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On Description of the Yrast Lines in IBM-1

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Abstract.

The geometric representation of IBM-1 is applyed for calculations of the energies of yrast lines in even-even deformed nuclei. The yrast lines can be successfully explained as a crossing of several number of rotational β-bands if the structure of the corresponding bands heads is taken into account.

The geometric properties of the interacting boson model are particularly important since they allow one to relate this model to the description of collective states in nuclei by shape variables [1]. There is a large variety of problems that can be attacked with this representation of IBM introduced in nuclear physics by Gilmore and Feng [2], Ginocchio and Kirson [3], Dieperink, Sscolten and Iachello [4], Bohr and Mottelson [5].

In this paper we apply the geometric representation of IBM-1 model following [6] in description of the yrast lines energies of the even-even deformed nuclei.

Recently it was shown that the energies of the yrast lines can be explained with an acceptable accuracy as the crossing of some number of the rotational $\beta-$bands even if the energies of the rotational $\beta-$bands are calculated within the framework of simple rigid-rotator model [7]. Let us remind that in this paper [7]. the distribution of the excited 0^+- states energies as a function of number of bosons p is described with simple formula:

$$E_p = Ap - Bp^2 \tag{1}$$

where p is the number of pure monopole bosons (b^+, b) connected with bosons R_+, R_-, R_0 through applying the T.Holstein-H.Primakoff [8] transformation:

$$R_- = \sqrt{2\Omega - b^\dagger b}\, b; \qquad R_+ = b^\dagger \sqrt{2\Omega - b^\dagger b}; \qquad R_0 = b^+ b - \Omega$$

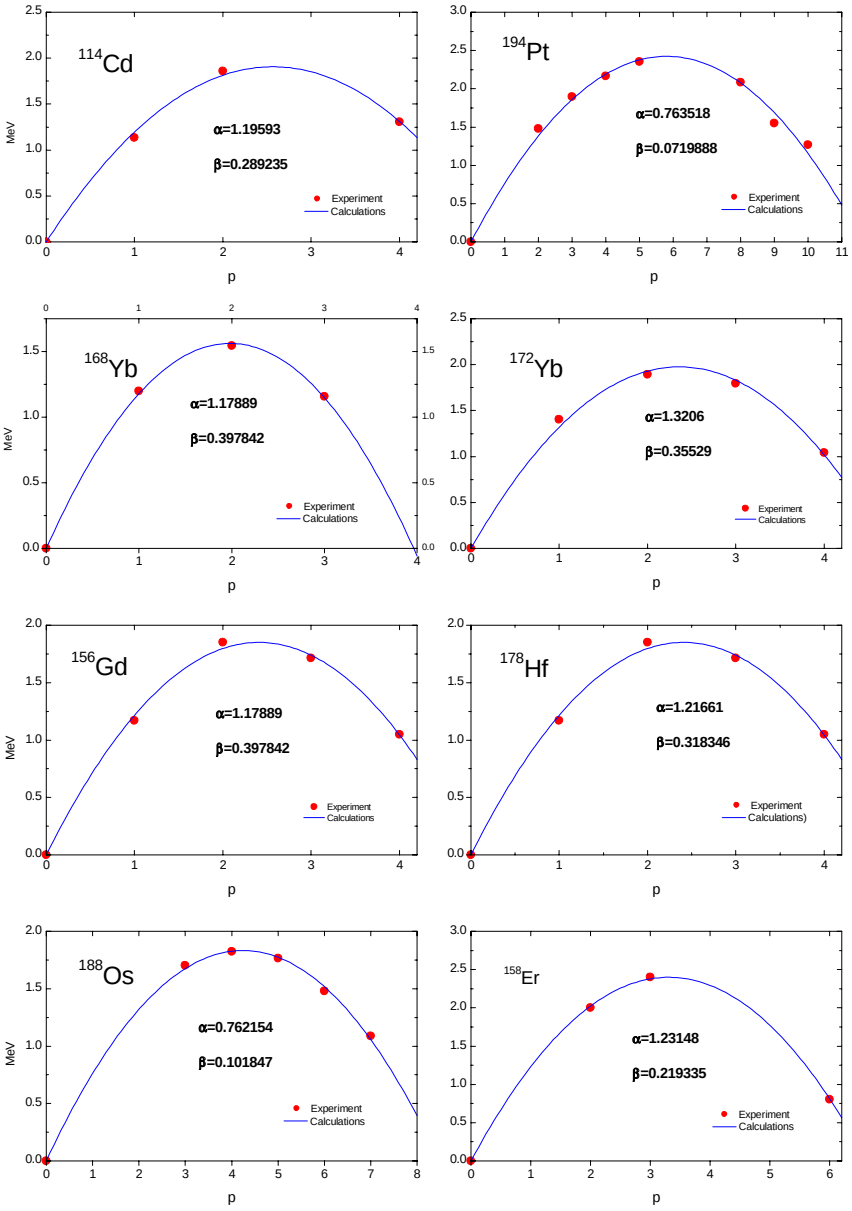

Figure 1. Comparison of calculated and experimental data of low-lying 0^+ excited states.

constructed from pairs of fermions

$$R_+ = \frac{1}{2}\sum_m (-1)^{j-m}\alpha^\dagger_{jm}\alpha^\dagger_{j-m}; \qquad R_- = \frac{1}{2}\sum_m (-1)^{j-m}\alpha_{j-m}\alpha_{jm};$$
$$R_0 = \frac{1}{4}\sum_m (\alpha^\dagger_{jm}\alpha_{jm} - \alpha_{j-m}\alpha^\dagger_{j-m}),$$

and commutating as:

$$[R_0, R_\pm] = \pm R_\pm; \qquad [R_+, R_-] = 2R_0; \qquad \Omega = \frac{2j+1}{2}$$

Figure 1 shows that formula (1) provides perfect description of the experimental data for large amount of nuclei. Further we use this classification in our calculations of the energies of the rotational $\beta-$ bands labeling each 0^+ state with its own number of bosons p. We also take the nucleus mean square radius $R_{ms}(p)$ to play a role of the carrier of this information about band head collective structure to formulae of the energies of rotational $\beta-$ bands.

From [7, 9] we have the expression of the mean square radius of the nucleus in any excited 0^+ state with the degree of collectivity determined by number of monopole bosons p:

$$R_{ms}(r_0, E_0, C_0, p) = \sqrt{\frac{3r_0^2\left(15E_0^2(p-1)p + 80E_0 p\pi C_0 + 32\pi^2 C_0^2\right)}{20C_0\left(3E_0 p + 8\pi^2 C_0\right)}} \qquad (2)$$

with equilibrium radius of the nucleus $r_0 = 1.287A^{\frac{1}{3}}, C_0$- nuclear surface compressibility parameter and E_0-one phonon excited 0^+ state energy.

Now for each of three decomposition of $U(6)$ symmetry chain following [6] we write the energies of the bands in terms of geometric representation parameters $e_0, \epsilon_1, \epsilon_2, k, k', \beta, \gamma$ and the eigenvalue of the first Casimir N as follow:

$$U(6) \supset U(5) \supset O(5) \supset O(3) \supset O(2):$$

$$E_1 = e_0 + \epsilon_1 N + \epsilon_2 N(5+N) + kN\frac{\beta^2}{1+\beta^2} + k'N(N-1)\frac{\beta 4}{(1+\beta^2)^2}$$

$$U(6) \supset SU(3) \supset O(3) \supset O(2):$$

$$E_2 = e_0 + \frac{6k'N\beta^2}{1+\beta^2} + \epsilon_1 N + \epsilon_2 N(5+N)$$
$$+2k\left(\frac{N\left(5+\frac{11\beta^2}{4}\right)}{1+\beta^2} + N(N-1)\frac{4\beta^2 + \frac{\beta^4}{2} + 2\sqrt{2}\beta^3\cos(3\gamma)}{(1+\beta^2)^2}\right)$$

$$U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2) :$$

$$E_3 = \epsilon_0 + \frac{kN\left(N-1\right)\left(1-\beta^2\right)^2}{4(1+\beta^2)^2} - \frac{kN\left(N+4\right)}{4} +$$

$$+ N\epsilon_1 + N\left(N+5\right)\epsilon_2 + k^{'}\frac{N\beta^2}{1+\beta^2}$$

As far as these energies being increasing with the increase of N we put into correspondence to each N the value of the angular momentum $L = 2N$ that gives us the minimal values of the energies for every chosen N. Further we redefine IBM-1 parameters making them depending on collective structure of corresponding $\beta-$ band head:

$$\varepsilon_0 = Ap - Bp^2, \qquad \text{if } p = 0, \quad \varepsilon_0 = 0$$

$$k = \frac{k_g r_0}{r_0 + \Delta R_{ms}(r_0, E_0/C_0, p)}, \qquad \text{if } p = 0, \quad k = k_g$$

$$k' = \frac{k'_g r_0}{r_0 + \Delta R_{ms}(r_0, E_0/C_0, p)}, \qquad \text{if } p = 0, \quad k' = k'_g$$

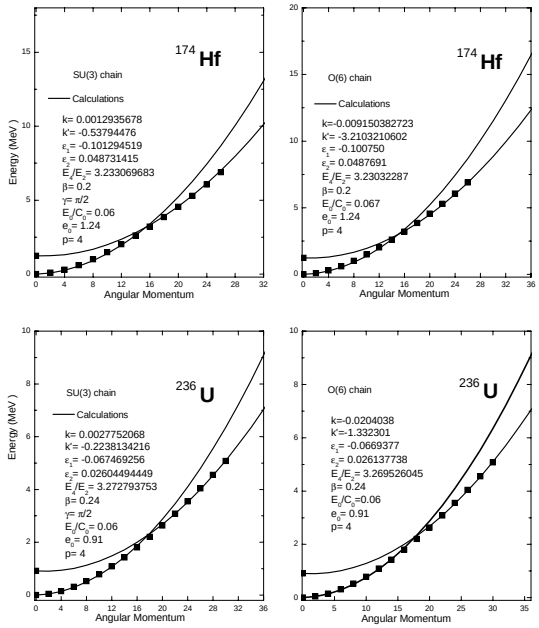

Figure 2. Comparison of the calculated energies with experiment.

$$\varepsilon_1 = \frac{\varepsilon_{g1}(r_0 + \Delta R_{ms}(r_0, E_0/C_0, p))}{r_0}, \qquad \text{if } p = 0, \quad \varepsilon_1 = \varepsilon_{g1}$$

$$\varepsilon_2 = \frac{\epsilon_{g2} r_0}{r_0 + \Delta R_{ms}(r_0, E_0/C_0, p)}, \qquad \text{if } p = 0, \quad \varepsilon_2 = \varepsilon_{g2}$$

with

$$\Delta R_{ms}(r_0, E_0/C_0, p) = \sqrt{R^2_{ms}(r_0, E_0/C_0, p) - R^2_{ms}(r_0, E_0/C_0, 0)}$$
$$= \sqrt{3} r_0 \sqrt{\frac{\frac{E_0}{C_0}\mathbf{p}\left(15\frac{E_0}{C_0}(-1+\mathbf{p}) + 4(-3+20\pi)\right)}{60\frac{E_0}{C_0}\mathbf{p} + 160\pi^2}}$$

Now we fit all model parameters $\epsilon_1, \epsilon_2, k, k', \beta, \gamma$ to the ground $\beta-$ band part of the yrast line states ($p = 0$) while the behavior of the rest bands will now depend only on the number of bosons p taken from 0^+ excited states energies classification (1). So calculated energies of $\beta-$ bands and comparison of our calcu-

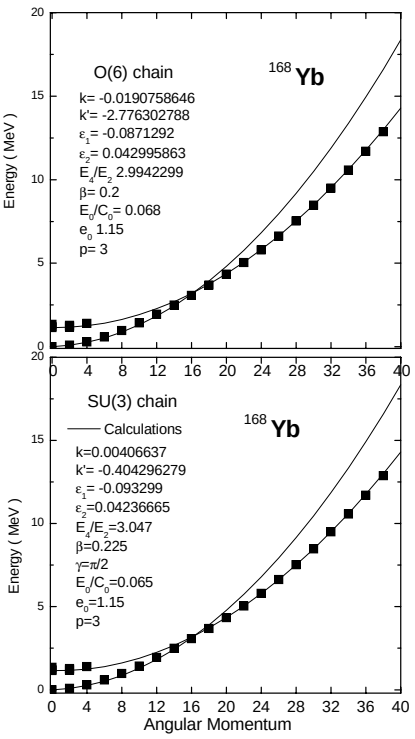

Figure 3. Comparison of calculated energies with experiment.

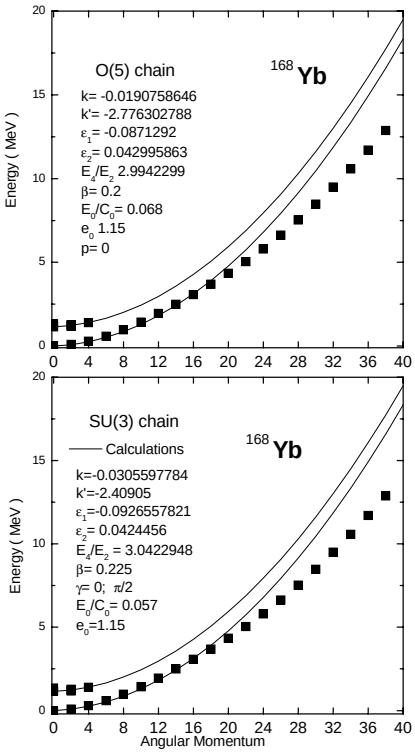

Figure 4. Calculations with $p = 0$.

lations with experiment are shown in Figures 2-3. All the fitting parameters and also the calculated values E_4/E_2 of the ground band are given in Table 1.

It is not surprising that the agreement between our calculations and experimental data is perfect (too many IBM-1 model parameters). But we have shown that the yrast line energy can be explained as a crossing of different $\beta-$ bands if the band heads structure is taken into account properly. In Figure 4 one can see how strong is the influence of number of bosons determining the corresponding band head on the behavior of corresponding $\beta-$ band.

And of course the transition probabilities $B(E2, L- > L-2)$ inside differ-

Table 1. The values of the fitting parameters for three decomposition of $U(6)$ symmetry chains.

^{168}Yb	$k_g = 0.7912489$	-0.0248049	0.0033024
	$k'_g = 24.196086$	-2.5815559	-0.43301
	$\epsilon_{1g} = 0.03043265$	-0.089947	-0.09052
	$\epsilon_{2g} = 0.0435757$	0.0427416	0.042683
	$E_4/E_2 = 2.70978$	3.1192	3.1248
	$\beta = 0.19$	0.22	0.22
	$\gamma =$	$\pi/2$	
	$E_0/C_0 = 0.06$	0.057	0.057
	$p = 3$	3	3
	$e_0 = 1.15$	1.15	1.15
^{174}Hf	$k_g = 0.7579$	-0.0091504	0.0012935
	$k'_g = 28.17$	-3.210321	-0.537946
	$\epsilon_{1g} = 0.02915$	-0.10075	-0.101294
	$\epsilon_{2g} = 0.0074$	0.048769	0.0487314
	$E_4/E_2 = 2.90$	3.23	3.233
	$\beta = 0.19$	0.2	0.2
	$\gamma =$	$\pi/2$	
	$E_0/C_0 = 0.2$	0.057	0.055
	$p = 4$	4	4
	$e_0 = 1.24$	1.24	1.24
^{236}U	$k_g = 0.329083$	-0.020403	0.0027752
	$k'_g = 15.72$	-1.3323	-0.22381
	$\epsilon_{1g} = 0.012657$	-0.066938	-0.06747
	$\epsilon_{2g} = 0.00386$	0.026138	0.026045
	$E_4/E_2 = 3.058$	3.269	3.273
	$\beta = 0.19$	0.24	0.24
	$\gamma =$	$\pi/2$	
	$E_0/C_0 = 0.2$	0.06	0.056
	$p = 4$	4	4
	$e_0 = 0.91$	0.91	0.91
	Chain 1	Chain 2 SU(3)	Chain 3 O(6)

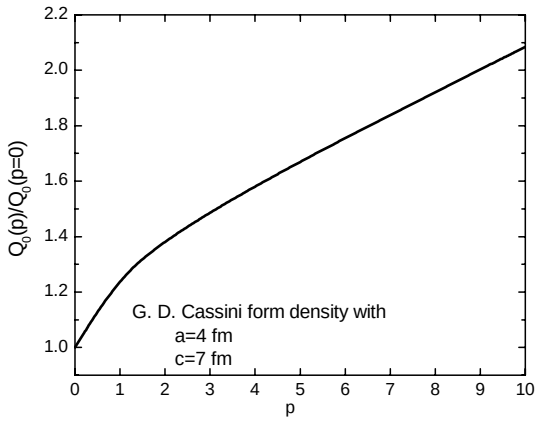

Figure 5. The intrinsic quadrupole moment as a function of number of bosons "p".

ent $\beta-$ bands now also depend on these bands heads collective structure because the intrinsic quadrupole moment Q_0 depends on the number of bosons p.

$$B(E2, L- > L-2; K=0) = \frac{5}{16\pi}\frac{6(L-1)^2L^2}{L(2L-2)(2L-1)(2L+1)}Q_0^2 \qquad (3)$$

We illustrate this in Figure 5 where we plot the dependence of Q_0 on number of bosons p. The intrinsic quadrupole moment is calculated for uniform density distribution with the nucleus shape of G. D. Cassini form [7] and depends on number of bosons through parameter a:

$$\begin{aligned}
Q_0 =& \frac{45c^8\log\left(4a(p)^2+2c^2+4a(p)R_{\max}\right)}{80a(p)^2\left(2a(p)\left(2a(p)^2-c^2\right)R_{\max}-3c^4\text{ArcSinh}(\frac{2a(p)}{c^2}R_{\max})\right)} \\
&- \frac{90a(p)c^6R_{\max}-45c^8\log(2c^2)}{80a^2\left(2a(p)(2a(p)^2-c^2)R_{\max}-3c^4\text{ArcSinh}(\frac{2a(p)}{c^2}R_{\max})\right)} \\
&+ \frac{(96a(p)^7-48a(p)^5c^2-324a(p)^3c^4)R_{\max}}{80a^2\left(2a(p)(2a(p)^2-c^2)R_{\max}-3c^4\text{ArcSinh}(\frac{2a(p)}{c^2}R_{\max})\right)} \\
&- \frac{a(p)\left(4a(p)^4-2a(p)^2c^2+24c^4-\frac{15a(p)c^4}{R_{\max}}\log(\frac{c^2+2a(p)(a(p)+R_{\max})}{c^2})\right)}{5\left(-4a(p)^3+2a(p)c^2+\frac{3c^4\text{ArcSinh}(\frac{2a(p)R_{\max}}{c^2})}{R_{\max}}\right)}
\end{aligned}$$

Inside one β-band the ratio $\dfrac{B(E2,4- > 2,p)}{B(E2,2- > 0,p)} = \dfrac{10}{7}$ conserves of course,

but to understand the mechanism of transition from one band to another it should be more beneficial to consider the ratios that connect these bands. For instance the ratio of probabilities in the large angular momentum region and $B(E2, 2- > 0, p = 0)$ of the ground band $\dfrac{B(E2, 16- > 14, p)}{B(E2, 2- > 0, p = 0)}$. Some values of this ratio are shown in the table below

p	0	1	2	3	4	5
$\dfrac{B(E2, 16- > 14, p)}{B(E2, 2- > 0, p = 0)}$	$\dfrac{600}{341}$	2.790065	3.368360	3.892864	4.400971	4.907720

Comparing these values with the experimental data one can determine directly the collective structure of the corresponding β - band head. Finally, the structure of 0^+ excited states seems to be very important in forming of the nuclear spectra. Also, the significant role in an investigation of nuclear structure may play $E0$ transitions between states related to different β - bands. These investigations are in progress.

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