

# Quasi-Elastic Electron Scattering from Light Nuclei

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## **Abstract.**

While the longitudinal quasi-elastic response function is satisfactorily understood, the excess in the transverse response remained a puzzle. For light nuclei, where this excess is largest, we determine the separated response functions from an analysis of the world data on quasi-elastic inclusive electron scattering. The corresponding Euclidean response functions are derived and compared to those calculated with Green's function Monte Carlo methods, using realistic interactions and currents. Large contributions associated with two-body currents are found for the transverse response, in agreement with data. The contributions of two-body charge and current operators in the  $^3\text{He}$ ,  $^4\text{He}$ , and  $^6\text{Li}$  response functions are also studied via sum-rule techniques. We find in particular that the large enhancement can only be understood if in the description of the initial (bound) and the final (continuum) state the tensor- and short range n-p correlations are included.

## **1 Introduction**

Over the past decades, much effort has gone into understanding quantitatively the roles that two-body components of the nuclear electromagnetic current play in the quasi-elastic response of nuclei at intermediate momentum transfers. Yet, despite the considerable attention that has been devoted to this topic, many open questions remain. Complications arise, in particular, as a consequence of the need of providing an accurate description for the initial bound- and the final scattering-state wave functions, based on realistic Hamiltonians.

The slow progress is also related to the confusing experimental situation that for some time obfuscated the interpretation of the data. The longitudinal and transverse response functions, obtained during the eighties from a Rosenbluth

separation of experimental cross sections, seemed to indicate that there was a gross (up to 40%) lack of *longitudinal* strength in the main quasi-elastic peak, and a correspondingly too low Coulomb sum rule [1, 2].

J. Jourdan [3] has performed an improved analysis of the data. The results are shown in Table 1. When analyzing the Saclay data alone, but using the correct e-p cross section, when including the well known relativistic corrections, when accounting for the fact that the upper cutoff of the integral over the longitudinal response is not  $\infty$  and when doing the Coulomb corrections using the exact calculations of the Ohio group, J. Jourdan finds that there is *no* violation of the Coulomb sum rule; within experimental errors the Coulomb sum, divided by Z, derived from the Saclay data alone is compatible with 1 within the experimental uncertainties.

A more precise Coulomb sum can be determined by including the SLAC data [4], which have been taken at more forward angle, and in a more reliable data acquisition mode (event mode, not histogram mode). This allows for a more reliable extraction of the longitudinal contribution. The result, shown in the last line of Table 1, shows that the larger angular range of the world data leads to a two times smaller error bar on the Coulomb sum. The sum rule is fulfilled for the case (the iron nucleus) where the biggest deviation had been claimed. Similar fulfillment is found for all other nuclei where the data base allowed a reliable extraction, see Table 2.

Table 1. Results for the Coulomb sum at  $q=570$  MeV/c for iron, divided by Z.

Result of Meziani ( $^{56}\text{Fe}$ )	$0.60\pm 0.20$
Dipole $G_{ep}$ replaced by Simon	$0.64\pm 0.20$
Relativistic correction added	$0.69\pm 0.20$
Tail contribution added	$0.75\pm 0.20$
Coulomb correction added	$0.81\pm 0.20$
.....	.....
SLAC-data added	$0.97\pm 0.12$

This apparent lack of longitudinal strength had absorbed much of the theoretical effort of the past two decades. The excess of transverse strength — presumably due to two-body currents — observed in the quasi-elastic region turns out to be the genuine problem. The experimental situation has been put in sharp

Table 2. Coulomb sum for upper integration limit of 355MeV

q	$\omega_{max}$	C	Fe	Ca
570	355	$0.88 \pm 0.13$	$0.91 \pm 0.12$	$0.91 \pm 0.15$
570	355		0.91	CBF theory

focus by the work on super-scaling by Donnelly and Sick [5] which allowed to systematically compare the longitudinal and transverse response functions. This work showed in the most clear way that the transverse strength for nuclei with mass number  $A=12, \dots, 56$  exceeds the longitudinal one already in the main quasi-elastic peak by 20-40%, in addition to the excess of strength occurring in the “dip” between the quasi-elastic and  $\Delta$ -peaks. This excess of strength in the region of the quasi-elastic peak is the main subject of this paper.

Theoretical calculations of two-body contributions in the region of the quasi-elastic peak have been performed by many groups [6]– [19] using different approaches. The overwhelming majority of these calculations find very small contributions due to the dominant two-body terms (pion contact and in-flight, and  $\Delta$ -excitation diagrams).

The calculations, based in general on an independent-particle initial state (shell model, Fermi gas model), give very small two-body contributions in the quasi-elastic peak [6]– [13], [16, 19] as the pion and  $\Delta$  terms tend to cancel. The model study of Leidemann and Orlandini [10], in which the nuclear response was expressed in terms of the response of deuteron-like pairs of nuclear density, first pointed out that it is important to account in the initial state for the *tensor* correlations between  $np$  pairs. Only when these (rather short-range) tensor correlations were included would the two-body terms give appreciable contributions to the quasi-elastic response. This insight was quantitatively confirmed by Fabrocini [17], who calculated the transverse response of infinite nuclear matter using correlated basis function theory including one-particle-one-hole intermediate states.

The calculation of Carlson and Schiavilla [14] was performed for  ${}^4\text{He}$  using Green’s function Monte Carlo (GFMC) techniques, a realistic (Argonne  $v'_8$ ) N-N interaction and consistently constructed two-body terms. The inelastic response could be accurately calculated in terms of the Euclidean response (an integral over the response function, see below). These “exact” calculations found that the charge-exchange character of the N-N interaction lead to substantial enhancements and in agreement with that observed experimentally. The study of Ref. [14] provided a qualitative understanding of the  ${}^4\text{He}$  quasi-elastic response, but did not identify those aspects of the calculation responsible for the successful prediction.

Here, we study the longitudinal and transverse response functions of light nuclei,  ${}^3\text{He}$  and  ${}^4\text{He}$ , and compare to GFMC theory. Accurate data for these responses in the region of the quasi-elastic peak can be determined via an analysis of the world data. A simultaneous study of  ${}^3\text{He}$  and  ${}^4\text{He}$  is particularly interesting as the excess in the transverse channel increases very rapidly between  $A=3$  and  $A=4$ , a feature which can give us a further handle for the understanding of two-body effects. The study of  ${}^4\text{He}$ , including the higher momentum transfers now available, is especially promising, since the available data [3] (as discussed

below) indicate that the relative excess of transverse strength in the quasi-elastic peak is largest for this nucleus.  ${}^4\text{He}$  thus is the ideal nucleus for a detailed study of the transverse excess.

In order to also include heavier nuclei, which allow us to follow the evolution of the excess with mass number, we also study via sum-rules the two-body contributions for p-shell nuclei, for which variational Monte Carlo (VMC) wave functions are available.

## 2 Response Functions from World Data

The longitudinal ( $L$ ) and transverse ( $T$ ) responses have been determined [20] by analyzing the ( $e,e'$ ) world data on  ${}^3\text{He}$  and  ${}^4\text{He}$ . A determination of the response functions in inclusive quasi-elastic scattering from the world cross section data has many advantages over the standard approach, which employs data from a single experiment only. For the extraction of the response functions, the difference of cross sections at high-energy/forward-angle and at low-energy/backward-angle is used. For a most accurate determination of the response functions the difference of the  $L$ - and  $T$ -contributions to the cross sections has to be maximized by including data over the largest possible angular range. This can only be achieved by including all available world cross section data. For  ${}^3\text{He}$  and  ${}^4\text{He}$  the use of the world data not only expands the range of available data in scattering angle but also increases the range of momentum transfer  $q$  where a separation can be performed.

At low momentum transfers  $q$  data sets are available from [21–24]. For high  $q$  the data come from [21, 22], which both cover the angular region from  $90^\circ$  to  $144^\circ$ ; they are combined with the cross sections from [25–28] which provide high-energy/forward-angle data with energies up to 7.2 GeV at scattering angles down to  $8^\circ$ .

In contrast to the analysis performed for medium- $A$  nuclei [3], Coulomb distortions play a negligible role for  ${}^3\text{He}$  and  ${}^4\text{He}$  and no corrections need to be applied. The following expression, valid in the plane wave Born approximation (PWBA), is used for the Rosenbluth or  $L/T$ -separation:

$$\Sigma(q, \omega, \epsilon) = \frac{d^2\sigma}{d\Omega d\omega} \frac{1}{\sigma_{Mott}} \epsilon \left(\frac{q}{Q}\right)^4 = \epsilon R_L(q, \omega) + \frac{1}{2} \left(\frac{q}{Q}\right)^2 R_T(q, \omega),$$

where the longitudinal virtual photon polarization  $\epsilon$  is defined as

$$\epsilon = \left(1 + \frac{2q^2}{Q^2} \tan^2 \frac{\vartheta}{2}\right)^{-1},$$

and varies between 0 to 1 as the electron scattering angle  $\vartheta$  ranges from  $180$  to  $0$  degrees. Here,  $d^2\sigma/d\Omega d\omega$  are the experimental cross sections,  $\omega$ ,  $q$  and  $Q$  are the energy transfer, 3- and 4-momentum transfers, respectively, and  $\sigma_{Mott}$  is the

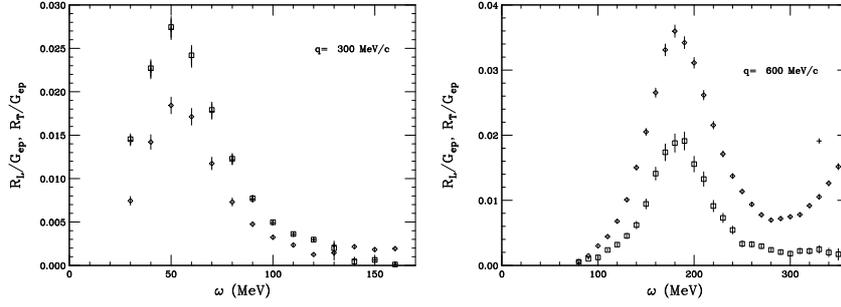


Figure 1. Longitudinal ( $\square$ ) and transverse ( $\diamond$ ) response functions of  ${}^3\text{He}$  at momentum transfers of 300 and 600 MeV/c. Indicated with a + are the upper integration limits used for the Euclidean response (Section 5).

Mott cross section. The structure of the equation for  $\Sigma$  shows, that measurements of the cross section at fixed  $\omega$  and  $q$  but different  $\epsilon$  allow for a separation of the two response functions  $R_L(q, \omega)$  and  $R_T(q, \omega)$ .

In general, the experimental data from the various experiments were measured at a given incident energy and scattering angle as a function of the energy loss of the scattered electron. To determine the cross section at given values of  $q$  and  $\omega$ , the data have to be interpolated. This traditionally was done by dividing out  $\sigma_{Mott}$  from the measured cross sections and interpolating the responses along  $\omega/E$ .

The responses in Figures 1,2 are obtained using an improved scheme by first dividing out an appropriate sum of elementary electron-nucleon cross sections, and removing kinematical dependencies. Essentially what is calculated from the data is the scaling function  $F(y, q)$ . The extracted  $F(y, q)$  are then used to de-

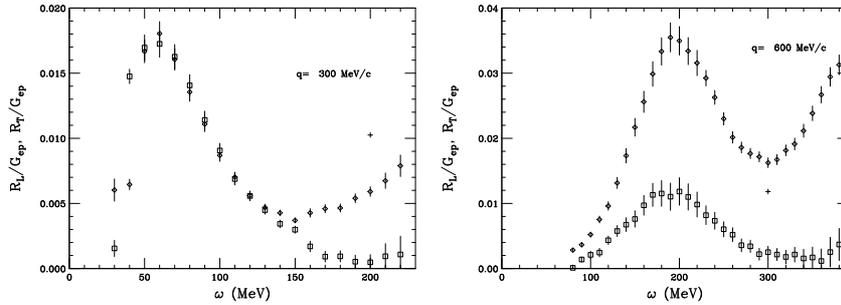


Figure 2. Longitudinal ( $\square$ ) and transverse ( $\diamond$ ) response functions of  ${}^4\text{He}$  at momentum transfers of 300 and 600 MeV/c. Indicated with a + are the upper integration limits used for the Euclidean response (Section 5).

termine  $F(y, q_o)$  at the desired value  $q_o$  by interpolating  $F(y, q)$  along lines of constant  $y$ .

The response functions have been extracted for  $q=300\text{--}700$  MeV/c in steps of 100 MeV/c for both nuclei. Representative examples for the longitudinal and transverse response functions resulting from this analysis of the *world* data are shown in Figures 1 and 2.

### 3 Scaling Functions

To demonstrate the excess strength of  $R_T(q, \omega)$ , we here study the scaling properties of the response functions. Guided by the approach of Sick and Donnelly [5], we use the variable  $\psi'$ . As discussed in [5],  $\psi'$ -scaling can also be studied for separated response functions. The dimensionless scaling functions  $f_{L,T}$  are defined in [5] as

$$f_{L,T} \equiv k_F \frac{R_{L,T}}{G_{L,T}},$$

with the factors  $G_{L,T}$  given in [5]. For the relativistic Fermi gas model and in IA, the universal relation

$$f_L = f_T = f$$

is predicted. We compare in Figure 3 the scaling functions  $f_L(\psi')$  and  $f_T(\psi')$  obtained for all response functions extracted from the global analysis of the  ${}^3\text{He}$  and  ${}^4\text{He}$  data. The longitudinal response functions approximately scale to a universal curve over the entire quasi-elastic peak, and do fulfill the Coulomb sum rule. The results for  $R_T(q, \omega)$  confirm, that the basic problem in quasi-elastic electron-nucleus scattering is the *excess strength in the transverse response*.  $R_T(q, \omega)$  does not scale to the same function as  $R_L(q, \omega)$ , but exceeds it considerably. The transverse excess is much larger for  ${}^4\text{He}$  than for  ${}^3\text{He}$ .

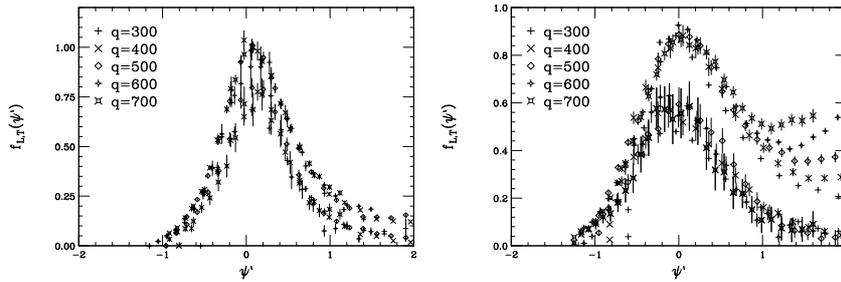


Figure 3. The scaling functions  $f_L$  and  $f_T$  are shown for all  $q$  values on the left for  ${}^3\text{He}$  and on the right for  ${}^4\text{He}$ . The upper bands of points correspond to  $f_T$ , the lower bands to  $f_L$ .

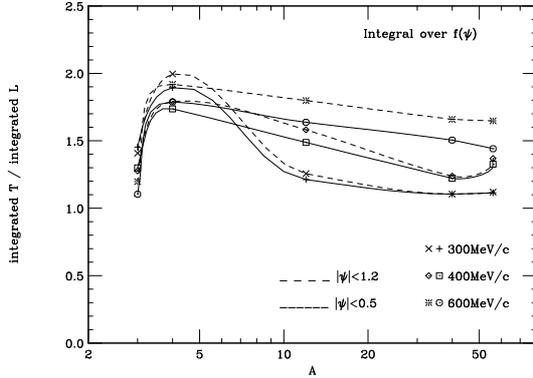


Figure 4. Ratio of transverse to longitudinal integrated strength for  ${}^3\text{He}$ ,  ${}^4\text{He}$ ,  ${}^{12}\text{C}$ ,  ${}^{40}\text{Ca}$ , and  ${}^{56}\text{Fe}$ : 300 MeV/c: x and +, 400 MeV/c:  $\diamond$  and  $\square$ , 600 MeV/c: \* and  $\circ$ . Points at the same  $q$  are joined by lines. The integrations are over the indicated ranges of  $\psi'$ .

The excess of transverse strength occurs at all momentum transfers, and does not seem to be limited to the “dip” region, but affects the whole quasi-elastic peak region, extending below the  $\pi$ -production threshold. The transverse strength in the dip, which increases with increasing  $q$ , is related to the growing overlap between the high-energy side of the quasi-elastic peak and the tail of the  $\Delta$ -peak.

In order to study the  $A$ -dependence of this excess, we can look at the longitudinal and transverse responses integrated over  $\psi'$  — those for  ${}^{12}\text{C}$ ,  ${}^{40}\text{Ca}$ , and  ${}^{56}\text{Fe}$  have been determined in Ref. [5]. We have integrated these responses over the region of  $\psi'$  that essentially covers the quasi-elastic peak ( $|\psi'| < 1.2$ ). When limiting the integration range to  $|\psi'| < 0.5$  much of the contribution from the tail of the  $\Delta$  is eliminated, at least for the light nuclei. The ratio of transverse to longitudinal integrated strength is shown in Figure 4.

Figure 4 makes it clear that: i) the excess of transverse strength rises very rapidly between  ${}^3\text{He}$  and  ${}^4\text{He}$ , and is indeed largest for  ${}^4\text{He}$ ; ii) it is already large at the lowest  $q$ , the increase at the larger  $q$  for the heavier nuclei is mainly due to the fact that the tail of the  $\Delta$  peak contributes appreciably despite the restricted range of integration in  $\psi'$ .

#### 4 Euclidean Response

For the study of the transverse excess, we are primarily interested in the overall strength of the longitudinal and transverse response. For this, we consider the Euclidean response functions, defined as [14, 29]

$$\tilde{E}_{T,L}(q, \tau) = \int_{\omega_{th}}^{\infty} \exp[-\omega\tau] R_{T,L}(q, \omega),$$

where the  $R_{T,L}(q, \omega)$  are the standard responses,  $E_0$  is the ground-state energy of the nucleus, and  $\omega_{\text{th}}$  is the threshold for the response of the system excluding the elastic contribution. The longitudinal and transverse Euclidean response functions represent weighted sums of the corresponding  $R_L(q, \omega)$  and  $R_T(q, \omega)$ : at  $\tau=0$  they correspond to the Coulomb and transverse sum rules, respectively. The derivatives with respect to  $\tau$  evaluated at  $\tau=0$  correspond to the energy-weighted sum rules. Larger values of  $\tau$  correspond to integrals over progressively lower-energy regions of the response.

For non-relativistic systems, the  $\tilde{E}_{T,L}$  can be obtained from:

$$\begin{aligned}\tilde{E}_L(q, \tau) &= \langle 0 | \rho^\dagger(\mathbf{q}) \exp[-(H - E_0)\tau] \rho(\mathbf{q}) | 0 \rangle \\ &\quad - \exp\left(-\frac{q^2\tau}{2Am}\right) |\langle 0(\mathbf{q}) | \rho(\mathbf{q}) | 0 \rangle|^2, \\ \tilde{E}_T(q, \tau) &= \langle 0 | \mathbf{j}_T^\dagger(\mathbf{q}) \exp[-(H - E_0)\tau] \mathbf{j}_T(\mathbf{q}) | 0 \rangle \\ &\quad - \exp\left(-\frac{q^2\tau}{2Am}\right) |\langle 0(\mathbf{q}) | \mathbf{j}_T(\mathbf{q}) | 0 \rangle|^2\end{aligned}$$

where  $|0(\mathbf{q})\rangle$  represents the ground state recoiling with momentum  $\mathbf{q}$ , and sums over spin projections are understood.

Below we consider the scaled Euclidean responses

$$E_{L,T}(q, \tau) = \frac{\exp[q^2\tau/(2m)]}{[G_{E,p}(\tilde{Q}^2)]^2} \tilde{E}_{L,T}(q, \tau),$$

where  $\tilde{Q}^2$  is the squared four-momentum transfer evaluated at the quasi-elastic peak. This removes the trivial energy dependence obtained from scattering off an isolated (non-relativistic) nucleon, and the  $q$  dependence associated with the nucleon form factors. The longitudinal response  $E_L(q, \tau)$  is unity for an isolated proton, and the transverse response  $E_T(q, \tau)$  is simply the square of its magnetic moment.

The great advantage of the use of the Euclidean response is the fact that it can be calculated exactly using Green's function or path integral Monte Carlo techniques. The Euclidean response can be calculated directly from the ground state, hereby including both final state interactions and two-nucleon currents. There is no need to quantitatively describe the final continuum state, which at present is not possible for  $A > 3$ .

For the nuclei considered here, the ground-state wave functions are obtained with variational Monte Carlo. They are of the general form [30]:

$$|\Psi_T\rangle = \prod_{i < j < k} [1 - \tilde{U}_0(ijk)] \mathcal{S} \prod_{i < j} \left[ \left[ 1 - \sum_{k \neq i, j} \tilde{U}_{2\pi}(ij; k) \right] F_{ij} \right] |\Phi\rangle,$$

where for 3- and 4-nucleon systems  $|\Phi\rangle$  is simply an anti-symmetrized product of spins and isospins. The central three-nucleon correlation  $\tilde{U}_0(ijk)$  is a scaled version of the repulsive central component of the Urbana-IX (UIX) three-nucleon interaction. The magnitude of the correlation and its range are scaled via variational parameters. The pair correlations  $F_{ij}$  depend upon the pair separation  $r_{ij}$  and the spins and isospins of the pair:

$$F_{ij} = f^c(r_{ij}) \left[ 1 + u^\sigma(r) \sigma_i \cdot \sigma_j + u^t(r) S_{ij} + u^{\sigma\tau}(r) \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j + u^{t\tau} S_{ij} \tau_i \cdot \tau_j \right].$$

The correlation  $\tilde{U}_{2\pi}(ij; k)$  is similarly scaled from the anti-commutator part of the two-pion exchange three-nucleon interaction. The anti-commutator depends upon the spins and isospins of only the two nucleons  $i$  and  $j$ , but the spatial positions of all three. Similarly, the magnitude of the spin-isospin dependent correlations  $u$  for pair  $ij$  are quenched by the presence of other nucleons. Both the two-nucleon correlation  $F_{ij}$  and the  $\tilde{U}_{2\pi}$  correlation arising from the three-nucleon interaction contain tensor-like terms correlating the spins and orientations of the nucleons. The contributions of these correlations to the response are discussed below.

The above wave functions are not exact, but they allow a rather precise characterization of the Euclidean response, as evidenced by comparisons with calculations using the correlated-hyperspherical-harmonics wave functions [31] in  $A=3$ . The Hamiltonian used in these studies is the Argonne model  $v'_8$  [30] N-N interaction plus the UIX three-nucleon interaction. This interaction reproduces many known properties of the alpha particle, including its binding energy and charge form factor.

The Euclidean response can be calculated with Green's function Monte Carlo techniques in an approach similar to the one used for the ground-state wave function. One needs to calculate matrix elements of the type:

$$\tilde{M}(\tau) = \frac{\langle 0|O_2 \exp[-(H - E_0)\tau] O_1|0\rangle}{\langle 0| \exp[-(H - E_0)\tau]|0\rangle}.$$

For a ground-state calculation of the energy ( $O_1=1$ ,  $O_2=H$ ) the matrix element is evaluated by a Monte Carlo sampling of the coordinate-space paths. The denominator is exactly one for an exact ground-state wave function, otherwise there is a correction for finite  $\tau$ . For a more general matrix element  $\tilde{M}$  one keeps another complete set of amplitudes for each operator  $O_1$ , each set of amplitudes corresponding to the full operator acting on the ground state. The paths are sampled precisely as in the ground-state calculation [32], and hence unaffected by the operators  $O_1, O_2$ . This allows one to calculate the response to a variety of operators (charge, current, different momenta, etc.) simultaneously.

The nuclear electromagnetic operator used in the calculation of Carlson and Schiavilla [20] consist of one- and two-body terms for the charge  $\rho(\mathbf{q})$  and cur-

rent  $\mathbf{j}(\mathbf{q})$  operators. The one-body operators  $\rho_i^{(1)}$  and  $\mathbf{j}_i^{(1)}$  have the standard expressions obtained from a relativistic reduction of the covariant single-nucleon current. The two-nucleon operators are the standard ones used successfully in explaining the elastic form factors of light nuclei [35–37]. They have been, as far as possible, derived in a model-independent way from the N-N interaction. Explicit expressions are given in Refs. [33, 34].

## 5 Comparison to Experiment

From the longitudinal ( $L$ ) and transverse ( $T$ ) experimental response functions of Figures 1 and 2 the corresponding experimental Euclidean responses have been calculated and are shown in part in Figures 5 and 6. The nucleon electromagnetic form factors [38] are already divided out. The integration has been performed up to the energy loss  $\omega$  where the  $T$ -response starts to increase significantly with  $\omega$  (the corresponding value of  $\omega$  is indicated in Figures 1 and 2 by a  $+$ ) such as to not include too much of the  $\Delta$  contribution. For the  $T$ -Euclidean response at very small  $\tau$  the tail of the  $\Delta$ -peak nevertheless plays a role, so the experimental response in this region is indicated by a dashed line only, and should not be compared to theory.

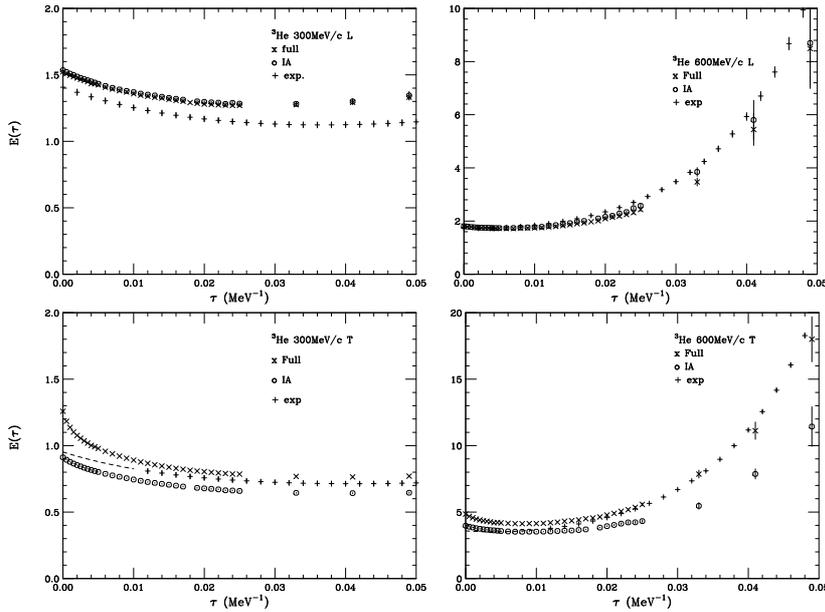


Figure 5. Longitudinal (upper half of figure) and transverse Euclidean response of  ${}^3\text{He}$  for momentum transfers 300 and 600 MeV/c.

We also show in Figures 5 and 6 the calculated Euclidean responses, obtained both in IA and when including the contributions associated with the two-body charge and current operators. The two-body contributions reduce by a small amount the  $L$ -responses, but they increase the  $T$ -responses by a very substantial amount at all momentum transfers. The enhancement in the transverse response is found already at quite low  $\omega$ , as demonstrated by the Euclidean response at large  $\tau$ . Two-body effects thus are important over the entire quasi-elastic peak, and not only in the “dip-region” on the large- $\omega$  side of the quasi-elastic peak which in the past was considered the main place where to see MEC.

The contribution of two-body currents (Figure 6) at low  $q$  are larger at low  $\omega$  (large  $\tau$ ) and smaller at large  $\omega$  (low  $\tau$ ). At large  $q$ , this situation is reversed. One can also see from Figure 5 and 6 that theory explains well the rapid increase of two-body contributions between  $^3\text{He}$  and  $^4\text{He}$ . In contrast to most published calculations (for a discussion see Section 1), the present calculation does give the sizeable two-body contribution required by the data.

As we pointed out above, the transverse excess is largest for  $^4\text{He}$ , the nucleus which therefore is optimal for a study of MEC. We find that the agreement between theory and experiment is excellent for the  $L$ -response, thus implying that an accurate treatment of the nuclear spectrum has been achieved as two-

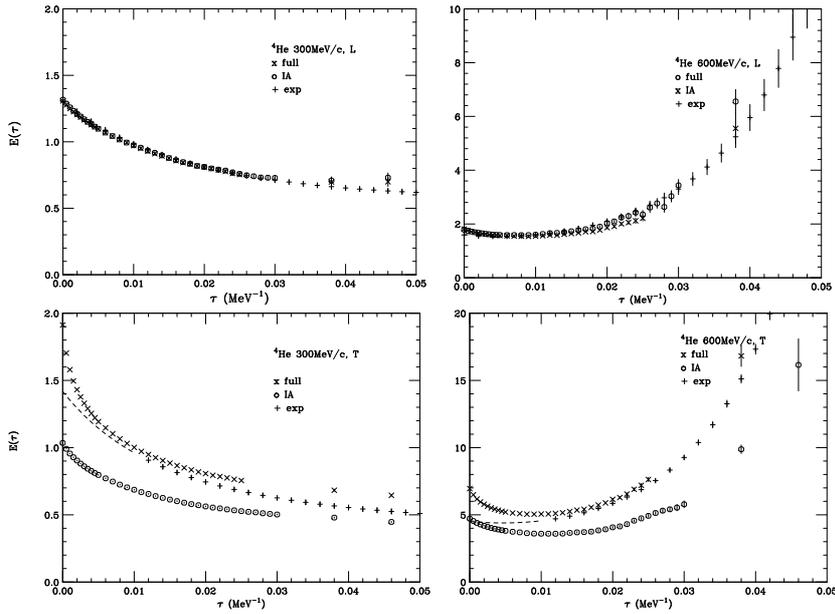


Figure 6. Longitudinal (upper half of figure) and transverse Euclidean response of  $^4\text{He}$  for momentum transfers 300 and 600 MeV/c.

body operators give small corrections in the  $L$ -channel. For the  ${}^4\text{He}$   $T$ -Euclidean response, the large two-body effects found by experiment are successfully predicted by theory, although the associated contributions are a bit too large in the  $q$ -range 400–500 MeV/c.

## 6 Sum Rules

Non-energy- and energy-weighted longitudinal sum rules have been extensively studied in the past (see Refs. [34, 39]). The number of studies of sum rules of the transverse response is much more limited [40]. Here we focus on the latter, in particular on the enhancement of transverse strength due to two-body currents. We also address, within the sum-rule context, the issue of the enhancement in the ratio of transverse to longitudinal strength, observed in the quasi-elastic response functions of nuclei. Finally, we attempt to provide a semi-quantitative explanation for the observed systematics in the excess of transverse strength, both as function of mass number and momentum transfer.

For the study of sum rules, the calculations employed [20] are based on the AV18/UIX Hamiltonian, and use correlated-hyperspherical-harmonics (variational Monte Carlo) wave functions for  $A=3-4$  ( $A=6$ ) nuclei.

The sum rules are defined as

$$S_\alpha(q) = C_\alpha \int_{\omega_{\text{th}}}^{\infty} d\omega S_\alpha(q, \omega) = C_\alpha \left[ \langle 0 | O_\alpha^\dagger(\mathbf{q}) O_\alpha(\mathbf{q}) | 0 \rangle - |\langle 0 | O_\alpha(\mathbf{q}) | 0 \rangle|^2 \right],$$

where  $S_\alpha(q, \omega)$  is the point-nucleon longitudinal ( $\alpha=L$ ) or transverse ( $\alpha=T$ ) response function,  $O_\alpha(\mathbf{q})$  is either the charge  $\rho(\mathbf{q})$  or current  $\mathbf{j}(\mathbf{q})$  operator divided by the square of the proton form factor  $|G_E^p(\tilde{Q}^2)|^2$  ( $\tilde{Q}^2$  is evaluated at the energy transfer corresponding to the quasi-elastic peak),  $|0\rangle$  denotes the ground state, and the elastic contribution to the sum has been removed. The constants  $C$  amount to  $C_L = 1/Z$ ,  $C_T = 2m^2/(Z\mu_p^2 + N\mu_n^2)q^2$ .

These sums have been calculated [20] for  $A=3,4,6$  using the full  $v_{18}$  interaction and the MEC discussed above. We here only quote some selected results that let us better understand the agreement between experiment and theory found above.

We first show in Table 3, for  $q=600$  MeV/c, the numerical effect of MEC on the transverse strength. The transverse excess is large, increases between  $A=3$

Table 3. The transverse sum rule obtained with one-body only and both one- and two-body current operators.

${}^3\text{He}$		${}^4\text{He}$		${}^6\text{Li}$	
1	1+2	1	1+2	1	1+2
1.01	1.25	1.01	1.49	1.01	1.41

and 4 and only gradually becomes smaller for  $A=6$ . Surprisingly, MEC have also a considerable effect on the longitudinal strength. As shown by Table 4 the integrated strength is *decreased* by about 7%, essentially independently of  $A$ . This shows, that the commonly made assumption that the Coulomb sum rule is negligibly affected by MEC, is not correct. As the effect of MEC is essentially the same for  $A=3,4,6$  one must assume that also for heavier nuclei a similar reduction of the Coulomb sum is occurring (an observation only partly accounted for in Tables 1,2).

Table 4. The longitudinal sum rule obtained with one-body only and both one- and two-body charge operators.

${}^3\text{He}$		${}^4\text{He}$		${}^6\text{Li}$	
1	1+2	1	1+2	1	1+2
0.982	0.908	0.973	0.910	0.990	0.924

When repeating the calculation of the two-body effects with simplified operators, one also finds that the most important two-body current contributions are those associated with the  $PS$  (pion-like) and  $\Delta$ -excitation currents.

Moreover, the transverse strength associated with two-body currents is almost entirely due to  $pn$  pairs. When only keeping the  $n - n$  and  $p - p$  correlations, the enhancement in the transverse strength is very small, as shown by Table 5, again for  $q=600$  MeV/c.

Table 5. The  ${}^4\text{He}$  transverse sum rule: contribution of  $pp$  and  $nn$  pairs.

1	1+2	1+2; $pp$ or $nn$ only
1.01	1.47	1.03

Lastly, we show in Table 6 the amount of excess strength due to MEC obtained when removing in the ground state all correlations, *i.e.* when using a Fermi gas. This corresponds to what has been done in most calculations of MEC in the literature [6]- [19]. Table 6 (again for  $q=600$  MeV/c) largely explains why most previous calculations have found much too small a transverse enhancement.

Table 6. Excess-strength contributions to the Fermi gas sum rules from terms involving two-nucleon currents.

$\Delta S_L$	$\Delta S_T$
0.017	0.060

## 7 Conclusions

We have discussed the understanding of the separated response functions as measured in inclusive electron-nucleus scattering in the region of the quasi-elastic peak. Much of the discussion of the past had been focused, without good reason as more careful work has shown, on the longitudinal strength (the Coulomb sum rule). The main question not understood, the strong enhancement of the transverse strength in the main quasi-elastic peak region, remained open.

The transverse response can be expected to get appreciable contributions from meson exchange currents. Actual calculations, however, produced MEC contributions that were far too small. Here we have focused on the transverse strength for  ${}^4\text{He}$ , the nucleus where this transverse excess strength is maximal.

We find that this excess can be understood once one uses a theoretical approach that does treat the short-range and tensor  $n - p$  correlations in both the initial and final (continuum) state. This is feasible for  $A=3,4,6$  as variational Monte Carlo calculations using modern N-N interactions can be performed for the bound states. By studying the Euclidian response (rather than the response as a function of electron energy loss) one can get around the difficulty of a similarly quantitative calculation for the continuum state: the Euclidian response can directly be calculated starting from the ground-state wave function.

The comparison of experimental and calculated response functions shows, that both the pronounced enhancement of the transverse strength and the smaller reduction of the longitudinal strength (the Coulomb sum) is due to MEC and can quantitatively be understood.

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