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Mixed-Symmetry States in Nuclei Near Shell Closure

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Abstract.

The quasiparticle-phonon model is adopted to investigate the microscopic structure of some low-lying states (known as mixed-symmetry states) recently discovered in nuclei around closed shells. The study determines quantitatively the phonon content of these states and shows that their main properties are determined by a subtle competition between particle-particle and particle-hole quadrupole interactions and by the interplay between orbital and spin-flip motion.

1 Introduction

Considerable effort has been devoted to the search and study of low-lying states in heavy nuclei after the discovery of the magnetic dipole (M1) excitation in the deformed ^{156}Gd through inelastic electron scattering experiments [1]. Such a mode, known as scissors mode, was predicted for deformed nuclei in a semiclassical two-rotor model (TRM) [2], in schematic microscopic approaches [3,4], and in the proton-neutron version of the interacting boson model (IBM-2) [5, 6]. As discussed in several reviews [7–9], this M1 mode is now well established in the different deformed regions of the periodic table and is also fairly well understood on experimental as well as theoretical grounds.

An important feature of the scissors mode is its isovector character. States of isovector nature were first considered in a geometrical model [10] as proton-neutron surface vibrational high-energy modes. These states were predicted to exist also at low energy in a revised version of the model [11, 12]. Low-lying isovector excitations are naturally predicted in the algebraic IBM-2 as mixed-symmetry states with respect to the exchange between proton and neutron bosons. They are distinguished from the symmetric ones by the F-spin quantum number [13], which is the boson analogue of isospin for nucleons.

In spherical nuclei, the M1 excitation mechanism does not permit to generate the scissors mode and, more generally, mixed-symmetry states from the $J^\pi = 0^+$ ground state, because of the conservation of the angular momenta of proton and neutron fluids. In these nuclei, the lowest mixed-symmetry state is predicted to have $J^\pi = 2^+$ and can be excited from the ground state via weak E2 transitions. Its signature, however, is its strong M1 decay to the lowest isoscalar $J^\pi = 2^+$ state.

Recently, unambiguous evidence in favor of mixed-symmetry states in spherical nuclei was provided by an experiment which combined photon scattering with a $\gamma\gamma$-coincidence analysis of the transitions following β decay of ^{94}Tc to ^{94}Mo [17]. Such a decay has populated several excited states among which it was possible to identify a two-phonon $J^\pi = 1^+$ and a one-phonon $J^\pi = 2^+$ mixed-symmetry states. The picture was enriched with the subsequent identification of two additional mixed-symmetry states, a $J^\pi = 3^+$ [18] and a $J^\pi = 2^+$ [19] two-phonon states. These experiments have also produced an almost exhaustive mass of information on low-lying levels and absolute transition strengths which made possible a rather accurate characterization of these low lying states. This analysis was carried out in IBM-2 and could test not only the isospin character of the states but also the multiphonon content of them. It was found that, while the lowest mixed-symmetry $J^\pi = 2^+$ state is composed of a single phonon, the other states lying at higher energy had a two phonon structure.

The collectivity and the energy of the low-lying excitations in nuclei near shell closure change considerably with mass number A. This reflects the close correlation of the simple (collective and non collective) modes with the detailed structure of the low-lying excited states. The phenomenological algebraic model is not suitable to clarify this structure. Such a study was carried with fairly good success through two microscopic calculations, one framed within the nuclear shell model [20], the other within the quasiparticle-phonon model (QPM) [21,22]. The two approaches are complementary under many respects. The shell model provides naturally information on the single particle content of the wave function. Moreover it is exact within the chosen model space. On the other hand, the space truncation induces uncertainties and, in this specific case, can account only effectively for the coupling between the low-lying, mainly orbital, states under study and the spin-flip configurations which are partly excluded from the model space.

The spherical QPM [23] is based on the quasi-boson approximation and, therefore, is reliable only in spherical nuclei with few valence nucleons. On the other hand, it allows to choose the configurations which are more relevant to the problem, including the high-energy spin-flip configurations, and has a clear phonon content which allows to state a bridge with the IBM-2 analysis.

2 Quasiparticle-Phonon Model

The QPM intrinsic Hamiltonian has the form

$$H = H_{sp} + V_{pair} + V_M^{ph} + V_{SM}^{ph} + V_M^{pp} \ . \tag{1}$$

H_{sp} is a one-body Hamiltonian, V_{pair} the monopole pairing, V_M^{ph} and V_{SM}^{ph} are respectively sums of separable multipole and spin-multipole interactions acting in the particle-hole, and V_M^{pp} is the sum of particle-particle multipole pairing potentials.

The QPM procedure goes through several steps. One first transforms the particle $a^\dagger_{jm}$ (a_{jm}) into quasiparticle $\alpha^\dagger_{jm}$ (α_{jm}) operators by making use of the Bogolyubov canonical transformation

$$a^\dagger_{jm} = u_{jm}\alpha^\dagger_{jm} + v_{jm}(-)^{j-m}\alpha_{j-m} \ . \tag{2}$$

In the second step one constructs the RPA phonon basis $Q^\dagger_{\lambda\mu i}\Psi_0$, where Ψ_0 is the RPA vacuum and

$$Q^\dagger_{\lambda\mu i} = \frac{1}{2}\sum_{\tau=n,p}\sum_{jj'}\left\{\psi^{\lambda i}_{jj'}[\alpha^\dagger_j\alpha^\dagger_{j'}]_{\lambda\mu} - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}[\alpha_{j'}\alpha_j]_{\lambda-\mu}\right\}_\tau \ , \tag{3}$$

is the phonon operator of multipolarity $\lambda\mu$. RPA equations are derived and solved to get the RPA energy spectrum and to determine the internal phonon structure, namely the coefficients $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ for each multipolarity λ and each root i.

It is worth to point out that the RPA basis includes collective as well as non collective phonons. The first ones are coherent linear combinations of many quasiparticle pairs configurations. The lowest $[2^+_1]_{RPA}$ and $[3^-_1]_{RPA}$ phonon states are appropriate examples. Most of the states, however, are non collective phonons, namely pure two-quasiparticle configurations. It is also important to stress that the particle-particle interaction V_M^{pp} is included in generating the RPA solutions. Such a term enhances the particle-particle correlations in the phonons and will be shown to play a crucial role.

In the third step, one expresses the Hamiltonian (1) in terms of quasiparticle and RPA phonon operators by making use of the above defining equations. Once this is done, the QPM Hamiltonian becomes

$$H_{QPM} = \sum_{\mu i}\omega_{\lambda i}Q^\dagger_{\lambda\mu i}Q_{\lambda\mu i} + H_{vq}, \tag{4}$$

where now the RPA eigenvalues $\omega_{\lambda i}$ and the $\psi^{\lambda i}_{q_1 q_2}$ and $\phi^{\lambda i}_{q_1 q_2}$ amplitudes entering into the corresponding phonon operators are well determined.

The quasiparticle-phonon coupling term H_{vq} is composed of a sum of multipole $H^{\lambda\mu}_{vq}$ and spin-multipole $H^{l\lambda\mu}_{vq}$ pieces, whose exact expressions can be found in Refs. [23]. The basic structure of the multipole term is

$$H^{\lambda\mu i}_{vq} \simeq \sum_{\tau j j'} V^{\lambda\mu}_{jj'} (Q^\dagger_{\lambda\mu i} + (-)^{\lambda-\mu} Q_{\lambda-\mu i})(\alpha^\dagger_j \otimes \alpha_{j'})_{\lambda\mu}. \tag{5}$$

No free parameters appear in the transformed Hamiltonian (4), once those appearing in the Hamiltonian (1) we started with have been fixed.

In the fourth step, one puts the quasiparticle-phonon Hamiltonian in diagonal form. This is done by using the variational principle with a trial wave function of total spin JM [24–26]

$$\Psi_\nu(JM) = \Bigg\{ \sum_i R_i(\nu J) Q^\dagger_{JMi} + \sum_{\substack{i_1\lambda_1\\ i_2\lambda_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(\nu J) \left[Q^\dagger_{\lambda_1\mu_1 i_1} \otimes Q^\dagger_{\lambda_2\mu_2 i_2} \right]_{JM}$$

$$+ \sum_{\substack{i_1\lambda_1 i_2\lambda_2\\ i_3\lambda_3 I}} T^{i_1\lambda_1 i_2\lambda_2 I}_{i_3\lambda_3}(\nu J) \left[\left[Q^\dagger_{\lambda_1\mu_1 i_1} \otimes Q^\dagger_{\lambda_2\mu_2 i_2} \right]_{IK} \otimes Q^\dagger_{\lambda_3\mu_3 i_3} \right]_{JM} \Bigg\} \Psi_0, \tag{6}$$

where Ψ_0 represents the phonon vacuum state and R, P, and T are unknown amplitudes, and ν labels the specific excited state. In computing the norm of the wave function as well as the necessary matrix elements the exact commutation relations for the phonons (7) [23–26] are used

$$[Q_{\lambda\mu i}, Q^\dagger_{\lambda'\mu' i'}]_- = \frac{\delta_{i,i'}\delta_{\lambda,\lambda'}\delta_{\mu,\mu'}}{2} \sum_{jj'} [\psi^{\lambda i}_{jj'}\psi^{\lambda i'}_{jj'} - \varphi^{\lambda i}_{jj'}\varphi^{\lambda i'}_{jj'}]$$

$$- \sum_{\substack{jj'j_2\\ mm'm_2}} \alpha^+_{jm}\alpha_{j'm'} \Big\{ \psi^{\lambda i}_{j'j_2}\psi^{\lambda' i'}_{jj_2} C^{\lambda\mu}_{j'm'j_2m_2} C^{\lambda'\mu'}_{jmj_2m_2}$$

$$- (-)^{\lambda+\lambda'+\mu+\mu'} \varphi^{\lambda i}_{jj_2}\varphi^{\lambda' i'}_{j'j_2} C^{\lambda-\mu}_{jmj_2m_2} C^{\lambda'-\mu'}_{j'm'j_2m_2} \Big\} . \tag{7}$$

While the first term corresponds to the boson approximation, the second one takes into account the internal fermion structure of phonons and insures the antisymmetrization of the multiphonon wave function (6). This has been successfully used to calculate the structure of the excited states in many spherical even-even nuclei [24–26].

The electromagnetic transition operators written in terms of quasiparticle and phonon operators have the form

$$M(\mathrm{X}\lambda\mu) = \sum_{\tau=n,p} \sum_{jj'} \frac{< j||\mathrm{X}\lambda||j' >}{\sqrt{2\lambda+1}} \left\{ \frac{u^{(\pm)}_{jj'}}{2} \sum_i (\psi^{\lambda i}_{jj'} + \varphi^{\lambda i}_{jj'}) \right.$$
$$\left. \times\ (Q^{\dagger}_{\lambda\mu i} + (-)^{\lambda-\mu} Q_{\lambda-\mu i}) + v^{(\mp)}_{jj'} \sum_{mm'} C^{\lambda\mu}_{jmj'm'} (-)^{j'+m'} \alpha^{+}_{j'm'} \alpha_{j'-m'} \right\}. \quad (8)$$

where $< j||\mathrm{X}\lambda||j' >$ is a reduced single-particle transition matrix element and

$$u^{(\pm)}_{jj'} = u_j v_{j'} \pm v_j u_{j'}, \qquad v^{(\pm)}_{jj'} = u_j u_{j'} \pm v_j v_{j'}. \quad (9)$$

The first term of Eq. (8) promotes the exchange of one-phonon between initial and final states, the second induces the so called *boson forbidden* transitions between components with the same number of phonons or differing by an even number of them. This second term was studied in detail in Ref. [25]. It was shown that, in order to describe these transitions, it is necessary to go beyond the ideal boson picture and take into account the internal fermion structure of the phonons and of the electromagnetic operator (8).

3 Numerical Details

We adopt a Woods-Saxon one-body potential U with parameters taken from [27, 28]. The corresponding single-particle spectra can be found in Ref. [29]. The radial component of the multipole fields entering into the particle-hole and particle-particle separable interaction is chosen to be $f(r) = dU(r)/dr$. The strengths $\kappa^{(2)}$ and $\kappa^{(3)}$ of the quadrupole-quadrupole and octupole-octupole particle-hole interaction were fixed by a fit to the energies of the first 2^+ and 3^- states. The strengths $\kappa^{(\lambda)}$ of the other multipole terms were adjusted so as to leave unchanged the energy of the computed lowest two-quasiparticle states [29]. Only the quadrupole pairing interaction in the particle-particle channel is important for our purposes. We have used for this force the strength parameters $\mathrm{G}^{(2)} = \mathrm{G}^{(2)}_{nn} = \mathrm{G}^{(2)}_{pp}$ and $\mathrm{G}^{(2)}_{np} = 0$. The role played by $\mathrm{G}^{(2)}$ will be discussed below.

We study first the properties of the the first and second 2^+ RPA states, which represent practically the building blocks of the low-lying multiphonon states. In order to test their isospin nature we compute the ratio [30]

$$B(2^+) = \frac{|\langle 2^+ \| \sum_k^{\mathrm{p}} r_k^2\, Y_{2\mu}(\Omega k) - \sum_k^{\mathrm{n}} r_k^2\, Y_{2\mu}(\Omega k) \| g.s.\rangle|^2}{|\langle 2^+ \| \sum_k^{\mathrm{p}} r_k^2\, Y_{2\mu}(\Omega k) + \sum_k^{\mathrm{n}} r_k^2\, Y_{2\mu}(\Omega k) \| g.s.\rangle|^2}. \quad (10)$$

This ratio probes the isoscalar ($B(2^+) < 1$) or isovector ($B(2^+) > 1$) properties of the 2^+ state under consideration. The calculation shows that the first $[2^+]_{RPA}$ state is isoscalar and will be denoted by $\left[2^+_{is}\right]_{RPA}$. Its properties are determined almost solely by the value of the isoscalar quadrupole strength $\kappa_0^{(2)}$. Those of the second $[2^+]_{RPA}$ state, which will be denoted by $\left[2^+_{iv}\right]_{RPA}$, depend critically on the ratio $G^{(2)}/\kappa_0^{(2)}$ between the strengths of the quadrupole pairing and particle-hole interactions. The example of ^{136}Ba shown in Table 1 is illustrative of all nuclei. The ratio $B(2^+)$ increases dramatically with $G^{(2)}/\kappa_0^{(2)}$, showing that $\left[2^+_{iv}\right]_{RPA}$ changes from isoscalar to isovector. The other properties of the state are also very sensitive to $G^{(2)}/\kappa_0^{(2)}$. Indeed, the strength $B(E2; g.s. \rightarrow \left[2^+_{iv}\right]_{RPA})$ increases with it thereby denoting an enhancement of the collectivity of the $\left[2^+_{iv}\right]_{RPA}$. Similarly the strength of the M1 transition between the $\left[2^+_{iv}\right]_{RPA}$ and the $\left[2^+_{is}\right]_{RPA}$ increases.

Table 1. The dependence of M1 and E2 transitions on the ratio $G^{(2)}/\kappa_0^{(2)}$ in ^{136}Ba.

$G^{(2)}/\kappa_0^{(2)}$	$B(E2; g.s. \rightarrow 2^+_{iv})_{RPA}$ (e^2b^2)	$B(M1; 2^+_{iv} \rightarrow 2^+_{is})_{RPA}$ (μ_N^2)	$B(2^+_{iv})$
0	0.0032	0.042	0.58
0.85	0.011	0.24	22.6

Another quantitative test of the isospin nature of the lowest $[2^+]_{RPA}$ states is provided by the relative signs of the neutron and proton amplitudes ψ entering the RPA phonons (3). As shown in Table 2 for ^{136}Ba, the proton-neutron amplitudes ψ of the main components of the $\left[2^+_{is}\right]_{RPA}$ phonon are in phase, while those of the $\left[2^+_{iv}\right]_{RPA}$ are in opposition of phase. Table 2 shows that, for an appropriate

Table 2. Structure of the first RPA phonons (only the largest components are given) and corresponding $B(2^+)$ ratios (see eq. (4)) for ^{136}Ba.

λ_i^π	$w_{\lambda_i^\pi}$ (MeV)	Structure	$B(E2)$ (e^2b^2)	$B(2^+)$
2^+_{is}	0.95	$0.76(1h_{11/2})^2_n + 0.72(2g_{7/2})^2_p$ $0.24(3s_{1/2}2d_{3/2})_n + 0.43(2d_{5/2})^2_p$ $0.31(2d_{3/2})^2_n + 0.23(1g_{7/2}2d_{3/2})^2_p$	0.51	0.0034
2^+_{iv}	2.009	$0.85(1h_{11/2})^2_n - 0.98(1g_{7/2})^2_p$ $0.37(2d_{3/2})^2_n - 0.17(2d_{5/2})^2_p$ $0.22(3s_{1/2}2d_{3/2})_n - 0.1(1h_{11/2})^2_p$	0.011	22.6

value of the ratio $G^{(2)}/\kappa_0^{(2)}$ ($= 0.8 \div 0.9$), the RPA basis contains a collective isoscalar $\left[2^+_{is}\right]_{RPA}$ and a slightly collective isovector $\left[2^+_{iv}\right]_{RPA}$ state. The two states are mutually coupled via a relatively strong M1 transition.

We now proceed with the QPM eigenvalue problem by diagonalizing the Hamiltonian (4) in the multiphonon basis (6). The choice of the phonon basis is dictated by the properties of the states to be determined. We included only phonons of positive parity, since negative parity phonons are not relevant to our low-lying positive parity QPM states. We considered phonons of multipolarities $\lambda = 1 \div 6$ and, for each λ, we included all phonons up to a given energy. Since the QPM Hamiltonian mix the multiphonon components differing by one phonon, the fragmentation of the two-phonon states is sensitive to the number of one- and three-phonon configurations. In the present calculation the one-phonon space was spanned by several RPA states of energy up to 5 MeV. Only in the case of 1^+ states, the one-phonon space was extended to an energy which includes the M1 resonance. The most important three-phonon components are : $[(2^+_{is} \otimes 2^+_{is})_{IK} \otimes 2^+_{is}]_{JM}$ and $[(2^+_{is} \otimes 2^+_{is})_{IK} \otimes 2^+_{iv}]_{JM}$.

4 Results

We adopted the outlined QPM formalism to generate the low-lying positive parity states and to computed the $E2$ and $M1$ mutual transition strengths in ^{136}Ba, ^{94}Mo and ^{112}Cd. In all these nuclei either the proton or the neutron open shell is occupied by two particles only.

4.1 ^{136}Ba

Energy and structure of the low-lying excited states in ^{136}Ba are given in Table 3. Most of the states have a component which exhausts more than 50% of the norm of the wave function. Due to the dominance of a single component, the states acquire a distinct nature. We can classify them according to isospin and, for a given isospin, according to the number of RPA phonons. In ^{136}Ba, the first and second 2^+ as well as the first 4^+ result to be isoscalar, while most of the remaining low-lying states fall into the isovector group. There are few other non collective (NC) states which are not characterized by the lowest RPA 2^+ phonons and therefore do not fall in either of the two groups. The 2^+_{nc} state shown in the table is an example.

The lowest isoscalar and isovector states have respectively the $\left[2^+_{is}\right]_{RPA}$ and the $\left[2^+_{iv}\right]_{RPA}$ single phonons as dominant components. The other isoscalar or isovector states are characterized by a $\left[2^+_{is} \otimes 2^+_{is}\right]_{RPA}$ or a $\left[2^+_{is} \otimes 2^+_{iv}\right]_{RPA}$ two-phonon components. For instance, the second $2^+_{2,is}$ is mainly an isoscalar two-phonon state with an admixture of the $\left[2^+_{is}\right]_{RPA}$ one-phonon and of the $[[2^+_{is} \otimes 2^+_{is}]_{IK} \otimes 2^+_{is}]_{JM}$ (10 %) three-phonon configurations.

Table 3. Energy and structure of selected low-lying excited states in ^{136}Ba. Only the dominant components are presented.

State		E (keV)		Structure,%
T	J $^{\pi}$	EXP.	QPM	
IS	$2^+_{1,is}$	810	760	$77\%[2^+_{is}]_{RPA}$ + $19\%[2^+_{is}\otimes 2^+_{is}]_{RPA}$
	$2^+_{2,is}$	1551	1640	$48\%[2^+_{is}\otimes 2^+_{is}]_{RPA}$ + $17\%[2^+_{is}]_{RPA}$
	$4^+_{1,is}$	1866	1630	$60\%[2^+_1\otimes 2^+_1]_{RPA}$
IV	$2^+_{1,iv}$	2129	1850	$73\%[2^+_{iv}]_{RPA}$
	$1^+_{1,iv}$	2694	2800	$85\%[2^+_{is}\otimes 2^+_{iv}]_{RPA}$
	$2^+_{2,iv}$		3120	$51\%[2^+_{is}\otimes 2^+_{iv}]_{RPA}$
	$4^+_{1,iv}$		3230	$41\%[2^+_{is}\otimes 2^+_{iv}]_{RPA}$
	$3^+_{1,iv}$		3040	$90\%[2^+_{is}\otimes 2^+_{iv}]_{RPA}$
NC	2^+_{nc}	2080	2370	non collective

The isospin nature combined with the phonon structure of the states leads to well defined $E2$ and $M1$ selection rules. Let us consider ^{136}Ba for illustrative purposes. We used the neutron and proton effective charges e^*_n = 0.1 and e^*_p = 1.1, respectively, to compute the $E2$ reduced transition probabilities. As shown in Table 4, the $E2$ strengths are quite large for the transitions between the members of the isoscalar group, fairly large for transitions between isovector states

Table 4. $E2$ transitions connecting some excited states in ^{136}Ba calculated in QPM. The experimental data are taken form Ref. [16]

	$B(E2; J_i \to J_f)(e^2b^2)$	EXP	QPM
$\Delta T = 0$	$B(E2; g.s \to 2^+_{1,is})$	0.400(5)	0.39
	$B(E2; g.s \to 2^+_{2,is})$	0.016(4)	0.08
	$B(E2; 2^+_{2,is} \to 2^+_{1,is})$	0.09(4)	0.15
	$B(E2; 4^+_{1,is} \to 2^+_{1,is})$		0.093
$\Delta T = 0$	$B(E2; 1^+_{1,iv} \to 2^+_{1,iv})$		0.066
	$B(E2; 3^+_{1,iv} \to 2^+_{1,iv})$		0.066
	$B(E2; 2^+_{2,iv} \to 2^+_{1,iv})$		0.04
	$B(E2; 4^+_{1,iv} \to 2^+_{1,iv})$		0.036
$\Delta T = 1$	$B(E2; g.s \to 2^+_{1,iv})$	0.045(5)	0.05
	$B(E2; 2^+_{1,iv} \to 2^+_{1,is})$		0.003
	$B(E2; 1^+_{1,iv} \to 2^+_{1,is})$		0.003
	$B(E2; 1^+_{1,iv} \to 2^+_{2,is})$		0.0004

$(1^+_{1,iv} \rightarrow 2^+_{1,iv})$, $(2^+_{2,iv} \rightarrow 2^+_{1,iv})$. In both cases, the $E2$ transitions are promoted by the exchange collective phonon term of the transition operator (8) and are therefore strong. The transitions between isovector and isoscalar states, which differ by an even number of phonons, are promoted instead by the scattering term and are therefore small.

The agreement with the experimental data [16] is generally good with few remarkable exceptions. The calculation yields a strength four times larger than the experimental value for the $E2$ transition from the ground to the isoscalar $2^+_{2,is}$ state. Such a large computed value is to be ascribed to the presence of a too large $[2^+_{is}]_{RPA}$ component (17 %). A strong suppression of such a component would reduce drastically the strength of the transition without practically affecting the rest of the $E2$ transition scheme. Such a suppression should actually yield a better overall agreement with experiments. It should produce in fact a modest reduction of the $B(E2; 2^+_{2,is} \rightarrow 2^+_{1,is})$ bringing this value closer to the experimental one. It must be pointed out, however, that this latter transition is almost totally determined by the isoscalar two-phonon $[2^+_{is} \otimes 2^+_{is}]_{RPA}$ component and is already close to the experimental value. Such a good agreement indicates that the estimate given here for the two-phonon amplitude (48%) is close to the true value. From inspecting the other results, one can actually infer that also for the other states the estimated amplitudes of the dominant components are reliably close to the true values.

A reverse pattern holds for the $M1$ transitions. As shown in Table 5, the strengths of the transitions between states of the same isospin character are strongly suppressed. On the other hand, the strengths of the $M1$ transitions between isoscalar and isovector states are large. It is to be pointed out that these transitions connect states differing by two RPA phonons, like the $g.s. \rightarrow 1^+_{1,iv}$ transition, or with an equal number of phonons. We also point out that the strengths of some transitions are close to $0.2\mu_N^2$, others are in the range $0.06 \div 0.09\mu_N^2$. These smaller values are due to the smaller amplitudes of the two-

Table 5. QPM versus experimental $M1$ transitions between some excited states in ^{136}Ba. The experimental data are taken from Ref. [16]

	$B(M1; J_i \rightarrow J_f)(\mu_N^2)$	EXP	QPM $g^s_{eff} = 0.7$
$\Delta T = 1$	$B(M1; 2^+_{1,iv} \rightarrow 2^+_{1,is})$	0.26(3)	0.25
	$B(M1; g.s. \rightarrow 1^+_{1,iv})$	0.13(2)	0.17
	$B(M1; 1^+_{1,iv} \rightarrow 2^+_{2,is})$	0.6(1)	0.18
	$B(M1; 2^+_{2,iv} \rightarrow 2^+_{2,is})$		0.06
	$B(M1; 3^+_{1,iv} \rightarrow 2^+_{2,is})$		0.07
	$B(M1; 3^+_{1,iv} \rightarrow 4^+_{1,is})$		0.09
$\Delta T = 0$	$B(M1; 1^+_{1,iv} \rightarrow 2^+_{1,iv})$		8.10^{-4}

phonon components present in the states involved in the transitions (see Table 3). The computed $M1$ transition scheme is in overall agreement with experiments with one puzzling exception. The experimental strength of the $1^+_{1,iv} \rightarrow 2^+_{2,is}$ $M1$ transition is three times larger than the computed one. We could reduce this discrepancy only by assuming that the two QPM $1^+_{1,iv}$ and $2^+_{2,is}$ states were pure $[2^+_{is} \otimes 2^+_{is}]_{RPA}$ and $[2^+_{is} \otimes 2^+_{iv}]_{RPA}$ two-phonon states. Such an assumption, however, would alter the scheme obtained for the $E2$ transitions. In particular we would get too large a strength for the isoscalar $E2$ $2^+_{2,is} \rightarrow 2^+_{1,is}$ transition.

As shown in Table 2, a second 1^+_2 state with excitation energy of 3370 keV was measured in Ref. [16]. This state is coupled to the ground state via a strong $M1$ transition. On the other hand, no $1^+_2 \rightarrow 2^+_{2,is}$ M1 transition was observed. If one assumes that, like the other $1^+_{1,iv}$, this is mainly an isovector two-phonon state, then, according to IBM, we should expect a strong transition to the $2^+_{2,is}$ isoscalar two-phonon state at variance with the experimental data. It is therefore more reasonable to assume that such a second 1^+_2 contains a sizeable spin-flip component.

4.2 ^{94}Mo

For this nucleus the experimental information is quite reach [17–19] and few theoretical microscopic studies are available. A calculation was performed in a truncated shell model space using a surface delta interaction [20], another was carried out by the present authors within the QPM [21, 22].

Energies and phonon structure of the calculated excited states are given in Table 6. As in ^{136}Ba, the states have one dominant component. The first $2^+_{1,is}$ is

Table 6. Energies and structure of selected low-lying excited states in ^{94}Mo. Only the dominant components are presented.

State		E (keV)		Structure,%
T	J $^\pi$	EXP	QPM	
IS	$2^+_{1,is}$	871	860	$93\%[2^+_{is}]_{RPA}$
	$2^+_{2,is}$	1864	1750	$82\%[2^+_{is} \otimes 2^+_{is}]_{RPA}$
	$4^+_{1,is}$	1573	1733	$82\%[2^+_{is} \otimes 2^+_{is}]_{RPA}$
IV	$1^+_{1,iv}$	3129	2880	$90\%[2^+_{is} \otimes 2^+_{iv}]_{RPA}$
	$2^+_{1,iv}$	2067	1940	$95\%[2^+_{iv}]_{RPA}$
	$2^+_{2,iv}$	2393	2730	$27\%[2^+_{is} \otimes 2^+_{iv}]_{RPA}$
	$2^+_{3,iv}$	2740	3014	$59\%[2^+_{is} \otimes 2^+_{iv}]_{RPA}$
	$4^+_{1,iv}$		3120	$64\%[2^+_{is} \otimes 2^+_{iv}]_{RPA}$
	$3^+_{1,iv}$	2965	2940	$87\%[2^+_{is} \otimes 2^+_{iv}]_{RPA}$
NC	$1^+_{2,iv}$		3550	$40\%[1^+_1]_{RPA}$

dominantly a $[2^+_{is}]_{RPA}$ one phonon isoscalar state, the second is an isoscalar two-phonon, the third an isovector one-phonon. They are actually more pure than in the case of ^{136}Ba. The two-phonon component $[2^+_{is} \otimes 2^+_{iv}]_{RPA}$ is dominant in $1^+_{1,iv}$, $3^+_{1,iv}$, and, with a smaller weight, in the $4^+_{1,iv}$. The same component is present with a significant amplitude in several 2^+ states belonging to the isovector group.

This simple phonon structure leads to an even more regular pattern in the transition scheme than in ^{136}Ba. The $E2$ reduced transition probabilities are shown in Table 7. They were computed with effective charges e^*=0.2 for neutrons and e^*=1.2 for protons. We notice once again strong $E2$ transitions between isoscalar states differing by one $[2^+_{is}]_{RPA}$ phonon, fairly strong transitions between isovector states again differing by one RPA phonon and very weak transitions between isovector to isoscalar states, differing by an even number of phonons. We get an overall agreement with experiments, also with respect to IBM [17]

Table 7. $E2$ transitions connecting some excited states in ^{94}Mo calculated in QPM. The experimental data are taken form Refs. [17, 18]

	$B(E2; J_i \to J_f)(e^2 fm^4)$	EXP	QPM	IBM-2
	$B(E2; g.s \to 2^+_{1,is})$	2030(40)	1978	2333
$\Delta T = 0$	$B(E2; g.s \to 2^+_{2,is})$	32(7)	35	0
	$B(E2; 2^+_{2,is} \to 2^+_{1,is})$	720(260)	673	592
	$B(E2; 4^+_{1,is} \to 2^+_{1,is})$	670(100)	661	592
	$B(E2; 2^+_{2,iv} \to 2^+_{1,iv})$		127	
	$B(E2; 2^+_{3,iv} \to 2^+_{1,iv})$		266	
$\Delta T = 0$	$B(E2; 1^+_{1,iv} \to 2^+_{1,iv})$	< 690	374	556
	$B(E2; 3^+_{1,iv} \to 2^+_{1,iv})$	$250\ ^{+310}_{-210}$	368	582
		$(1.5^{+1.2}_{-0.6}) \times 10^3$		
	$B(E2; 4^+_{1,iv} \to 2^+_{1,iv})$		274	
	$B(E2; g.s \to 2^+_{1,iv})$	230(30)	150	151
	$B(E2; g.s \to 2^+_{2,iv})$	27(8)	18	0
$\Delta T = 1$	$B(E2; g.s \to 2^+_{3,iv})$	83(10)	10	0
	$B(E2; 1^+_{1,iv} \to 2^+_{1,is})$	30(10)	13	49
	$B(E2; 3^+_{1,iv} \to 2^+_{1,is})$	9^{+25}_{-8}	12	

The $M1$ transitions are shown in Table 8. The measured $1^+_{1,iv} \to g.s$ and $1^+_2 \to g.s$ $M1$ strengths are both reproduced by our calculation. The structure of these two states however is totally different. Their different nature emerges more clearly than in ^{136}Ba. As shown in Table 6, the first one is basically a two-phonon isovector state, consistently with the IBM picture. In this algebraic approach the $M1$ transition of this mixed-symmetry state to the ground state is forbidden in the $U(5)$ spherical vibrational limit and is allowed only in the $O(6)$ limit. In our RPA

Table 8. $M1$ transitions connecting some excited states in ^{94}Mo calculated in QPM. The experimental data are taken from Refs. [17, 18]

	$B(M1; J_i \to J_f)(\mu_N^2)$	EXP	QPM		IBM-2
			$g^s_{eff} = 0.7 g^s_{free}$	$g^s_{eff} = 0.0 g^s_{free}$	
	$B(M1; 1^+_{1,iv} \to 2^+_{2,is})$	0.43(5)	0.75	0.22	0.36
	$B(M1; 2^+_{1,iv} \to 2^+_{1,is})$	0.48(6)	0.72	0.23	0.30
	$B(M1; 2^+_{2,iv} \to 2^+_{2,is})$		0.10	0.034	
$\Delta T = 1$	$B(M1; 2^+_{3,iv} \to 2^+_{2,is})$	0.35(11)	0.24	0.072	0.1
	$B(M1; 3^+_{1,iv} \to 2^+_{2,is})$	$0.24^{+0.14}_{-0.07}$	0.34	0.10	0.18
	$B(M1; 3^+_{1,iv} \to 4^+_{1,is})$	$0.074^{+0.044}_{-0.019}$	0.26	0.08	0.13
	$B(M1; 4^+_{1,iv} \to 4^+_{1,is})$	0.8(2)	0.75	0.23	
	$B(M1; 1^+_{1,iv} \to g.s.)$	0.16(1)	0.14	0.09	0.16
	$B(M1; 1^+_{1,iv} \to 2^+_{1,is})$	0.007^{+6}_{-2}	6.10^{-4}	5.10^{-3}	0
$\Delta T = 1$	$B(M1; 2^+_{2,iv} \to 2^+_{1,is})$	0.07	0.001	0.002	0
	$B(M1; 2^+_{3,iv} \to 2^+_{1,is})$	0.03	0.013	0.005	0
	$B(M1; 3^+_{1,iv} \to 2^+_{1,is})$	$0.01^{+0.012}_{-0.006}$	0.006	0.0025	0
	$B(M1; 1^+_{1,iv} \to 2^+_{1,iv})$	< 0.05	3.10^{-6}	2.10^{-5}	0
$\Delta T = 0$	$B(M1; 3^+_{1,iv}) \to 2^+_{1,iv})$	$0.021^{+0.035}_{-0.014}$ $0.09^{+0.07}_{-0.03}$	2.10^{-5}	9.10^{-6}	0
	$B(M1; 2^+_{2,is} \to 2^+_{1,is})$	0.06	0.006	0.004	0
	$B(M1; 1^+_{nc} \to g.s.)$	0.046(18)	0.04	0.009	

and QPM approaches this is a boson forbidden transition promoted only by the scattering term of the $M1$ operator (8). The second 1^+_2 instead has a composite structure and contains a sizeable $[1^+]_{RPA}$ with the dominant spin-flip quasiparticle configuration $(2p_{3/2} \otimes 2p_{1/2})$ responsible for the transition to the ground state. This transition is outside the domain of the algebraic IBM.

The theoretical scheme of the $M1$ transitions is in remarkable good agreement with the experimental picture. We get strong transitions between members of the isovector and isoscalar groups having an equal number of phonons. In particular, the strong $B(M1; 2^+_{1,iv} \to 2^+_{1,is})$ supports the isoscalar and isovector nature of the $2^+_{1,is}$ and $2^+_{1,iv}$ states. We also obtain weak or very weak transitions between states of different isospin but with different number of phonons and, between states belonging to the same isospin group.

The only noticeable discrepancy concerns the two $2^+_{1,iv} \to 2^+_{1,is}$ and $1^+_{1,iv} \to 2^+_{2,is}$ transitions, which are somewhat overestimated. The same transitions are underestimated by the IBM-2 [17], where spin is ignored, and are instead practically reproduced by the shell model calculation of Ref. [20], carried out in a severely truncated space which excluded part of the spin contribution and using a gyromagnetic factor $g^{(s)}_{eff} = 0.57 g^{(s)}_{free}$. This comparative analysis indicates that the spin contribution to these two M1 transitions is sizeable but smaller than what the QPM calculation predicts. It should be not too difficult to find a mechanism for partly suppressing the spin contribution. In fact, the strong $M1$ strengths are quite sensitive to variations of either the orbital or spin transition amplitudes, while the small ones are much less sensitive. Just to illustrate this sensitivity, we have compute the $1^+_{1,iv} \rightarrow 2^+_{2,is}$ and the $1^+_{1,iv} \rightarrow g.s$ M1 transitions for $g_s = 0.6 g_s^{(free)}$ getting respectively $B(M1) = 0.66\mu_N^2$ and $B(M1) = 0.13\mu_N^2$. While the latter value practically coincides with the one obtained for $g^{(s)}_{eff} = 0.7 g^{(s)}_{free}$, the first one is appreciably smaller though still larger than the experimental value. Quenching the gyromagnetic ratio, however, is not an appropriate prescription in view of the general good agreement between QPM and measured $M1$ transitions. This is specially noticeable for the $4^+_{1,iv} \rightarrow 4^+_{1,is}$ $M1$ transition. The corresponding experimental strength, largely overestimated by the shell model calculation [20], is practically reproduced by ours due to the appreciable spin contribution. It seems therefore more appropriate to try to modify the spin-flip content of these states by acting on the parameters of the one-body Hamiltonian. On the other hand, we need sufficiently detailed experimental information on the $M1$ giant resonance as a guide for a fine tuning adjustment of the parameter of the one-body potential. Independently of these marginal details, the generally close agreement between QPM and experimental data indicates that spin plays an important role in determining the structure of these low-lying states and the magnitude of these low-lying $M1$ transitions.

4.3 ^{112}Cd

^{112}Cd was one of the first nuclei explored experimentally for the search of mixed-symmetry states in spherical or weakly deformed nuclei [14, 15]. Its low-lying spectrum is more complex than in ^{94}Mo and ^{136}Ba. The vibrational band includes up to three-phonon levels [15]. There are however intruder 0^+ and 2^+ states falling at the energy of the two-phonon vibrational multiplet.

According to our calculation, the picture of the lowest state is analogous to the one found in the other nuclei under exam. The lowest 2^+ is predominantly one-phonon. The $[2^+_{is}]_{RPA}$ phonon accounts for 81 % of its norm. We reproduce the experimental strength of the $E2$ transition to the ground state by using an effective charge e^*=0.5 for neutrons and e^*=1.5 for protons. The second 2^+ falls at the excitation energy 1.66 MeV and has a two-phonon component $[2^+_{is}\otimes 2^+_{is}]_{RPA}$ which accounts for 53% of its norm. For this reason it is $E2$ strongly coupled to the lowest $2^+_{1,is}$. The third $2^+_{1,iv}$ is an isovector one-phonon state. It falls at an excitation energy 1.931 MeV, close to the energy of the corresponding mixed-symmetry states measured in Ref. [14]. According to our calculation, the $M1$ strength of the transition to the isoscalar $2^+_{1,is}$is $B(M1; 2^+_{1,iv} \rightarrow 2^+_{1,is})$ = 0.25 μ_N^2 which is close to the IBM corresponding strength but five times larger than the experimental value [14].

The lack of experimental information on the strong transitions does not allow a detailed analysis. We like to mention only that the calculation predicts also a 1^+ state with 95 % of the $[2_1^+ \otimes 2_2^+]_{RPA}$ two-phonon component at 2.97 MeV which carries the $M1$ strength $B(M1; 1^+_{1,iv} \to g.s.)$ = 0.17 μ_N^2. We also like to point out that, because of the large number of valence neutrons, this nucleus is on the border of the range of validity of the QPM. The large effective charges needed to reproduce the lowest $E2$ transition strength suggests that we are indeed close to this limit. Its predictions have therefore only a semiquantitative validity.

QPM calculations yielding a similar $M1$ and $E2$ transition scheme have also been carried out for ^{144}Nd [31] and $^{122-130}$Te [32]. Unfortunately the experimental information is rather scarce for those nuclei.

5 Conclusions

According to our findings, the building blocks of our QPM multiphonon low-lying states in nuclei near shell closure are the first and second $[2^+]_{RPA}$ states. The first couples strongly to the ground state through the isoscalar $E2$ operator, the second through the isovector one. This occurs at low energy only for a sufficiently strong proton-proton and neutron-neutron quadrupole pairing interaction.

The resulting low-lying QPM states can be classified into two groups, composed respectively of isoscalar and isovector states. All these states have a single dominant component with a given number of phonons. This feature makes possible a further classification of the states of each group according to the number of phonons and leads to well defined selection rules. We obtain appreciable $E2$ strengths only for transitions connecting states differing by one-phonon. They are very large when the states involved in the $E2$ transition are isoscalar, large for transitions between isovector states, small for transitions between states of different isospin. On the contrary, the $M1$ operator couples strongly only states of different isospin with an equal number of phonons. We should point out that these transitions are promoted by the scattering piece of the $M1$ operator ignored in most multiphonon calculations.

The picture emerging from the present calculation seems to be a general feature of nuclei near shell closure and is consistent with the IBM scheme. Our isoscalar (isovector) states correspond to fully symmetric and mixed-symmetry IBM states.

With respect to the algebraic approach, the QPM provides information on the spin correlations present in these states. We have found, specially in ^{94}Mo, that the spin contribution is comparable to the orbital one in the strongest $M1$ transitions. The overestimation of the strengths of two $M1$ transitions with respect to the experiments suggests that one should change slightly and selectively the parameters entering into the one-body potential in order reduce slightly the amplitudes of the spin-flip components of some selected QPM wave functions. Doing now such a fine tuning adjustment would be premature. We need first experimental information on the detailed structure of the $M1$ resonance.

The spin degree of freedom plays a dominant role in some states, like the second 1^+, which has a composite structure and includes spin-flip configurations with appreciable amplitudes through which it can be coupled to the ground state. Such a state is outside of the multiphonon picture drawn above.

In summary, the generally good agreement with experiments suggests that the microscopic structure of the low-lying states obtained in the present QPM calculation is close to the true one. The level and transition scheme obtained is consistent with the picture provided by the algebraic IBM and appears to be a general feature of nuclei near shell closure.

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