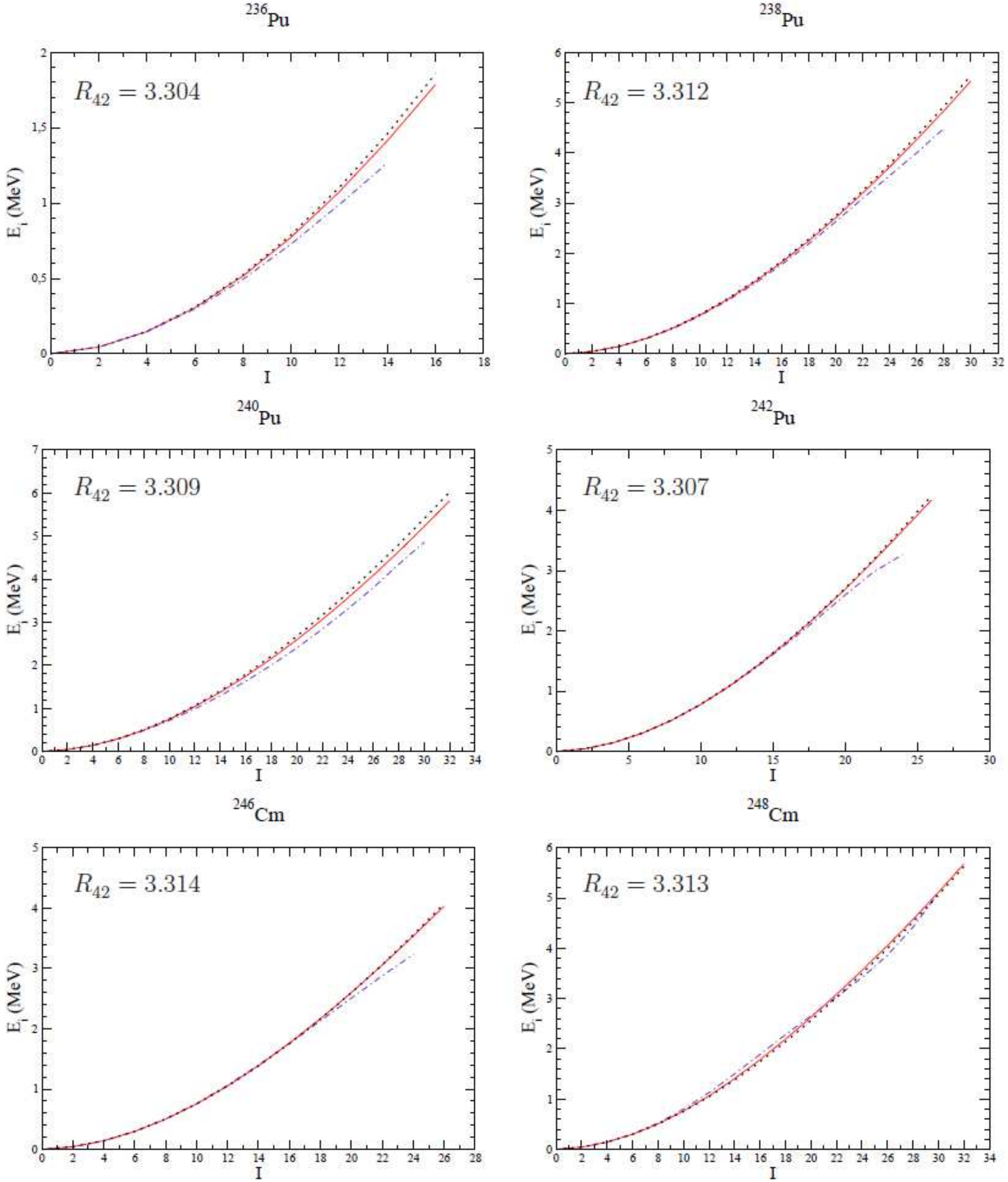
Rare Earth Region

Backbending !

$E(\Omega_{\exp}(I))$

Our model

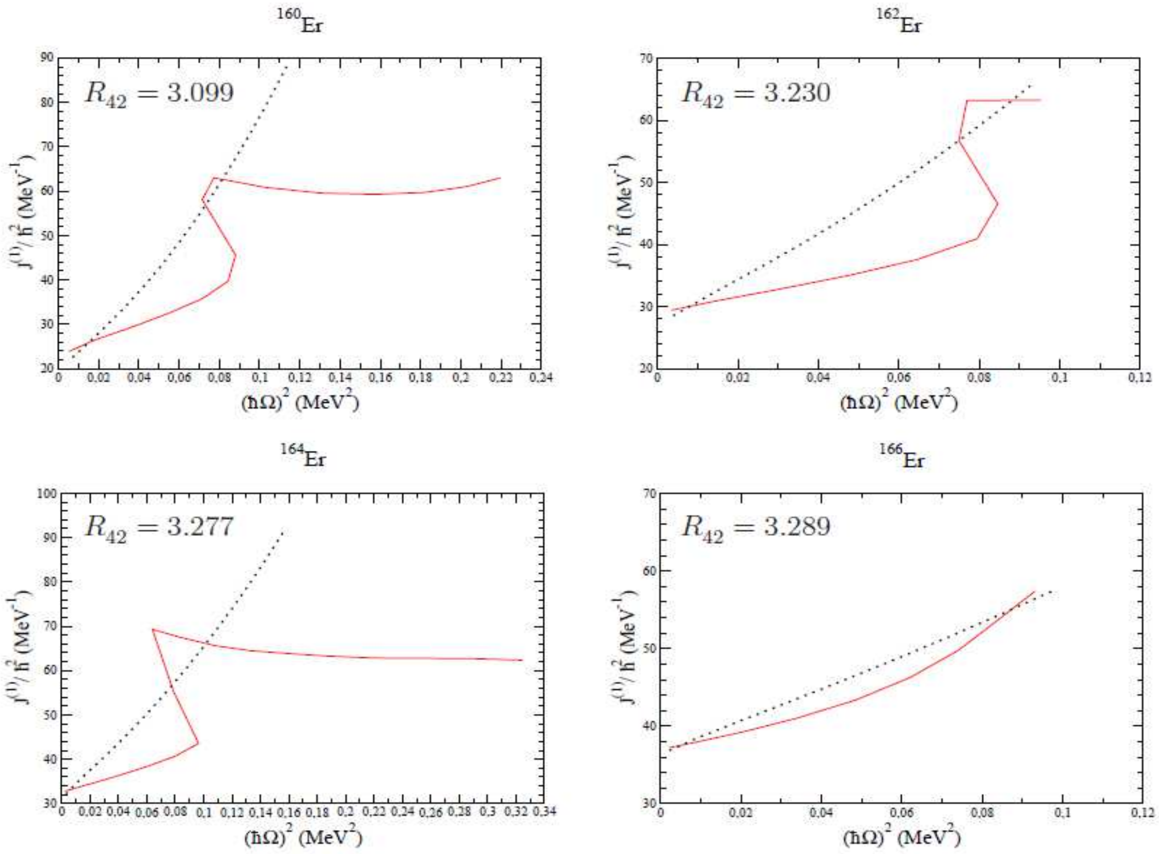
$$E(I)$$



Actinide Region

$$J^{(1)}(\Omega)$$

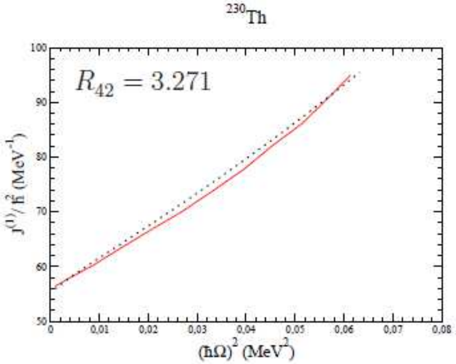
Rare Earth Region

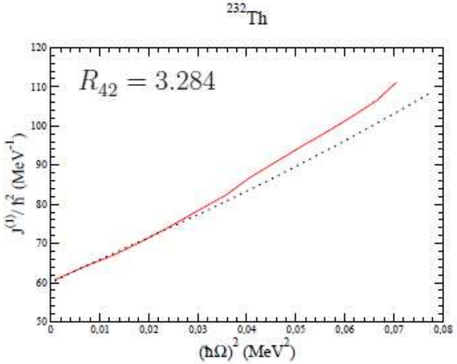


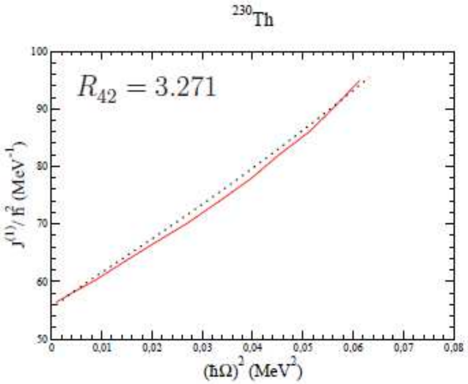
Backbending is clearly beyond the scope of our CAP model !

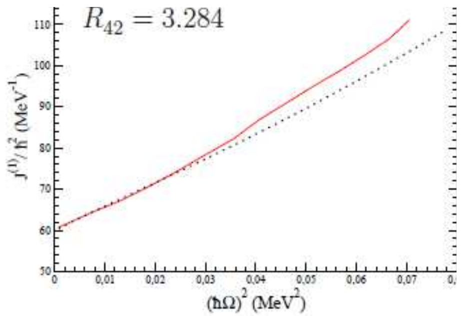
$J^{(1)}(\Omega)$

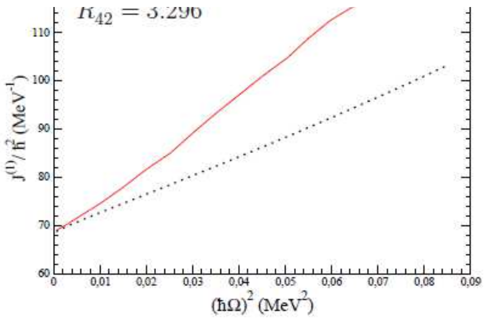
Actinide region

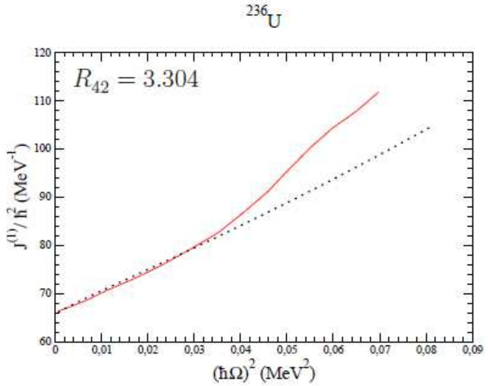


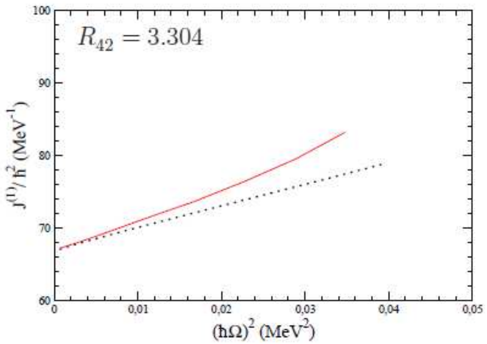


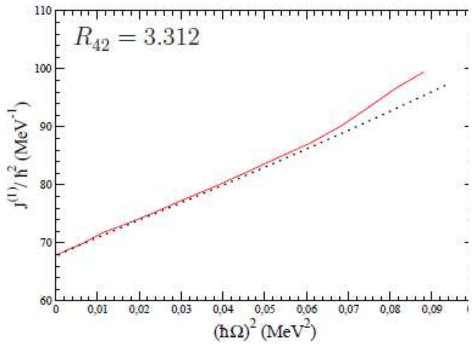


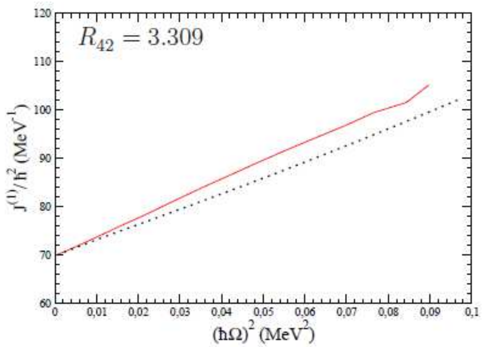


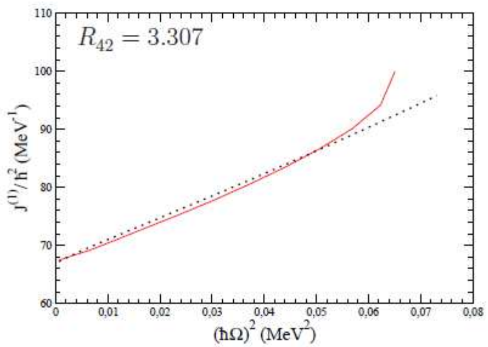


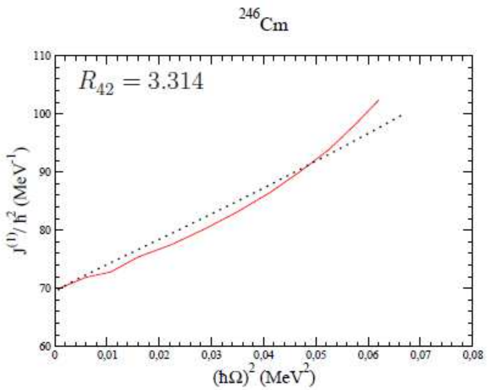


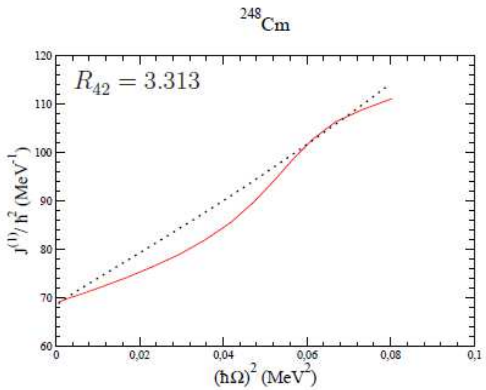












Straight line: Ω^4 terms
Beyond: Ω^6 terms

2) CONSISTENCY OF A FIT OF A SIMPLE PAIRING RESIDUAL INTERACTION FROM TWO INDEPENDENT POINTS OF WIEV

(Koh Meng-Hock, L. Bonneau , P.Q.)

Nurhafiza M. Nor et al., Phys. Rev C 99 (2019) 064306

Bohr, Mottelson and Pines have mentioned in particular two consequences of the existence of pairing

1) Moments of inertia appreciably smaller than rigid body values

2) Odd-even mass differences

We have computed both in a relevant sample of well deformed rare earth nuclei within the framework of Hartree-Fock + (self-consistent blocking) BCS calculations

We use the well-seasoned SIII Skyrme interaction
for the particle-hole channel
and a particle-number dependent seniority interaction
for the particle-paticle hole-hole channels

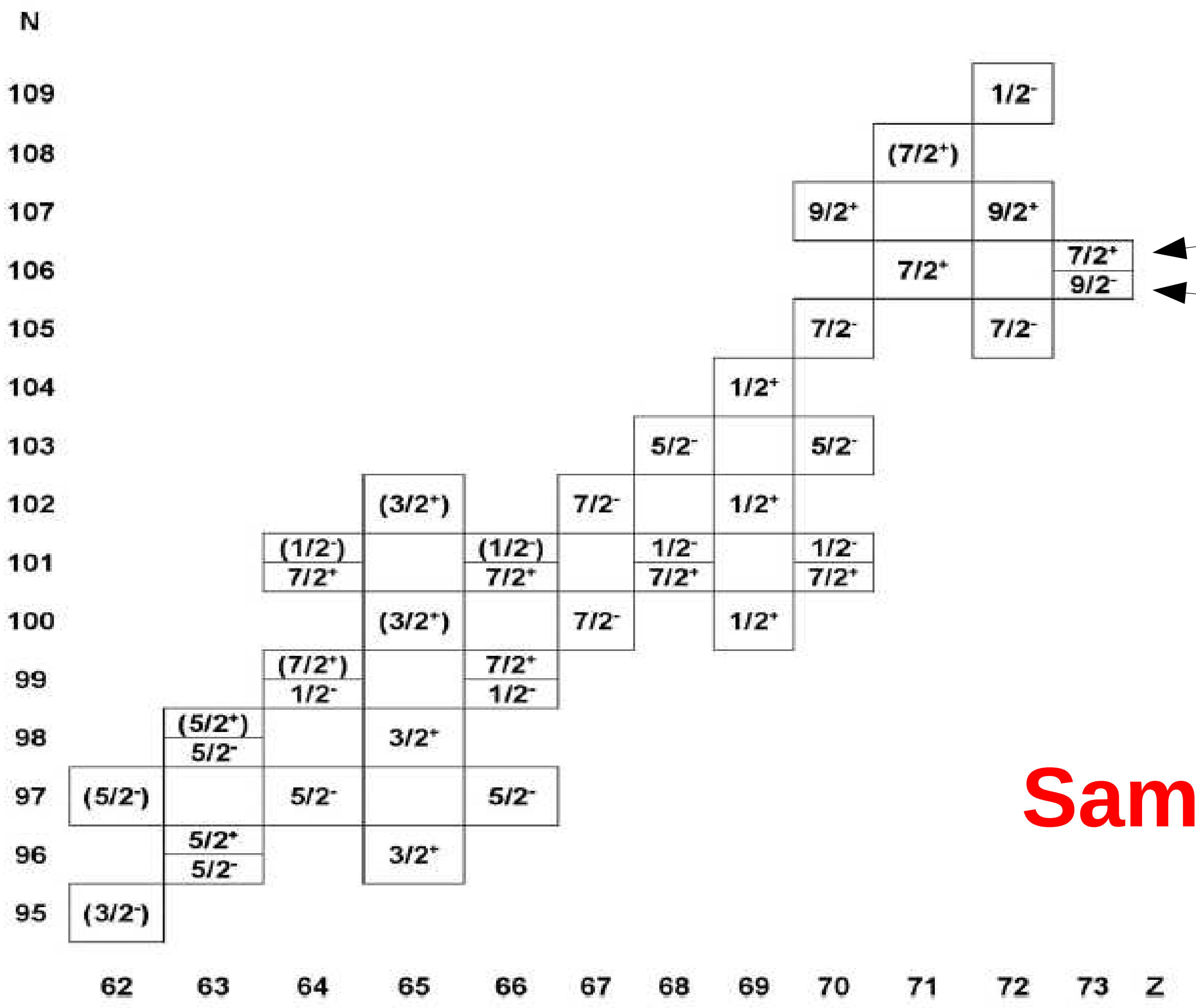
$$\forall \, |i>, |j> \in \{ \text{ canonical basis } \} \quad \langle i \, \bar{i} \, | v^{antisym.}_{residual} | \, j \, \bar{j} \rangle = \delta_{q_i, q_j} \frac{G_{q_i}}{11 + N_{q_i}}$$

Two quantities to be fitted G_q with q standing for n and p

Our sample includes 24 even-even rare earth nuclei
(most of our even-even selected nuclei fulfill $R_{42} \geqslant 3.3$ **)**

$$^{156,158,160}Sm \quad ^{160,162,164,166}Gd \quad ^{162,164,166,168}Dy \quad ^{168,170,172}Er$$
$$^{170,172,174,176,178}Yb \quad ^{176,178,180,182}Hf \quad ^{180}W$$

together with 17 odd-neutron and 14 odd-proton neighbouring nuclei



Experimental I^{π}

Calculated K^{π}

Sample of odd-A nuclei

Ground state spins and parities reasonably well reproduced (agreement - when only one entry shown - for ~ 77% cases)

Fit on M.o.I.

Calculated with the Inglis-Belyaev formula
plus approximate Thouless-Valatin correction *
(* including the T-odd density response
to the T-odd part of the HF+BCS Routhian)

2D Mesh in the $\left(G_n, G_p\right)$ **plane**
Minimal r.m.s. deviation by a cubic regression approach

$$G_n = 16.27 \ \text{MeV} \quad \text{and} \quad G_p = 15.26 \ \text{MeV}$$

$$\sqrt{\left\langle \left(J^{calc.} - J^{exp.}\right)^2 \right\rangle} \approx 1.7\hbar^2 \text{MeV}^{-1} \quad \text{for } J \approx 40\hbar^2 \text{MeV}^{-1}$$

Fit on Odd-Even mass differences

Here two quantities to be fitted $\left(\Delta_n^{(3)}, \Delta_p^{(3)}\right)$
by two parameters $\left(G_n, G_p\right)$

Actually the two fits (on neutrons and protons) are rather well decoupled

Yet we have searched for a minimal r.m.s. deviation by a cubic regression approach for a quantity where the squared deviation for n and p have been simply added

$$G_n = 16.10\ (16.27)\ \text{MeV} \quad \text{and} \quad G_p = 14.84\ (15.26)\ \text{MeV}$$

$$\sqrt{\left\langle \left(\Delta_n^{(3)calc.} - \Delta_n^{(3)exp.}\right)^2 \right\rangle} \approx 90\, keV$$

$$\sqrt{\left\langle \left(\Delta_p^{(3)calc.} - \Delta_p^{(3)exp.}\right)^2 \right\rangle} \approx 180\, keV$$

Agreement between the two fits : 1% for n, 3% for p

CONCLUSIONS

1) The physical assumptions that the M.o.I. decrease from the rigid body value and the odd-even mass differences stem from pairing correlations are documented here
Not much of a surprise but an illustration …

2) Our rather crude way to handle it (e.g. BCS, seniority force, approximate Thouless-Valatin corrections …) seems sufficient to describe these physical effects

3) Our choices of relevant samples seem appropriate

4) The description of the Coriolis Anti-Pairing is very well described within the Chandrasekhar's S - ellipsoids frame up to spins where another physical effect switches on
A clean-cut confirmation of the collective modes at work

5) The 44 years old SIII force not so bad for spectroscopy...