Revisiting the concept of nuclear rotation: 
quantum and classical viewpoints

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Abstract

Nuclear rotation is one of the most important concept for understanding nuclear many-body systems. It assumes a possibility of separation of the Hamiltonian to the internal and collective degrees of freedom, i.e., $H = H_{\text{int}}(\vec{p}, \vec{q}) + H_{\text{coll}}(J, \Omega)$. The canonical pair of intrinsic dynamical variables $(\vec{p}, \vec{q})$ are represented in the first term as a multi-dimensional vector, and the second term corresponds to the quantized rigid-body motion. However, as implied in the Bohr model \(^1\), the interplay between the internal and collective degrees of freedom may not be neglected under certain circumstances.

In particular, Beryllium 8 ($^8\text{Be}$), which can be regarded as a two-alpha system, provides an interesting physical case challenging the concept of nuclear rotation. The nucleus is known to have a rotational-like band, but the system itself does not possess a bound state. The “rotational states” of $^8\text{Be}$ need to be built on a resonance. From a phenomenological two-alpha potential (the Buck potential) \(^2\), the resonant state is expected to be “soft” unlike molecular structures of O$_2$, in which the system is tightly bound to form a hard “crystal”. When we imagine “rotational motion” of a rigid body or similar kinds in our mind, the motion depends on time, but quantized rotation of a nucleus does not require time to flow. The connection of classical (time-dependent) rigid-body motion and its quantized counterpart (time-independent) was discussed originally by Casimir and Wigner \(^3\), so that it is most important to read their results correctly for a proper interpretation of “nuclear rotation” of “soft” nuclear many-body systems.

Comparing with the Bohr model, I attempt to extract an unnoticed point in the “nuclear rotation” based on the study of the $^8\text{Be}$ structure, which might be interesting as a possible extended application to the similar elongated systems, such as deuteron, axial symmetric nuclei with super- and hyper-deformation, and so on.

References

\(^1\) A. Bohr, Mat. Fys. Medd. Dan. Vid. Selsk. 26 (1952) 9, 18, 21, 97, 336.
\(^3\) H.B.G. Casimir, Rotation of a rigid body in Quantum Mechanics (1931); E. Wigner, Gruppentheorie und ihre Anwendungen auf die Quantenmechanik der Atomspektren (1931).