

## Description of Alternating Parity Bands in a Quadrupole-Octupole Rotation Model

**N. Minkov<sup>1,2</sup>, S. Drenska<sup>1</sup> and P. Yotov<sup>1</sup>**

<sup>1</sup>Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

<sup>2</sup>RCNP Osaka University, 10-1 Mihogaoka, Ibaraki city, Osaka 567-0047, Japan

### **Abstract.**

We apply a point-symmetry based Quadrupole-Octupole Rotation Model to study the collective motion of nuclei with simultaneous presence of octupole and quadrupole deformations. We demonstrate that it describes successfully the energy levels of alternating parity bands and reproduces their odd-even staggering structure. On this basis we are capable to determine quite accurately the regions of reflection asymmetry correlations in nuclear collective spectra.

Recently we have proposed a model formalism applicable to rotation motion of nuclei with octupole deformations [1]. As a basic ingredient of the model we introduce a collective octupole Hamiltonian

$$\hat{H}_{oct} = \hat{H}_{A_2} + \sum_{r=1}^2 \sum_{i=1}^3 \hat{H}_{F_r(i)} \quad (1)$$

constructed by the irreducible representations  $A_2$ ,  $F_1(i)$  and  $F_2(i)$  ( $i = 1, 2, 3$ ) of the octahedron ( $O$ ) point-symmetry group, where

$$\hat{H}_{A_2} = a_2 \frac{1}{4} [(\hat{I}_x \hat{I}_y + \hat{I}_y \hat{I}_x) \hat{I}_z + \hat{I}_z (\hat{I}_x \hat{I}_y + \hat{I}_y \hat{I}_x)], \quad (2)$$

$$\begin{aligned}
\hat{H}_{F_1(1)} &= \frac{1}{2} f_{11} \hat{I}_z (5\hat{I}_z^2 - 3\hat{I}^2), \\
\hat{H}_{F_1(2)} &= \frac{1}{2} f_{12} (5\hat{I}_x^3 - 3\hat{I}_x \hat{I}^2), \\
\hat{H}_{F_1(3)} &= \frac{1}{2} f_{13} (5\hat{I}_y^3 - 3\hat{I}_y \hat{I}^2), \\
\hat{H}_{F_2(1)} &= f_{21} \frac{1}{2} [\hat{I}_z (\hat{I}_x^2 - \hat{I}_y^2) + (\hat{I}_x^2 - \hat{I}_y^2) \hat{I}_z], \\
\hat{H}_{F_2(2)} &= f_{22} (\hat{I}_x \hat{I}^2 - \hat{I}_x^3 - \hat{I}_x \hat{I}_z^2 - \hat{I}_z^2 \hat{I}_x), \\
\hat{H}_{F_2(3)} &= f_{23} (\hat{I}_y \hat{I}_z^2 + \hat{I}_z^2 \hat{I}_y + \hat{I}_y^3 - \hat{I}_y \hat{I}^2).
\end{aligned} \tag{3}$$

The different terms in the above Hamiltonian (cubic combinations of angular momentum operators in body fixed frame) generate rotation degrees of freedom for the system in correspondence to various octupole shapes with magnitude determined by the model parameters  $a_2$  and  $f_{r,i}$  ( $r = 1, 2$ ;  $i = 1, 2, 3$ ).

We consider that the octupole degrees of freedom are superposed on the top of the leading quadrupole deformation of the system so that the standard quadrupole rotation Hamiltonian

$$\hat{H}_{rot} = A\hat{I}^2 + A'\hat{I}_z^2, \tag{4}$$

provides the general energy scale for rotation motion of the nucleus. In addition we assume the presence of a high order quadrupole–octupole interaction

$$\hat{H}_{qoc} = f_{qoc} \frac{1}{I^2} (15\hat{I}_z^5 - 14\hat{I}_z^3 \hat{I}^2 + 3\hat{I}_z \hat{I}^4), \tag{5}$$

and a phenomenological band head term

$$\hat{H}_{bh} = E_0 + f_k \hat{I}_z. \tag{6}$$

Eqs. (2)–(6) represent the total Hamiltonian of the collective Quadrupole–Octupole Rotation Model (QORM) [1]. The yrast rotational spectrum of the system is obtained by minimizing the energy in the diagonal Hamiltonian terms with respect to the third projection,  $K$ , of the collective angular momentum  $I$  in the states  $|I, K\rangle$ , and subsequently diagonalizing the total Hamiltonian.

Generally the structure of the spectrum depends on the quadrupole and octupole shape parameters  $A$ ,  $A'$  and  $f_{1i}$ ,  $f_{2i}$  ( $i = 1, 2, 3$ ) respectively, on the high order interaction parameter  $f_{qoc}$  and the band head parameters  $E_0$  and  $f_k$ . However, for a given nucleus only few of them can be considered as free model parameters, while the others could vary in very narrow limits. So,  $A$  and  $A'$  are kept reasonably close to the known quadrupole shapes,  $E_0$  and  $f_k$  are determined to reproduce the energy and the angular momentum projection in the beginning of the spectrum, and furthermore (as it will be discussed below) three parameters of the off-diagonal octupole matrix elements can be excluded since in the

intrinsic frame of reference three octupole degrees of freedom are related to the orientation angles.

The so determined energy spectrum is built on different intrinsic  $K$ -configurations which provide a  $\Delta I = 1$  staggering behavior of rotational energy. The changing quantum number  $K$  implies the presence of a wobbling type collective motion resulting from the complicated shape characteristics of the system.

Based on the above properties, in present work we apply the model to describe experimental energy levels in nuclear octupole bands together with the spectacular  $\Delta I = 1$  staggering patterns [2] observed there. As a relevant region of applicability of our formalism we consider the states with angular momentum higher than  $I \sim 7 - 8$  where the octupole structure of the band is well developed. This is an important limitation of the study which provides physically reasonable basis for further analysis and conclusions. The point is that for  $I < 7 - 8$  the negative parity states are shifted up with respect to the positive parity states and both together do not form a single rotational band. The reason is that at low angular momenta the potential barrier that separates the two reflection asymmetric shape orientations of the system (up and down) is not high enough. As a result some tunnelling between the two mirror orientations of the system is possible which lowers the even angular momentum levels with respect to the odd levels. For the higher angular momentum  $I > 7 - 8$  the barrier becomes higher and the tunnelling effect sharply decreases. Then a well formed single alternating parity band can be considered. This process is explained reasonably in terms of a Dinuclear System Model [3].

Here we present results of our Quadrupole–Octupole Rotation Model (QORM) description of the alternating parity levels in the light actinide nuclei  $^{220-222}\text{Rn}$ ,  $^{218-226}\text{Ra}$ ,  $^{224,226}\text{Th}$ , together with the respective theoretical results for the  $\Delta I = 1$  staggering effect, which are compared with the experimental observations. The staggering patterns are presented through the fourth (discrete) derivative of the energy difference  $\Delta E(I) = E(I + 1) - E(I)$  in the form

$$Stg(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2). \quad (7)$$

The parameters of model fits are given in Table 1. A sample comparison between theoretical and experimental results for energy levels and the quantity  $Stg(I)$  is given in Table 2 for  $^{226}\text{Ra}$ , while the theoretical and experimental staggering patterns for all considered nuclei are presented in Figures 1-3. In all cases a very good agreement between theory and experiment is observed.

It is important to remark that our model procedure provides consistent description for the different nuclei although their collective properties change considerably from good rotators (as  $^{224}\text{Ra}$ ) to nuclei near the vibration region ( $^{218}\text{Ra}$ ). As it is seen from Table 1 the model parameters change consistently from nucleus to nucleus being kept in physically reasonable regions. For example the inertia parameter of the quadrupole shape  $A$  gradually decreases in the

Table 1. Parameters (in keV) of QORM energy fits.

Nucl.	$E_0$	$f_k$	$A$	$A'$	$f_{11}$	$f_{12}$	$f_{21}$	$f_{22}$	$f_{qoc}$
<sup>220</sup> Rn	1241.43	-144.50	12.88	3.19	0.445	0.065	—	-0.098	0.228
<sup>222</sup> Rn	1098.7	-218.7	20.05	3.65	1.02	-0.038	0.277	0.057	0.678
<sup>218</sup> Ra	3305.22	-1024.32	41.39	53.29	3.699	-0.198	—	0.299	1.288
<sup>220</sup> Ra	3007.48	-877.84	23.17	54.69	1.98	-0.874	-0.0001	1.321	0.956
<sup>222</sup> Ra	360.79	-93.55	15.33	0.69	0.77	-0.007	—	0.01	0.157
<sup>224</sup> Ra	400.02	-79.64	10.87	4.28	0.47	-0.021	—	0.0314	0.176
<sup>226</sup> Ra	224.25	-42.60	9.51	3.86	0.438	-0.026	—	0.039	0.179
<sup>224</sup> Th	496.05	-129.96	16.09	1.36	0.79	-0.036	—	0.055	0.115
<sup>226</sup> Th	207.12	-30.00	9.83	3.68	0.422	0.0039	—	-0.0059	0.162

Ra isotope group holding the values of about 10 keV typical for good rotators. It should be also mentioned that the parameters of the octupole terms obtain values at least one order in magnitude smaller than the leading quadrupole term. As it will be shown below the obtained octupole parameter values provide a detailed information about the octupole shape deformations that contribute to the fine structure of the spectrum. Also, we see in Table 1 that the parameter  $f_{21}$

Table 2. Energy levels (in keV) and the respective values of the quantity  $Stg(I)$  (in keV), Eq. (7), at given angular momentum  $I$  for the octupole band in <sup>226</sup>Ra (QORM description and experiment). The values of the quantum number  $K$  which minimize the diagonal part of QORM Hamiltonian are also given.

$I$	$K$	$E_{th}$	$E_{exp}$	$Stg(I)_{th}$	$Stg(I)_{exp}$
8	5	689.007	669.600		
9	6	823.769	858.200		
10	6	959.832	960.300	61.674	393.400
11	7	1115.572	1133.500	-42.584	-186.800
12	7	1274.354	1281.600	18.650	23.200
13	8	1446.208	1448.000	10.262	99.700
14	8	1625.238	1628.900	-44.256	-184.200
15	9	1808.278	1796.500	83.379	233.800
16	9	2005.154	1998.700	-127.648	-254.100
17	10	2194.424	2174.900	163.518	250.600
18	10	2406.760	2389.800	-163.897	-228.000
19	11	2597.301	2579.300	125.763	191.300
20	12	2809.171	2801.100	-70.162	-145.900
21	12	3009.579	3006.700		
22	13	3215.583	3232.700		
23	13	3423.929	3454.900		

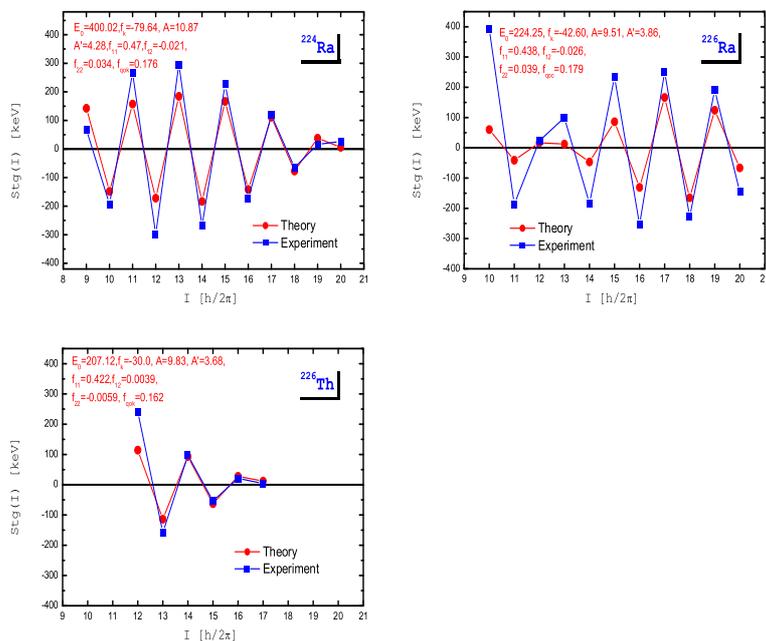


Figure 1.  $\Delta I = 1$  staggering patterns for the octupole bands in  $^{224,226}\text{Ra}$  and  $^{226}\text{Th}$ , experiment and theory (the parameter values are given in keV).

which provides essentially non axial octupole interaction is generally rejected by the fitting procedure especially in nuclei near rotational regions. In addition the other parameters of non diagonal terms  $f_{12}$  and  $f_{22}$  are also relatively small compared to  $f_{11}$  (the diagonal term) which indicates the leading role of the axial part in the octupole deformation.

Another important characteristic of our model description is the correct reproduction of the major "beat" points in the respective staggering patterns. First of all the theoretical pattern clearly indicates the regions where the alternating parity sequence is formed as a stable octupole band. For example in the case of  $^{226}\text{Ra}$  this is the region near  $I \sim 10$ . The second beat regions in the experimental staggering patterns (for example  $I \sim 20$  in  $^{224}\text{Ra}$ ) are also correctly reproduced indicating the respective change in the intrinsic structure of the band.

Our present formalism allows us to propose some general features of the electromagnetic transitions in a rotating quadrupole–octupole system. As the simplest step in this direction we consider the set of  $K$ -values (the third projection of the total angular momentum) obtained by minimizing the energy in the diagonal part of the Hamiltonian. These values are involved in the general expression

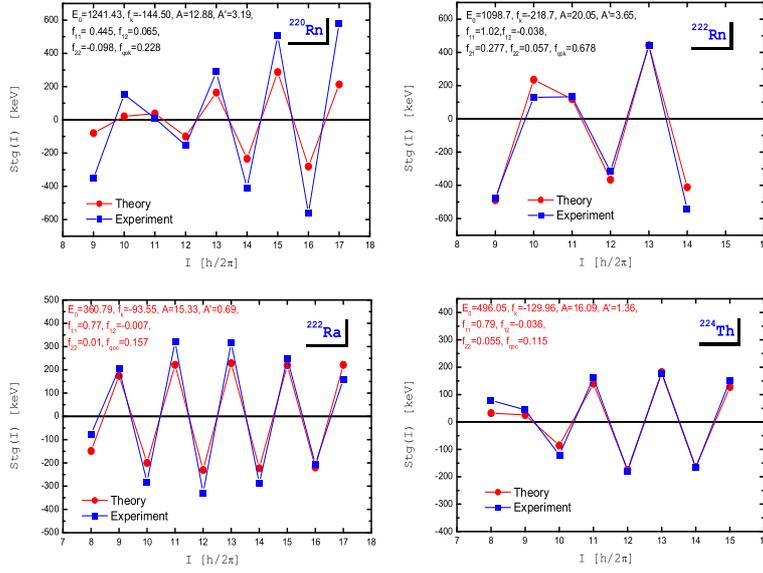


Figure 2.  $\Delta I = 1$  staggering patterns for the octupole bands in  $^{220,222}\text{Rn}$ ,  $^{222}\text{Ra}$  and  $^{224}\text{Th}$ , experiment and theory (the parameter values are given in keV).

for the reduced transition probabilities as follows [4]

$$B(E\lambda; I_1 \rightarrow I_2) \sim \langle I_1 K_1 \lambda \mu | I_2 K_2 \rangle^2 \langle K_2 | T_\mu^\lambda | K_1 \rangle^2, \quad (8)$$

where  $T_\mu^\lambda$  is the transition operator with multipolarity  $\lambda$  and  $\mu = K_2 - K_1$ . The first term in Eq. (8) is the kinematic (Clebsch-Gordan) factor which for the case of E1, E2 and E3 transitions can be written as

$$CG_{E\lambda}^2 = \langle I_1 K_1 \lambda K_2 - K_1 | I_2 K_2 \rangle^2, \quad \lambda = 1, 2, 3 \quad (9)$$

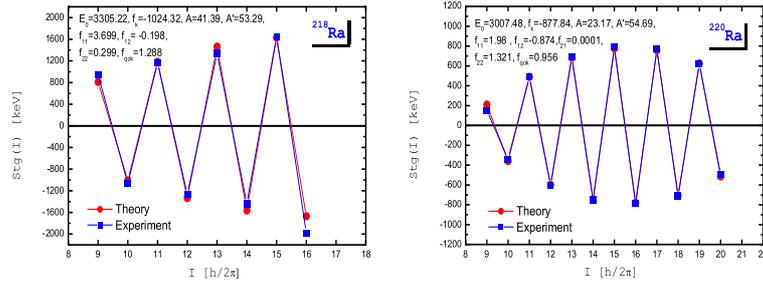


Figure 3.  $\Delta I = 1$  staggering patterns for the octupole bands in  $^{218,220}\text{Ra}$ , experiment and theory (the parameter values are given in keV).

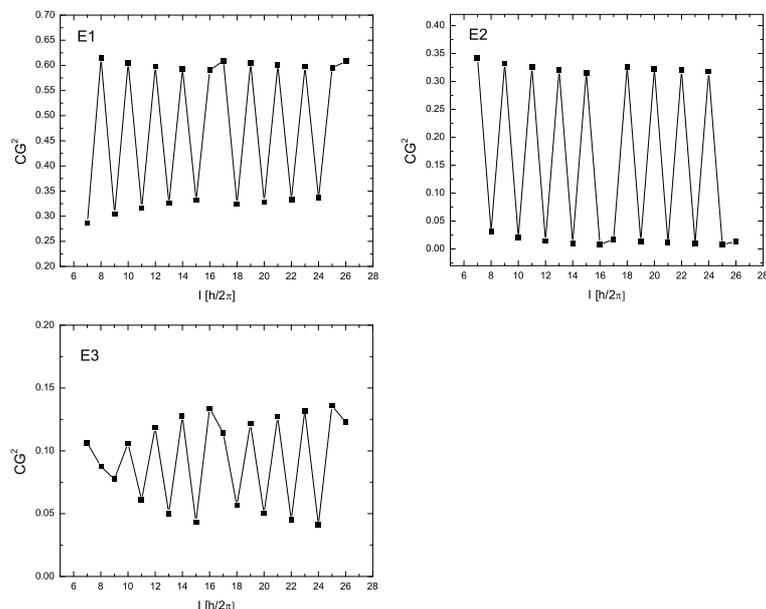


Figure 4. The square of the Clebsch-Gordan coefficients for E1, E2 and E3 transitions, Eq. (9), as a function of the angular momentum  $I$  for the set of  $K$ 's in Table 2.

with  $I_1 = I + 1$  and  $I_2 = I$  being the angular momenta of two neighboring states of the band. The changing quantum number  $K$  provides a staggering behavior of this factor as a function of angular momentum. This is demonstrated in Figure 4 for the set of  $K$ 's in Table 2.

The second term in Eq. (8), which depends on the intrinsic states of the system also depends on  $K$  but it is considered to be slightly changed along the band [4]. So in our present analysis we take it constant. (Its treatment in a microscopic extension of present considerations is envisaged.) In this way our model predictions for the quantity (9) suggest a possible staggering behavior of the reduced electric transition probability in nuclear octupole bands. This result is in consistency with the considerations in Ref. [5]. For example, an experimental indication for such a behavior is reported in [6] for the octupole band of  $^{150}\text{Sm}$ .

As a further step in our analysis it is important to estimate the physical significance of the different deformation modes in the collective motion of the system. The problem is that we consider a combination of two shape fields (quadrupole and octupole) which in general could not be fixed (parameterized) in purely geometric way so as to determine the collective dynamics of the system uniquely. Moreover, it is known, that even alone the octupole field can not be parameterized [7, 8] appropriately due to the lack of a natural "principal" axes of the shape.

A reasonable way for addressing this problem is to make some physical assumptions about the dynamical properties of the combined quadrupole–octupole shape as follows: 1) both fields are not independent; 2) the quadrupole field is the leading mode in the collective motion; 3) as a result of 1) and 2) the body fixed frame can be fixed with respect to the principal axes of the quadrupole shape which is well determined; 4) hence the total number of degrees of freedom can be reduced so as to determine the dynamical behavior of the system in a unique way. In our formalism we apply the above assumptions by diagonalizing the total Hamiltonian in the basis of collective states determined with respect to the “quadrupole” body fixed frame. An important consequence of this approach is the fact that the non-diagonal Hamiltonian terms  $\hat{H}_{A_2}$ ,  $\hat{H}_{F_1(3)}$ ,  $\hat{H}_{F_2(3)}$  [see Eqs (2) and (3)] appear to be redundant and the respective model parameters  $a_2$ ,  $f_{13}$ ,  $f_{23}$  should be set zero. From geometrical point of view it reflects the circumstance that in the intrinsic frame of reference three octupole degrees of freedom, from the total of seven ones, are related to the orientation (Euler) angles.

So the above physical assumptions provide a well determined geometrical structure of the model. To illustrate this we consider the relation between the parameters of our model and the amplitudes of the octupole deformation which can be easily derived by using Eqs (2), (3)–(7) and (18) of ref. [1]

$$\alpha_{30} = \sqrt{\frac{4\pi}{7}} f_{11} , \quad (10)$$

$$\alpha_{3\pm 1} = \pm \left( \sqrt{\frac{\pi}{21}} f_{22} + \sqrt{\frac{3\pi}{28}} f_{12} \right) + i \left( \sqrt{\frac{3\pi}{28}} f_{13} - \sqrt{\frac{\pi}{21}} f_{23} \right) , \quad (11)$$

$$\alpha_{3\pm 2} = \sqrt{\frac{8\pi}{105}} f_{21} \pm i \sqrt{\frac{2\pi}{105}} a_2 , \quad (12)$$

$$\alpha_{3\pm 3} = \pm \left( \sqrt{\frac{\pi}{35}} f_{22} - \sqrt{\frac{5\pi}{28}} f_{12} \right) + i \left( \sqrt{\frac{\pi}{35}} f_{23} + \sqrt{\frac{5\pi}{28}} f_{13} \right) . \quad (13)$$

Then the octupole deformation parameters

$$\beta_3 = \left[ \sum_{\mu=-3}^3 \alpha_{3\mu}^2 \right]^{\frac{1}{2}} , \quad \beta_{3\mu} = (\alpha_{3\mu}^2 + \alpha_{3-\mu}^2)^{\frac{1}{2}} \quad (\mu = 0, 1, 2, 3) \quad (14)$$

can be related to the QORM parameters as

$$\beta_3 = \sqrt{\frac{4\pi}{105}} [15(f_{11}^2 + f_{12}^2 + f_{13}^2) + 4(f_{21}^2 + f_{22}^2 + f_{23}^2) + a_2^2]^{\frac{1}{2}} , \quad (15)$$

$$\beta_{30} = \sqrt{\frac{8\pi}{7}} f_{11} , \quad (16)$$

$$\beta_{31} = \sqrt{\frac{2\pi}{7}} \left[ \frac{3}{4}(f_{12}^2 + f_{13}^2) + \frac{1}{3}(f_{22}^2 + f_{23}^2) + f_{12}f_{22} - f_{13}f_{23} \right]^{\frac{1}{2}}, \quad (17)$$

$$\beta_{32} = \sqrt{\frac{4\pi}{105}} [4f_{21}^2 + a_2^2]^{\frac{1}{2}}, \quad (18)$$

$$\beta_{33} = \sqrt{\frac{2\pi}{7}} \left[ \frac{5}{4}(f_{12}^2 + f_{13}^2) + \frac{1}{5}(f_{22}^2 + f_{23}^2) - f_{12}f_{22} + f_{13}f_{23} \right]^{\frac{1}{2}}. \quad (19)$$

Now we see that the removal of the redundant Hamiltonian terms ( $a_2, f_{13}, f_{23} = 0$ ) provides a set of real  $\alpha$ 's in (10)–(13) and has the same meaning as the standard transition to the body fixed frame in the case of a pure quadrupole deformation. The respective octupole deformation parameters then have the form

$$\beta'_3 = \sqrt{\frac{4\pi}{105}} [15(f_{11}^2 + f_{12}^2) + 4(f_{21}^2 + f_{22}^2)]^{\frac{1}{2}}, \quad (20)$$

$$\beta'_{30} = \sqrt{\frac{8\pi}{7}} f_{11}, \quad (21)$$

$$\beta'_{31} = \sqrt{\frac{2\pi}{7}} \left[ \frac{3}{4}f_{12}^2 + \frac{1}{3}f_{22}^2 + f_{12}f_{22} \right]^{\frac{1}{2}}, \quad (22)$$

$$\beta'_{32} = \sqrt{\frac{16\pi}{105}} f_{21}, \quad (23)$$

$$\beta'_{33} = \sqrt{\frac{2\pi}{7}} \left[ \frac{5}{4}f_{12}^2 + \frac{1}{5}f_{22}^2 - f_{12}f_{22} \right]^{\frac{1}{2}}. \quad (24)$$

Eqs (20)–(24) provide model predictions for the particular octupole deformations that play role for the alternating parity band in any one of the considered nuclei on the basis of the fitted parameter values given in Table 1. In this way the QORM formalism allows one to extract important information about the complicated shape properties of the system just from the fine structure of its collective energy spectrum. This model capability can be extended essentially by involving the electromagnetic transitions in consideration.

Analysis in the opposite direction is also possible. Given in the left side of (20)–(24) the deformation parameters determined by analysis of experimental data or another model estimations for the ground state (or lowest excited states), then we are capable to predict the collective dynamics of the system at the higher angular momentum regions providing detailed information about the respective fine structure of the spectrum. As a very promising example in this direction we consider the region of exotic  $N = Z$  nuclei, where some microscopic calculations already suggest the presence of octupole deformations in the ground state indicating the need of information about the possible collective modes that can be excited in these systems [9].

In conclusion, we demonstrated that the QORM successfully reproduces the fine structure of alternating parity bands in light actinide nuclei allowing a relevant analysis of the collective interactions associated with the quadrupole and octupole degrees of freedom. Further studies in the directions outlined above would provide a useful tool in understanding the relation between the complicated shape properties and the intrinsic structure of nuclei. In this respect an appropriate microscopic extension of the QORM formalism appears to be important. Work in this direction is in progress.

## References

- [1] N. Minkov, S. Drenska, P. Raychev, R. Roussev and D. Bonatsos, (2001) *Phys. Rev.* **C63** 044305.
- [2] D. Bonatsos, C. Daskaloyannis, S. Drenska, N. Karoussos, N. Minkov, P. Raychev and R. Roussev, (2000) *Phys. Rev.* **C62** 024301.
- [3] T.M. Shneidman, G.G. Adamian, N. V. Antonenko, R. V. Jolos and W. Scheid, (2002) *Phys. Lett.* **B526** 322.
- [4] A. Bohr and B. R. Mottelson, (1975) *Nuclear Structure* vol. II (Benjamin, New York).
- [5] W. Urban, R. M. Lieder, W. Gast, G. Hebbinghaus, A. Kramer-Flecken, K. P. Blume and H. Hubel, (1987) *Phys. Lett.* **B185** 331.
- [6] W. Andrejtscheff, C. Doll, F. Bečvář, H. G. Börner, (1998) *Phys. Lett.* **B437** 249.
- [7] I. Hamamoto, X. Z. Zhang and H. Xie, (1991) *Phys. Lett.* **B257** 1.
- [8] P. A. Butler and W. Nazarewicz, (1996) *Rev. Mod. Phys.* **68** 349.
- [9] M. Yamagami, K. Matsuyanagi and M. Matsuo, (2001) *Nucl. Phys.* **A693** 579.