Nuclear Theory'21
ed. V. Nikolaev, Heron Press, Sofia, 2002

Pre-Equilibrium Complex Particle Emission*

E. Běták

Institute of Physics, Slovak Academy of Sciences, 84228 Bratislava, Slovakia**
Faculty of Philosophy and Sciences, Silesian University, 74601 Opava, Czech Rep.

Abstract.

Semi-classical (phenomenological) pre-equilibrium emission of clusters of nucleons (complex particles) such as deuterons, tritons, helions and α particles from reactions induced by light projectiles (nucleons to α's) is addressed. The main attention is given to the hard components in the emission energetic spectra, which play an increasing role at incident energies above 20 MeV, and are currently attributed to a presence of some kind of pre-equilibrium processes. In addition, the mechanisms of cluster reactions show special features such as the competition between pickup and knockout processes and the contributions of several successive steps in the reaction. The main frame used here to illustrate the processes and interplays of the competing mechanisms of pre-equilibrium cluster formation and emission, namely the coalescence, pick-up and knock-out, is the pre-equilibrium exciton model. It obviously contains the process of clusterisation itself as its organic part.

The most important case of complex particles with the largest amount of experimental data is that of alpha emission, which therefore naturally attracts most of the attention and where the widest range of possible mechanisms is available on the market. The loosely bound ejectiles, on the other side, are usually not able to demonstrate all features of the whole spectrum of contributing mechanisms, but they are nevertheless an important link between the nucleon emission and the cluster one.

*Partially based on the review article written together with P.E. Hodgson [1].
**URL:http://nic.savba.sk/∼fyzibeta

1 Introduction

There is considerable evidence that nucleons in the nucleus spend part of their time in clusters, particularly on the nuclear surface [2]. Such clusters — or complex particles, as they are alternatively refered to — can participate in nuclear reactions and this enables their properties to be studied. The most likely cluster, due to its spin and isospin symmetry and hence its high binding energy, is the α-particle, but there is the possibility of other clusters as well.

The reactions involving such clusters can be treated by considering the individual nucleons but in some circumstances the cluster behaves as a single entity and it is therefore more convenient to treat it as such, without explicit reference to its internal structure or its constituent neutrons and protons.

The mechanism of a nuclear reaction depends on the incident energy. At energies below about 20 MeV, it is likely that the incident particle is captured by the target nucleus to form an excited compound nucleus which then decays statistically by sequential emission of particles until its energy falls below the particle emission threshold, when it returns to the ground state by γ emission. As the incident energy increases, direct processes, which take place in the time taken by the incident particle to traverse the target nucleus, become increasingly likely. At still higher energies pion (and other meson) productions become possible.

For many years it was assumed that nuclear reactions take place either by the direct process or by the compound nucleus process or by both processes together. The direct process occurs rapidly, whereas the compound nucleus process takes much longer, after full statistical equilibrium has been established. In recent years this sharp division into direct and compound nucleus processes has proved inadequate, and there is much evidence for the presence of pre-equilibrium reactions that take place after the direct stage but long before the attainment of full statistical equilibrium (see, e.g. the review by Gadioli and Hodgson [3]). These pre-equilibrium reactions take place in a number of stages or steps, corresponding to the excitation of successive particle-hole pairs by the interaction of the projectile with nucleons in the target. At each stage it is possible for emission to occur, and these are the pre-equilibrium reactions.

The emission of a particle from a nucleus does not imply that it is present in the nucleus before emission, so that the success of the cluster approximation in the treatment of a nuclear reaction does not imply that a cluster was indeed present in the nucleus before the reaction. Even if the constituents are already present, the cluster may be formed during the reaction. If there are clusters in the nucleus, they are affected by the surrounding nucleons and so do not have the same properties as in the free state.

We review some of theories used to analyse cluster reactions. Here, we restrict ourselves to the semi-classical (phenomenological) pre-equilibrium approaches. The microscopic pre-equilibrium approaches would require more space and the reader is refered to our recent review [1].

2 Compound Nucleus

The theory of compound nucleus reactions was proposed by Bohr [4]. He suggested that a nuclear reaction begins with the capture of the projectile by the target nucleus followed by the sharing of the energy among all the nucleons of the compound system. The compound nucleus lives long enough for complete statistical equilibrium to be established. Subsequently, nucleons or clusters of nucleons are emitted by a statistical process similar to evaporation of molecules from a liquid drop, until finally the residual nucleus reaches its ground state by γ emission.

According to this theory, the processes of formation and decay of the compound nucleus are independent of each other; this is the Bohr independence hypothesis. It was first tested experimentally by Ghoshal [5] by forming the same compound nucleus in two different ways and then verifying that they decay in the same way. More accurate tests followed both within the compound nucleus concept and also outside of it, i.e. allowing for the pre-equilibrium emission [6–9].

The Bohr independence hypothesis enabled Weisskopf and Ewing [10] to develop a theory of compound nucleus reactions that enabled the total cross sections of reactions to all outgoing reaction channels to be calculated. Subsequently, Hauser and Feshbach [11] formulated a more detailed theory that includes spin dependence and enables the differential cross sections of reactions to discrete and continuum states to be calculated. These theories are described in detail in literature (e.g. [3, 12]).

Many calculations of reaction cross sections using these theories have shown their basic correctness, but very frequently other reaction mechanisms also contribute and these must be taken into account when analysing any data. It is then essential to evaluate the cross sections of these processes as well.

The cross section for the formation of a compound nucleus (or a composite system in general) may be written

$$\sigma_c^{J\Pi} = \frac{\pi}{k^2} \frac{2J+1}{(2J_{\rm targ}+1)(2i_x+1)} T_i^{J\Pi} R_i^{J\Pi}, \tag{1}$$

where i stands for all quantum numbers of the incident channel ($i \equiv A_{\rm targ}$, x, s, l), s is the channel spin ($\mathbf{s} = \mathbf{J}_{\rm targ} + \mathbf{i}_x$) and $T_i^{J\Pi}$ is the transmission coefficient. Up to the last factor, the above equation coincides with that given by Hauser and Feshbach [11]. The factor $R_i^{J\Pi}$, used by [13], accounts for the difference between the optical model cross section and that which remains after depletion by direct and pre-equilibrium processes. Now, the rate of creation of the compound nucleus within channel i is $W_c^{J\Pi} = \sigma_c^{J\Pi} v/\Omega$, where v is the relative velocity and Ω is the volume, in which the particle is confined [14]. Using the principle of detailed balance, we get the averaged decay rate into i-th chan-

nel $W_i^{J\Pi} = W_c^{J\Pi}\rho_i^{J\Pi}(\varepsilon)/\rho^{J\Pi}$ and correspondingly the full decay rate a summation over all open channels j is $W^{J\Pi} = \sum_j W_j^{J\Pi}$. The Bohr independence hypothesis [4] enables the creation and decay of the compound nucleus to be separated, giving the cross section for the compound nucleus reaction from channel i to channel j in the form $\sigma_{ij}^{J\Pi} = \sigma_i^{J\Pi}W_j^{J\Pi}/W^{J\Pi}$ [11]. There are no cluster preformation probabilities in this formulation because they are taken into account by using detailed balance.

3 Semi-Classical (Phenomenological) Pre-Equilibrium Theories

3.1 Pre-equilibrium Particle Emission — Basis of the Model

Pre-equilibrium models (see [3, 15–17]) are widely used to describe nuclear reactions in the energy range 10 to 200 MeV. So far, reactions involving only nucleons are reasonably understood, whereas theories of cluster (complex particle) and gamma emission are still partially deficient. Nevertheless, reasonable progress has been made in these directions.

The first analyses of pre-equilibrium reactions were made using the exciton model, and this proved able to account for the total cross sections in many reaction channels in a unified way ([16,18,19]). This model, and other semi-classical models, were however less successful in accounting for the angular distributions of the emitted particles. The Boltzmann master equation theory of Harp, Miller and Berne [20] has also been used to calculate the energy spectra of particles emitted in nucleon-induced reactions and also in heavy-ion reactions [21].

Pre-equilibrium emission of light clusters (d to α) is treated in several distinctive ways. The broadest range of model assumptions has been developed for α particles, the most frequent cluster ejectiles. Already in the initial period, two opposite mechanisms have been suggested. The concept of pre-formed α particles [22] assumes that the α particle is a strongly coupled object, and assumes that it can be treated as a single (special) exciton. On the other hand, coalescence models initiated * by Cline [24] and Ribanský and Obložinský [25] assume forming a cluster (not necessarily the α particle) in the course of a reaction from excitons, or — in its later modifications (see [26,27]) — also from already unexcited nucleons.

The coalescence models are of more general nature that the pre-formed ones and they are currently applied to all types of light complex particles. Apart from these two groups of models with straightforward physical backgrounds, phenomenological descriptions are also popular [28], which are employed e.g. in the well-known code GNASH [29] or in the recent version of EMPIRE [30]. In fact, their predictive power is higher than those of the former group, though they are handicapped by more parameters.

*In fact, the attempts to calculate pre-equilibrium complex particle emission can be traced back to [23].

The pre-equilibrium model assumes the reaction to proceed via a sequence of relatively simple states characterized by their exciton number. The energy spectrum of the emitted particles and/or γ quanta in the spin-independent formulation of the model is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon_x} = \sigma_{\mathrm{R}} \sum_n \tau_n \lambda_x^c(n, E, \varepsilon_x), \tag{2}$$

where $\lambda_x^c(n, E, \varepsilon_x)$ is the particle (or γ) emission rate from an n-exciton state ($n = p + h$) of excitation energy E to continuum, the energy of the ejectile of type x is ε_x, and τ_n and σ_{R} are the time spent in an n-exciton state and the cross section of creation of the composite system, respectively *.

The nucleon (x denotes either the proton π or the neutron ν) emission rate (see, e.g. [3]) is

$$\lambda_x^c(n, E, \varepsilon_x) = \frac{2s_x + 1}{\pi^2 \hbar^3} \mu_x \varepsilon_x \sigma^*_{\mathrm{INV}}(\varepsilon_x) \frac{\omega(p-1, h, U)}{\omega(p, h, E)} R_x(p)\ , \tag{3}$$

where μ_x and s_x are the ejectile reduced mass and spin, respectively, σ_{INV} is the inverse cross section, which is, in fact, replaced by the optical model cross section describing the capture of a projectile x by the nucleus in its ground state, and $U = E - B_x - \varepsilon_x$ is the energy of residual nucleus which is produced in an $(n-1)$-exciton state. The charge factor $R_x(p)$ takes into account the charge composition of the excitons with respect to the ejectile, but is not generally accepted ** (see also [3]).

3.2 Parameters of the Model

The calculations within pre-equilibrium models (e.g. within the exciton model, but similarly also in other formulations, like the hybrid model [32]) depend on a whole set of quantities and parameters. Some of them are common to other fields of nuclear physics, like the level density parameters (e.g. single-particle level density g and pairing δ), inverse cross sections σ_{INV} (usually replaced by the reaction cross sections σ_{R}) or the transmission coefficients T_l, while other are specific to the pre-equilibrium problems. Two of the most pronounced of them applicable to all types of reactions are the initial exciton number n_0 and the average squared transition matrix element of the residual interaction $|M|^2$. The intensity of the intranuclear transitions may be alternatively expressed using the optical potential, if we allow for scaling results by some normalizing factor.

*Some authors prefer using closed expressions. In such case, the summation in (2) stops at the equilibrium exciton number $\bar{n}$, and the compound nucleus contribution (reduced by the pre-equilibrium emission) is added to the truncated sum.

**We follow here the charge factor $R_x(p)$ of [24]. Explicitly, the relation $R_\pi(p) + R_\nu(p) = 1$ holds for nucleons at each stage of the reactions. A recent discussion of various forms of the charge factors is given by [31].

The matrix element $|M|^2$ can, in principle, be derived from our knowledge of nuclear matter and of nucleon-nucleon cross sections, but the values needed to reproduce observed cross sections differ significantly from those obtained from the basic knowledge. Because of that, only the energy and mass-number dependence is kept, but the absolute value is fixed as to describe reasonably a large set of observed data. The most popular form used is either $|M|^2 = K \cdot A^{-3} E^{-1}$ [33] (K is the constant determined from fit to the data) or in a slightly more refined form where the dependence on the total excitation energy E is replaced by the average excitation energy per exciton, $e = E/n$. For e between 7 and 15 MeV its form coincides with the preceding one $|M|^2 = K' \cdot A^{-3} e^{-1}$ [34], where $K' = 100 - 110$ MeV^3 gives reliable overall results over a reasonable set of reactions, if we use the nucleon emission rates (3) with the charge factor $R(p)$ (see [35] and so-called *"Blind Intercomparison"* of pre-equilibrium models and codes [36] and also the more recent one [37]).

The initial exciton number n_0 can be obtained from the analysis of the slopes of particle energy spectra at their high-energy edge (see [16]). Simple arguments would lead to $n_0 = A_{\rm proj}$ for light-projectile induced reactions (at least on even-even targets) [24, 38, 39] etc.), which would imply $n_0 = 1$ for nucleon- and $n_0 = 4$ for α-induced reactions, respectively. The slope analyses prefer $n_0 = 3$ in the former case, but as no emission (up to a tiny correction due to the γ emission) can proceed from the $n = 1$ state, its whole strength is transferred to the 3-exciton one which is therefore "seen" as the first emitting stage in the slope analysis. Obviously, target structure, possible Coulomb interaction between the target and the projectile and/or other interactions may cause departures from this simple rule. Generally, the values of n_0 in nucleon- and light-ion-induced reactions do not show large deviations from the *a priori* values stated above (cf. [17, 40, 41]).

3.3 Cluster Coalescence

In the simplest case, the cluster emission rate can be written formally in exactly the same way as for nucleons, just by replacing the exciton number of the residual nucleus $(p-1, h)$ by $(p - p_x, h)$ (Blann and Lanzafame [23], Cline [24]), where we assume that the cluster x is formed by p_x of the total of p excited particles. Originally, this has been the only difference with respect to the neutron emission [23]. All the details associated with the cluster formation or its "dissolution" into p_x nucleons in the case of the reversed process, must be therefore already contained in the inverse cross section $\sigma^*_{\rm INV}(\varepsilon_x)$ [42]. When making the approximation of $\sigma^*_{\rm INV}(\varepsilon_x) \approx \sigma_{\rm R}$, we expect that the de-clusterisation would be the main feature of the process of the cluster capture by the target nucleus. The analyses of particle energy spectra do not contradict this idea, but the absolute value is however too low with respect to the data. In order to bring them closer, Cline [24] multiplied the emission rates pragmatically by the energy-independent factor $p_x!$, which brought the calculations closer to the data. However, the shapes

of the energy spectra showed also some discrepancies at higher energies: they were too soft. Moreover, the new factor $p_x!$ could not be derived. Ribanský and Obložinský [25] replaced this artificial factor by $\gamma_x \times \omega(p_x, 0, \varepsilon_x + B_x)/g$, which has straightforward physical interpretation: its second part is simply the number of configurations of the p_x excitons forming the cluster, and γ_x is the formation probability which expresses the fact that these (non-interacting) excitons really form the cluster, so that their product is the number of clusters of given type with proper energy. This approach led to both reasonable absolute values and for some clusters even rather good spectra shapes. The full expression for the particle emission rate (see, e.g. [3, 25]) is now

$$\lambda_x^c(n, E, \varepsilon_x) = \frac{2s_x + 1}{\pi^2 \hbar^3} \mu_x \varepsilon_x \sigma^*_{\mathrm{INV}}(\varepsilon) \frac{\omega(p - p_x, h, U)}{\omega(p, h, E)} \times \frac{\omega(p_x, 0, \varepsilon_x + B_x)}{g_x} R_x(p) \gamma_x. \quad (4)$$

Obviously, nucleon emission is a special case of the cluster one (the nucleon formation probability is equal to 1 by definition).

A commonly used approach employed for the formation probability γ_x is to consider this quantity to be merely a parameter ($\gamma_x \leq 1$) to be obtained from the fit to the data. Figure 1 shows a comparison of basic coalescence ideas for four kinds of the emitted clusters, from d to α. The original coalescence model [24] is drawn as a full line, and that based on Eq. (4) by the dashed curves, with the value of the cluster formation probabilities (considered just as fitting parameters, which do not influence the spectra shape, but only its absolute value) indicated on each curve [25].

Several frequently analysed sets of double-differential cross sections of complex particles (and protons) from bombardment of various targets by protons and charged projectiles up to the α's at incident energies from 15 to 200 MeV can be found in the literature (for a partial list, see e.g. [1]), and also in reactions induced by neutrons starting with energy of 14 MeV or higher.

Many of these papers also contain analyses and interpretations of the data, either within the coalescence model or using the pre-formed α's (see the next section). Strictly speaking, not all the analyses stemmed from the exciton model, and some of them are based on so-called hybrid model. The mechanism of the cluster emission is nevertheless essentially the same in both models. The most important analyses of the data have been made and new model ingredients introduced by many physicists in the course of years (see again [1] for more details).

As for the formation probabilities γ_x, considered as a fitting parameter, their values of [25] are given in Figure 1, later calculations ([40, 43, 44]) analysed within this approach reactions on targets starting with ^{12}C up to ^{209}Bi and incident energies 14 to 90 MeV. The formation probabilities depend somewhat on the energy and in general they decrease with increasing mass number A of the com-

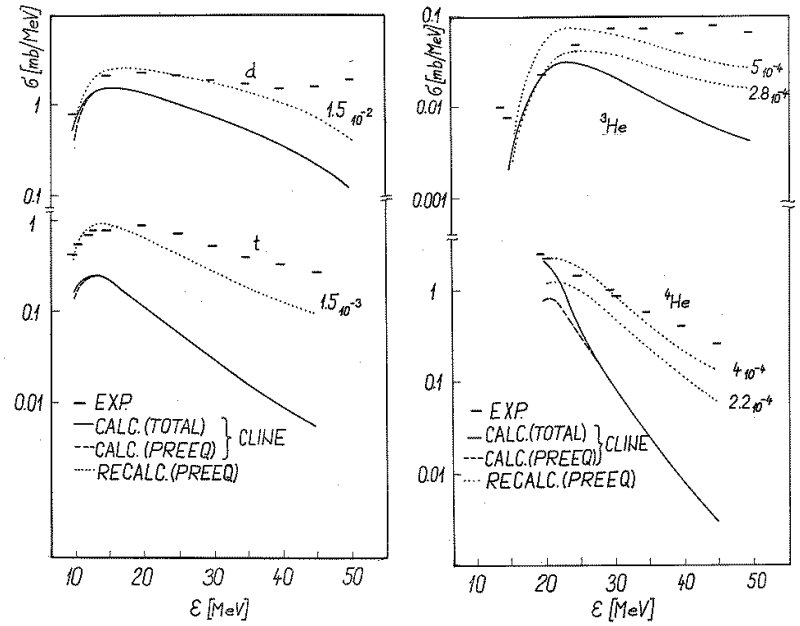

Figure 1. Cluster energy spectra for the ^{197}Au+p reaction at 62 MeV in the coalescence model: The full lines are the original calculations [24], the dotted curves are those of Ribanský and Obložinský [25]. The corresponding formation probabilities are indicated by numbers for individual clusters [25].

posite system (or of the target). Here, Wu and Chang [44] prefer $\gamma_x \approx 1/A^m$ above mass $A \approx 27$, with $m \approx 1$ for deuterons and $m \approx 4/3$ for α's, tritons and ^{3}He.

A simple theoretical estimate of the formation probability of the coalescence model ([43] and also [45] which pioneered this way) yields $\gamma_x \approx {p_x}^3 (p_x/A)^{p_x - 1}$. In addition to that, other suggestions for the formation probabilities can be found in literature, e.g. the empirical expression given by [46]). More justified is coupling the formation probabilities to the radius P_0 of the sphere in the momentum space containing the nucleons which are picked up $\gamma_x = [\frac{4}{3}(P_0/mc)^3]^{p_b - 1}$ [47], where m is the nucleon mass and c is the velocity of light. The coalescence radius P_0 is then determined from the fit to the data, and it was found to be close to the standard Fermi momentum, namely about 200 to 250 MeV/c for deuterons, 240 to 290 MeV/c for tritons and 310 to 370 MeV/c for ^{3}He and α in the analysis of decay of ^{65}Zn* [7].

As the values of the formation probabilities extracted from the data are strongly scattered, it is not possible to make a clear judgment among the above

expressions only on the basis of comparing calculations and the experiments *.

However, the presence of formation probabilities and/or other additional functions is not strictly justifiable by detailed balance, and it is therefore rejected by some groups (e.g. [28,48,49]), even though it means worsening the quality of the agreement between theory and experiment.

In fact, Eq. (4) still contains still one not very well defined quantity, namely the single-cluster density g_x. The single-particle level density is g, i.e. the minimal "energy quantum" gained or released by the exciton is $1/g$. In the cluster coalescence model, when one of those excitons which compose the cluster is shifted by this minimal energy quantum up or down, the energy of the whole cluster is changed by the same amount **. This — to some extent — justifies the common approach of replacing the single-cluster density g_x by the single-particle one g within the cluster coalescence model.

3.4 Angular Momentum and Angular Distributions

Generally, the pre-equilibrium models ignore the influence of angular momentum. This is easily shown to be rather small for the nucleon emission (see Běták [52]), but is surely greater for clusters. Here, the effect arises from two facts: *i)* cluster emission is usually enhanced at higher angular momenta, which means increased role of the nuclear surface and consequently effective lowering of the Coulomb barrier, especially in the case of deformed nuclei [53] and *ii)* many of quantities entering the pre-equilibrium reactions are spin- (or precisely both spin- and energy-) dependent, and their simple contraction to one variable necessarily affects the results. Of these two effects, the first one has been included in sufficient detail (see the quoted papers by Blann). The consistent incorporation of the angular momentum, however, is much more complicated *. Some attempts in this direction can be found in [9, 54], but, anyway, a consistent incorporation of angular momentum into the pre-equilibrium calculations is necessary. Some preliminary steps have been undertaken in [55], but the full formulation of the set of master equations with spin-dependent intranuclear transition and emission rates has been enabled by paper of Obložinský [56] and incorporated into computer code PEGAS [57]. Only nucleon (and γ) emission has been considered

*Recent paper [50] tries even to establish some stage-dependence of the formation probabilities. However, the precision of the data is not sufficient to distinguish clearly among different possibilities considered.

**If we distinguish between the neutrons and the protons from the very beginning, we have to consider separately the single-neutron and the single-proton densities g_ν and g_π, respectively, with $g = g_\nu + g_\pi$. As a rough approximation, $g_\nu = (N/A)g$ and similarly for g_π, which yields $g_\nu = g_\pi = g/2$ in the symmetric case ($Z = N$). In the real two-component description, the minimal energy quantum of either the neutron or the proton is not $1/g$, but rather $1/g_\nu$ or $1/g_\pi$. (See also the discussions of the role of the single-cluster density [27, 51]).

*One has also keep in mind the different isospin of a cluster (usually α's were studied) from that of a nucleon. This introduces some degree of violation of the Bohr independence hypothesis, but does not show large effects on the emitted spectra themselves.

there, and the full adaptation of the spin-dependent formalism to the coalescence (or some other model for the cluster emission) still has to be done.

Closely connected to the contribution of the angular momentum is the angular distribution. Initially, pre-equilibrium models were used only to describe the energy spectra of the emitted particles. The angular distributions have been added somewhat artificially using the idea of a fast (or leading) particle [58]: in nucleon-induced reactions, the original exciton (in fact, the incident neutron) is assumed to be the most energetic one for the most of the time and as such it keeps the notion of its direction, which is slowly smeared out in the course of the reaction. In the energy range of the pre-equilibrium models, the nucleon-nucleon differential cross section is nearly isotropic in the c.m. system, so that in the laboratory system it is proportional to $\cos\theta \times \Theta(\frac{\pi}{2} - \theta)$, where θ is the angle measured with respect to the original direction. After the first interaction, one thus arrives to the initial condition for the occupation probabilities of n exciton states with specified direction Ω of the fast particle

$$P_n(\Omega, 0) = \delta_{nn_0} \pi^{-1} \cos\theta \, \Theta\left(\frac{\pi}{2} - \theta\right), \tag{5}$$

and with each interaction the angular distribution gets flatter. Thus, the emitted nucleon at the very early stage of the reaction is now just the leading particle, and the angular distribution is that which corresponds to the degree of smearing out the originally sharp value during the time interval from the creation of the composite system to the particle emission.

Because of its physical transparency and simplicity, this model became very popular for calculations of pre-equilibrium angular distributions of nucleons. For many practical purposes, and also strongly supported by analyses of large number of nuclear reactions, Kalbach and Mann [49] proposed simple systematics, which has been later on justified within a more general approach [59]. Therein, the cross section of a reaction can be written as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}\epsilon}(a,b) = a_0(\mathrm{MSD}) \sum_{l=0}^{l_{\max}} b_l P_l(\cos\theta) + a_0(\mathrm{MSC}) \sum_{\substack{l=0 \\ \Delta l=2}}^{l_{\max}} b_l P_l(\cos\theta), \tag{6}$$

where $P_l(\cos\theta)$ are the Legendre polynomials, the a's determine the portion of the cross section arising from the multistep direct and multistep compound processes, and the coefficients b_i are determined from the fit to the data (or from systematics), and — as shown later — can be in principle derived from the model [59].

Though the model of fast particle became rather popular, its application is not without problems: the emitted particle need not be just the leading one, and — moreover — the model was suggested for the *nucleon* emissions and its general-

ization of the model to the case of clusters is not straightforward [*].

A possible way for the cluster angular distributions to be used has been suggested in [60] who considered the linear momentum shared by all the excitons together. The approach became more transparent and feasible due to Chadwick and Obložinský [61], who also delivered a method to calculate the necessary densities with specified linear momentum [62]. However, even then the model has been too demanding for the computer time needed, so that approximate methods were preferred, which were, in fact, developed shortly after the model of the leading particle appeared. Thus, Kalbach and Mann [49] suggested — based on analyses of many reactions in the energy range from 18 to 80 MeV — systematics of angular distributions. Later on, it has been extended toward higher energies, up to 700 MeV incident energy and up to about 200 MeV ejectile energy [63].

A very popular way to express the angular distributions within the exciton (and similar) model(s) uses the systematics [49] which spread its original range of validity to wider range of projectiles and ejectiles and it is currently used in many computer codes, the best known of them is probably GNASH [29].

3.5 Pre-Formed Alphas

The approach of the Milano group of pre-formed α emission [22, 65] uses the fact that the α particle is a very strongly coupled object, whose existence (as a cluster) has been frequently demonstrated in nuclear studies. Thus, it need not be formed in the course of a reaction, and may enter the process as a pre-formed entity, which can be considered as a single exciton, though of properties (like mass, spin, etc.) specific for the α particle. One has to stress out that this approach is not a general one for different clusters, but it has been formulated *solely* for the case of the α emission. Instead of formation probability we have now the pre-formation factor γ_α, which is the probability that such an entity is present in the target nucleus and that it becomes involved in the reaction by interaction of the projectile with the target. The corresponding probability (pre-formation factor) γ_α enters the emission rates, which are (apart from this factor) *formally* identical to those for the nucleon emission (Eq. (3) or — equivalently — to the coalescence cluster emission given by eq. (4), where we put $p_\alpha = 1$). Obviously, the specific ejectile properties, like mass, spin, etc., refer now to the α particle and not to nucleons.

However, γ_α now has a completely different interpretation: it is that fraction of the reaction cross section σ_{R} which leads to the excitation of cluster states. Additionally, one assumes that once a cluster has been preformed and took part in the initial reaction stage, it does not dissolve any more. Though such an assumption is not realistic, introduction of some average cluster lifetime (within

[*]The ideas of coalescence angular distributions have been also applied to the Iwamoto-Harada model (see below), see [64].

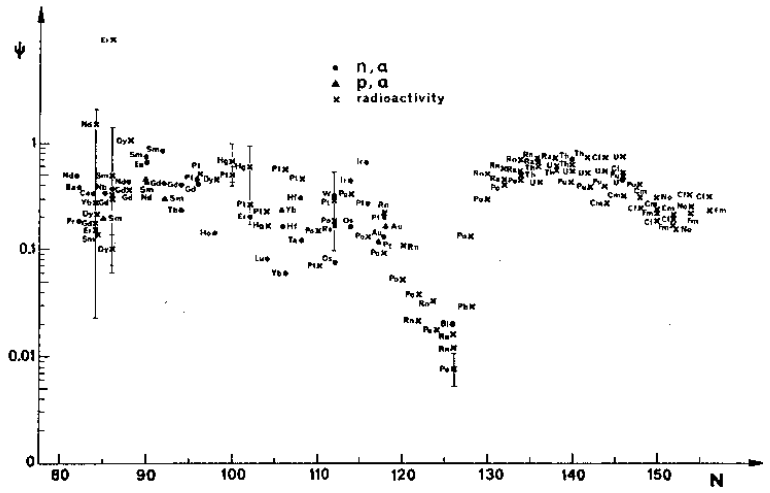

Figure 2. The α preformation probability ϕ as given by (n,α) and (p,α) reaction analyses compared with the same quantity as given by the analysis of α radioactivity (even-even nuclei) as a function of neutron number N of the decaying nucleus [66].

acceptable range of its possible values) cannot significantly influence the calculated particle spectra.

The use of the concept of the pre-formed α particles has a striking consequence for the single-cluster density. Whereas in the cluster coalescence approach the cluster was allowed to change its energy in multiples of $1/g$, the strongly coupled quatro-exciton can change its energy only when *each* of its constituent excitons changes its position by $1/g$, so we obtain $4/g$ for the α particle as a whole [51].

One can compare the preformation probabilities to similar values obtained from analyses of α radioactivity. The first such comparison [66] showed nearly perfect agreement between the values obtained from (p,α) and (n,α) reactions and those from radioactivity (about 50 "reaction" values and some more from radioactivity above the $N = 82$ shell, see Figure 2). However, as pointed out by Ribanský and Obložinský, the consistent consideration of the two-component nature of the cluster level density in the emission rates should imply not the equality of these two values, but rather $\gamma_{\alpha\mathrm{decay}} \geq k\gamma_{\alpha\mathrm{reaction}}$ with $k > 1$ (see [42]) *.

*A simple and straightforward application of separate neutron and proton single-particle level densities and of the cluster level density when one takes into account its composition from neutrons and protons even leads to $k = 6$.

3.6 The Iwamoto-Harada Coalescence (Pickup) Model

The original simple coalescence model has been made more sophisticated to allow the cluster to be formed not only of excitons, but also from unexcited nucleons below the Fermi level. This approach became known as the Iwamoto-Harada model [26, 67], even though it has been suggested and applied five years earlier [27]. Mathematically, it means replacing the density product $\omega(p-p_x,h,U)\times\omega(p_x,0,\varepsilon_x+B_x)$ in (4) by

$$\sum_{p^*=1}^{p_x}\int_{\varepsilon_x+B_x}^{E}\omega(p-p^*,h,E-\varepsilon_1)\omega(p^*,0,\varepsilon_1)\omega(0,p_x-p^*,\varepsilon_2)\,\mathrm{d}\varepsilon_1\,, \qquad (7)$$

where p^* is the number of excitons contributing to forming the cluster, while the remaining (p_x-p^*) nucleons are picked up from the Fermi sea. Now, the cluster density is

$$g_x = g\frac{[g(\varepsilon_x+B_x+p_xE_F)]^{p_x-1}}{p_x!(p_x-1)!}\cdot\gamma_x, \qquad (8)$$

making this approach parameterless for cluster emission [27].

The model became rather popular, even though its computational aspects are more complicated than in the case of the pure coalescence model or for preformed α particles [64, 68–70] and has been also adapted to heavy ions [71].

Bisplinghoff suggested that not all nucleons be available for the cluster formation within the model, but only those close to the Fermi energy, and the energy width of the "band of availability" is determined by the binding energy of nucleons inside the cluster [72].

If we denote the energies of individual excitons entering the cluster as ε_i, the mean exciton energy over the cluster as ε_m, the cluster binding energy in the composite system as B_x and the binding energy of nucleons within that cluster (i.e. 28.3 MeV for α and 2.2 MeV for deuteron) as $\mathcal{B}_x$, one has for alphas $\varepsilon_m=(\varepsilon_\alpha+B_\alpha)/4$ [72] and $\sum_{i=1}^{4}|\varepsilon_i-\varepsilon_m|\le\mathcal{B}_\alpha$ [72]. For the case when three of four nucleons are excited, and only the fourth one is taken from the Fermi sea (and only this case has been considered in fact in the original paper), this requires $\varepsilon_h+\varepsilon_m\le\mathcal{B}_\alpha$ [72]. However, this relation implicitly assumes that all three excited nucleons contributing to the cluster have the same energy. If we allow for some energy spread, the above condition becomes more stringent.

3.7 Cluster Binding Energy and the Fermi Motion – IH Model

It is natural to generalize the Bisplinghoff idea to arbitrary combinations of excited and unexcited nucleons on one hand, and to all types of clusters on the other one. As the binding energy of nucleons in the deuteron is very small, the generalization of the Bisplinghoff idea to loosely bound ejectiles is hardly likely to be observed in practice.

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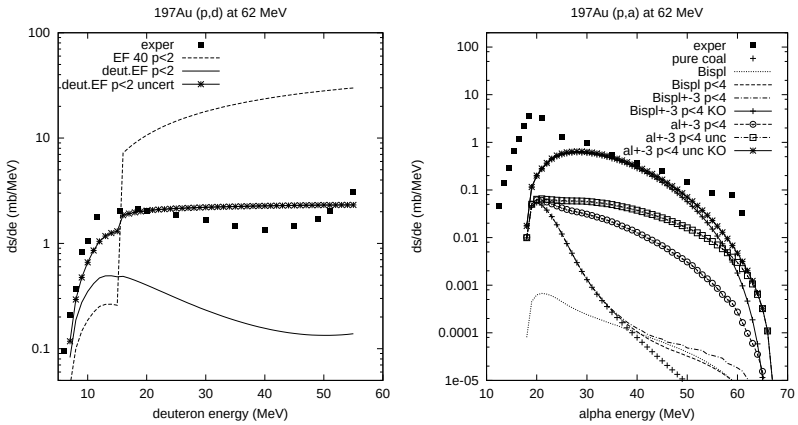

Figure 3. Different mechanisms and their corresponding deuteron (*left*) and α (*right*) energy spectra in the case of ^{197}Au+p at 62 MeV using the updated PEQAG code [73]. For details, see the text. From [74].

Thus, strongly bound entities, like α's, have large energy space available for their creation (which makes the approach close to the original Iwamoto-Harada ideas [26, 27], and loosely coupled objects (e.g. deuterons) practically get close to the standard coalescence model. This idea has been developed in [74]. The resulting α and deuteron spectra therefrom are presented in Figure 3. In both Figures, the points are the experimental data, the dashed line is the Iwamoto-Harada model (with the potential depth of $E_{\rm F} = 40$ MeV), but applied only when the number of excited nucleons is not sufficient to form the required cluster (deuteron or α). For the deuteron spectra, it practically coincides above 15 MeV with unrestricted original Iwamoto-Harada model (not drawn). The full line shows the Bisplinghoff idea of limiting the depth of "valence nucleons" to the cluster binding energy, generalized here for arbitrary kind of clusters [*]. The full line with stars contains possible admixture of the pickup process due to the time-energy uncertainty. For more compact ejectiles, like α, also the possibility of knockout is considered. The pure coalescence calculation for α's is drawn by isolated crosses, and the improvement on the Bisplinghoff idea by the knockout inclusion in its straightforward generalization as a full line with crosses and with incorporation of time-energy uncertainties as a full line with stars.

[*] In fact, this approach yields nearly exactly the same spectra as the pure coalescence calculation without specific clusterisation in the case of deuteron emission, because of very low binding energy of nucleons in the deuteron.

3.8 Statistical (or Thermodynamical) Formalism

Completely another approach to the particle emission, including that of clusters, has been proposed and later on developed by Friedman, *et al.* [75]. Therein, the emision rates are expressed essentially in the same form as it was in the case of compound nucleus; the principal difference and a possibility to include pre-equilibrium effects is that the densities of states are expressed as proportional to the $\exp(S)$, where S is the entropy. Thus, the ratio of densities in the emission rate converts to the entropy change, as $\rho_i(\varepsilon)/\rho = \exp(\Delta S)$, and the entropy change ΔS is evaluated as a function of the thermal energy $E_{\rm th}$ defined as the excitation energy lowered by the rotation one, of the number of particles N and of the volume per particle v. In the first order in $E_{\rm th}$ and N, the volume per particle v is assumed to be constant. Now, the temperature can be introduced by the well-known relation $\partial S/\partial E_{\rm th} = 1/T$ of statistical mechanics. The practical application of this scheme needs some additional inputs, like the Fermi energy, binding energies for the emitted particles (clusters) etc. For practical reasons of feasibility of the approach, the standard empirical mass formulae are converted to their conterparts allowing for non-integer values of A and Z, and the transmission coefficients T_l's are taken in a sharp cut-off model, i.e. $T_l = 1$ if the thermal energy exceeds the Coulomb barries, and $T_l = 0$ in the other case.

This statistical approach smeares out majority of details. In practice, this formalism is applied practically solely to high excitations (typical initial temperature ranges from 5 to 20 MeV, which corresponds to excitations close to or above 1 GeV) and to angular momenta of several hundreds of $\hbar$.

3.9 Purely Phenomenological Approaches

The practical use of the models described above has some limitations and moreover they are not able to describe the data perfectly. For special purposes is therefore more popular to use purely phenomenological description of the pre-equilibrium cluster emission. Thus, Kalbach [28] suggested admixtures of direct components in the complex particle spectra and used phenomenology to fit the data. Therein, the contribution to the cross section of the reaction (a, x) due to the pickup/stripping processes can be expressed semiempirically as

$$\frac{{\rm d}\sigma^{\rm PS}}{{\rm d}\varepsilon}(a,x) = (2s_x+1)p_x\varepsilon\sigma_x(\varepsilon)\omega_{\rm F}^{\rm PS}(U)\ (20)^{\delta_\alpha} \times f\left(\frac{E_a}{p_a}\right)^{-2\Delta}\left(\frac{780}{A}\right)^{\Delta}\ 1.4\times10^{-4}({\rm MeV})^{2\Delta-1}, \quad (9)$$

where Δ is the number of transferred particles, $\delta_\alpha = 1$ if an α is formed, $\delta_\alpha = -1$ if destroyed, and $\delta_\alpha = 0$ otherwise, and the function f is

$$f(N,Z,\Delta_\nu,\Delta_\pi) = \left(\frac{2Z}{A}\right)^{6\Delta_\pi}\left(\frac{2N}{A}\right)^{(1-\Delta_\pi)\Delta_\nu(\Delta_\nu+1)/2}, \quad (10)$$

and Δ_π and Δ_ν are the numbers of transferred protons and neutrons, respectively. The density of final accessible states is determined for this process from

$$\omega_{\rm F}^{\rm PS}(U) = \frac{\Delta!}{\Delta_\pi!\Delta_\nu!}\sum_{i=1}^{\Delta}\omega(0,i,U). \tag{11}$$

Similarly, the knockout and inelastic processes involving α are given by

$$\frac{{\rm d}\sigma^{\rm KI}}{{\rm d}\varepsilon}(a,x) = \frac{\sigma_a}{p_a\varepsilon_a^3}(2s_x+1)p_x\varepsilon\sigma_x(\varepsilon)\frac{\omega_{\rm F}^{\rm KI}(U)}{A^2}F_a\ [0.12{\rm MeV}^2{\rm mb}^{-1}], \tag{12}$$

where $F_\alpha =$ and $F_n = F_p = fZ/2A$. The density function $\omega_{\rm F}^{\rm KI}(U)$ depends on the process; it is

$$\omega_{\rm F1}^{\rm KI}(U) = g_i g_\alpha\left(U - \frac{1}{2g_i} - \frac{1}{2g_\alpha}\right) \tag{13}$$

for (p,α), (n,α), (α,n) and (α,p) knockout ($i =$ n or p), $\omega_{\rm F2}^{\rm KI}(U) = g_\alpha^2 U$ for elastic scattering with excitation of an α particle-hole pair and finally $\omega_{\rm F3}^{\rm KI}(U) = g_n^2 U + g_p^2 U$ for (α,α') inelastic scattering exciting a nucleon pair. In the above equations, one usually takes $g_p = Z/(13$ MeV), $g_n = N/(13$ MeV), and $g_\alpha = A/(52$ MeV).

4 Model Intercomparisons and Flavours and the Consistency of Pre-Equilibrium Approaches with the Compound Nucleus

Alpha-particle emission is the most frequently studied case of complex particles. The high binding energy of nucleons in the α particle justifies considering the latter as a single object [22,72]. However, if we consider complex particle emission as a whole, we have to take into account general mechanisms, not a specific one (even if proved to be very successful) tailored to one type of ejectile only (e.g. the pre-formed α's). The coalescence model in its pure form [23, 24] obviously fails to reproduce the data. Two other competing models, namely the Ribanský-Obložinský coalescence model [25] and the Iwamoto-Harada model [26, 27, 67] describe some cases well but fail for others, with no simple rule about their applicability. There is still one model, rather successful for a wide range of reactions, namely the phenomenological one of Kalbach [28], that also describes pickup and stripping. It contains many parameters without clear physical justification, just to fit the data. Therefore, it can be (and it is) successfully applied to calculate cross sections and other quantities, but there is not much sense in discussing the underlying physics. The coalescence model [25] often works well for deuteron emission, usually fails for alphas, and there is a 50% chance of reasonable description for tritons and ^{3}He. The Iwamoto-Harada model does not contain any free parameter, such as the formation probability in the former case. The overall

fit is of similar quality (though it may be significantly different for a specific reaction) than in the coalescence model, but one cannot find any drastic discrepancies with respect to the data here.

There are two basic principles which can be used to bridge the pre-equilibrium approaches with the compound nucleus theory. One of them is the principle of microscopic reversibility applied to the emission rates and to the particle capture, and the other is requirement of reaching the compound nucleus theory as the limit (equilibrium) case of the pre-equilibrium emission when one goes to sufficiently long times.

The key problem of the majority of approaches within the Iwamoto-Harada model is that they use different formalisms for describing various stages of the reaction, and the equilibrium (i.e. compound nucleus) emission cannot be reached as the limit of the pre-equilibrium description. The exceptions here within the Iwamoto-Harada approach are the papers by Běták, *et al*. [27,74]. In the older of the two, however, the fit to the data is significantly worse than in [26,67,68,72].

The formula of the the pre-equilibrium exciton model in the case of nucleon emission [76] and with some additional approximation also for cluster coalescence model in its pure version [48] can yield the Weisskopf-Ewing formulae within the model by summation over all exciton states.

We suggest that the pickup is effective *only* when the number of excited particles is insufficient to form the cluster of the required type. When the exciton number is large enough, the excitons do not show the need to pickup their partner(s) from the Fermi sea. This suggestion does not influence the high-energy part of the spectrum, but is able to yield the proper equilibrium limit.

5 Quantum-Mechanical (Microscopic) Theories

Quantum mechanical theories of pre-equilibrium reactions range from very detailed formalisms that are difficult or impracticable to calculate numerically to phenomenological models or parametrisations that are easy to calculate and to fit to the data but contain little physics and so lack generality. Over the years, pre-equilibrium theories have been continually improved to incorporate as much physics as possible in a formally correct way while still allowing numerical calculations to be made. These theories are divided into the multistep compound reactions (MSC) where all the particles remain bound and the multistep direct reactions (MSD) where at least one particle, usually the incident one, but possibly also some others, remain in the continuum. The theory of Feshbach, Kerman and Koonin (FKK) [77] enables both processes to be calculated.

These theories use completely different approaches and methods than those presented and discussed above. We do not feel suitable to explain in detail so unsimilar groups of theories within one lecture of the Workshop and the reader is refered to the cornerstones of the microscopic approaches [38,77,78] or to our

recent review [1], where all approaches gained their adequate space. Below, we summarize just the basic ideas of these theories.

The FKK theory distinguishes between multistep compound and multistep direct reactions: in a multistep compound reaction all the particles remain bound during the equilibration cascade, whereas in multistep direct reactions at least one particle remains in the continuum.

In the following years many analyses were made of MSC and MSD reactions and these established the validity of the FKK theory over a wide range of energies and target nuclei. Many of the reactions described in the next section have been analysed using the FKK theory. Extensive reviews of the FKK formalism have been published for both the MSC [80] and the MSD [81] reactions.

In multistep compound reactions the phases of the matrix elements involving different total angular momenta J, parity, and other quantum numbers required to specify a channel are assumed to be random so that no interference terms remain after averaging, and the energy-averaged cross sections are symmetric about 90°. In the multistep direct reactions, there is constructive interference between matrix elements involving the same change in the momentum of the particle in the continuum and so the cross sections are forward-peaked. This distinction between multistep compound and multistep direct reactions was subsequently used in empirical parametrization of pre-equilibrium cross sections [49].

At each stage in the excitation process the states with at least one particle in the continuum and the states with all particles bound are considered separately; these are formally described by the projection operators $\mathcal{P}$ and $\mathcal{Q}$ acting on the total wave function, with $\mathcal{P} + \mathcal{Q} = 1$. The set of states P contributes to the multistep direct process and the complementary set of states Q to the multistep compound process (see e.g. [77, 79]).

These states are shown in Figure 4, with the arrows indicating transitions from one configuration to another. If only two-body interactions are present, these transitions can only take place between neighbouring stages; this is the *chaining hypothesis*. At each stage, there are three possibilities: excitation of one additional particle-hole pair, de-excitation of a particle-hole pair, and emission into the continuum.

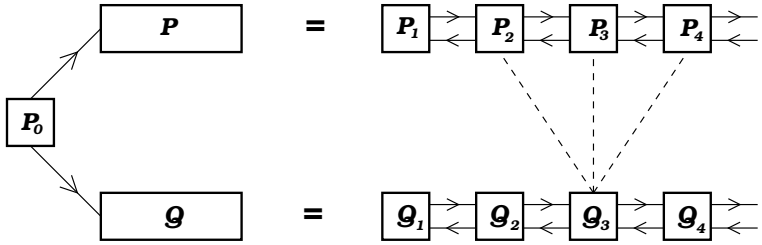

Figure 4. Multistep description of a nuclear reaction (Feshbach, *et al.* 1980).

Pre-equilibrium emission can take place directly at each stage from the P-chain, or indirectly from the Q-chain. In the latter case the emission process goes through states in the P-chain; this can happen in three different ways, as shown in the Figure 4.

6 Conclusions

Relatively naive and surely oversimplified view on the cluster emission can be found in the context of various pre-equilibrium models. Two basic approaches to the cluster emission can be traced here. The first of them is the concept of the pre-formed α's, where the α particle is considered as a special entity which may be involved into the reaction in addition to the neutrons and protons usually considered as the components of a nucleus. The opposite views are contained in a range of subspecies of coalescence models. These models are much more general than the concept of pre-formed α's and they are formulated and used for all types of clusters. The pre-equilibrium models may be completed by a presence of direct-type reactions, like the pickup and/or knockout, expressed for this purpose rather phenomenologically than microscopically. All these approaches, although simple, are rather useful to yield reliably the overall trends and also correctly predict the magnitude of cross sections and related quantities. However, for deeper understanding and more precise description, one has to incorporate details of cluster formation and emission and this is obviously beyond the scope of simple statistical phenomenological models.

Acknowledgments

The author is grateful to many colleagues, especially to A.N. Antonov, J. Bisplinghoff, R. Čaplar, M.B. Chadwick, A.A. Cowley, S.S. Dimitrova, J. Dobeš, P. Obložinský and K. Spasova for discussions. This lecture is heavily based on a review on cluster reactions prepared together with P.E. Hodgson. I would like to express him my special gratitude for our fruitful collaboration. The author is highly indebted to the Royal Society for its support of his visit to the University of Oxford and to its Department of Physics for warm hospitality. The work has been supported in part by the VEGA Grant No. 2/1124/21.

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