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Description of the Ground and Octupole Bands as Yrast Bands in the Symplectic Extension of the Interacting Vector Boson Model

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1 Introduction

The rotational spectra of some of the even-even nuclei exhibit next to the ground band a negative parity band which consists of states with $I^{\pi} = 1^{-}, 3^{-}, 5^{-}, ...$ traditionally considered as vibrational one. The observation of negative parity states near the ground state, was related to a shape asymmetry under reflection. The reflection symmetry breaking is associated with a static octupole deformation, which may play a role in the negative band structure. The presence of octupole deformations for some isotopes from the light actinide and rare earth region is firmly established [1].

There is a large variety of models that try to describe the behaviour of the low laying states of the deformed nuclei. Particularly successful in this respect are the algebraic models, based on symmetry principles. For the description of the negative parity bands, the introduction of additional octupole degrees of freedom is common feature of the mentioned models. But as another example, the phenomenological Interacting Vector Boson Model (IVBM) [2] does not introduce additional building blocks into it's algebraic structure. This model was an $U(6)$ generalization of the phenomenological model of the broken $SU(3)-$symmetry [3], which provides a good description of the experimentally observed low-lying collective states of the well deformed even-even nuclei. The rotational limit of the IVBM [4], contained sequences of $SU(3)$-multiplets and some of them proved to be convenient for the description of the low-lying negative parity bands [5] .

With the recent advance of the experimental technique, the collective bands are investigated to rather high angular momenta [1]. This motivates a new approach in the framework of the model aiming at the description of the first positive and negative bands, up to very high spins. In this new application we make use of the symplectic extension of the IVBM [6]. This allows the consideration of these bands as yrast bands in respect to the total number N of vector bosons that build the collective states for each L. As a result, the specific properties of these two bands are very well reproduced up to measured high values of the angular momenta L.

2 The Algebraic Basis of the Symplectic Extension of IVBM.

IVBM is based on the introduction of two kinds of vector bosons (called p - and n-bosons), that "built up" the collective excitations in the nuclear system. The creation operators $u_m^+(\alpha)$ of these bosons are assumed to be $SO(3)$-vectors and they transform according to two independent fundamental representations $(1,0)$ of the group $SU(3)$. The annihilation operators $u_m(\alpha) = (u_m^+(\alpha))^\dagger$ transform according to the conjugate representations $(0,1)$. These bosons form a "pseudospin" doublet of the group $U(2)$ and differ in their "pseudospin" projection $\alpha = \pm\frac{1}{2}$. The introduction of this additional degree of freedom leads to the extension of the $SU(3)$ symmetry to $U(6)$ so that the two kind of bosons $u_m^+(\alpha = \pm\frac{1}{2})$ transform according to the fundamental representation $[1]_6$ of the group $U(6)$. The bilinear products of the creation and annihilation operators of the two vector bosons generate the noncompact symplectic group $Sp(12, R)$ [2]:

$$\begin{aligned} F_M^L(\alpha,\beta) &= \sum_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m^+(\beta), \\ G_M^L(\alpha,\beta) &= \sum_{k,m} C_{1k1m}^{LM} u_k(\alpha) u_m(\beta), \\ A_M^L(\alpha,\beta) &= \sum_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m(\beta), \end{aligned} \tag{1}$$

where C_{1k1m}^{LM} are the usual Clebsh-Gordon coefficients and L and M define the transformational properties of (1) under rotations.

We consider $Sp(12, R)$ to be the group of the dynamical symmetry of the model [2]. Hence the most general one- and two-body Hamiltonian can be expressed in terms of its generators . Using commutation relations between $F_M^L(\alpha,\beta)$ and $G_M^L(\alpha,\beta)$, the number of bosons preserving Hamiltonian can be expressed only in terms of operators $A^L_M(\alpha,\beta)$:

$$H = \sum_{\alpha,\beta} h_0(\alpha,\beta) A^0(\alpha,\beta) + \sum_{M,L} \sum_{\alpha\beta\gamma\delta} (-1)^M V^L(\alpha\beta;\gamma\delta) A^L_M(\alpha,\gamma) A^L_{-M}(\beta,\delta), \quad (2)$$

where $h_0(\alpha,\beta)$ and $V^L(\alpha\beta;\gamma\delta)$ are phenomenological constants.

Being a noncompact group, the representations of $Sp(12,R)$ are of infinite dimension, which makes it rather difficult to diagonalize the most general Hamiltonian. The operators $A^L_M(\alpha,\beta)$ generate the maximal compact subgroup of $Sp(12,R)$, namely the group $U(6)$:

$$Sp(12,R) \supset U(6)$$

So the even and odd unitary irreducible representations /UIR/ of $Sp(12,R)$ split into a countless number of symmetric UIR of $U(6)$ of the type $[N,0,0,0,0,0] = [N]_6$, where $N = 0,2,4,...$ for the even one (see Table 1) and $N = 1,3,5,...$ for the odd one [7]. These subspaces are of finite dimension, which simplifies the problem of diagonalization. Therefore the *complete* spectrum of the system can be calculated only trough the diagonalization of the Hamiltonian in the subspaces of *all* the UIR of $U(6)$, belonging to a given UIR of $Sp(12,R)$.

The rotational limit [4] of the model is further defined by the chain:

$$U(6) \supset SU(3) \times U(2) \supset SO(3) \times U(1) \quad (3)$$

$$[N] \quad (\lambda,\mu) \quad (N,T) \;\; K \quad L \quad T_0 \quad (4)$$

where the labels below the subgroups are the quantum numbers (4) corresponding to their irreducible representations. Their values are obtained by means of standard reduction rules and are given in [4]. In this limit the operators of the physical observables are the angular momentum operator

$$L_M = -\sqrt{2} \sum_{M,\alpha} A^1_M(\alpha,\alpha)$$

and the truncated (”Elliott”) quadrupole operator

$$Q_M = \sqrt{6} \sum_{M,\alpha} A^2_M(\alpha,\alpha),$$

which define the algebra of $SU(3)$.

The "pseudospin" and number of bosons operators:

$$T_{+1}=\sqrt{\frac{3}{2}}A^0(p,n); \qquad T_{-1}=-\sqrt{\frac{3}{2}}A^0(n,p);$$
$$T_0=-\sqrt{\frac{3}{2}}[A^0(p,p)-A^0(n,n)]; \qquad N=-\sqrt{3}[A^0(p,p)+A^0(n,n)],$$

define the algebra of $U(2)$.

Since the reduction from $U(6)$ to $SO(3)$ is carried out by the mutually complementary groups $SU(3)$ and $U(2)$, their quantum numbers are related in the following way:

$$T=\frac{\lambda}{2},\; N=2\mu+\lambda \tag{5}$$

Making use of the latter we can write the basis as

$$\mid [N]_6;(\lambda,\mu=\frac{N}{2});K,L,M;T_0\rangle =\mid (N,T);K,L,M;T_0\rangle \tag{6}$$

The ground state of the system is:

$$\begin{aligned}\mid 0\;\rangle =&\mid (0,0);0,0,0;0\rangle\\ &=\mid (N=0,T=0);K=0,L=0,M=0;T_0=0\;\rangle\end{aligned} \tag{7}$$

which is the vacuum state for the $Sp(12,R)$ group.

Then the basis states [7] associated with the even irreducible representation of the $Sp(12,R)$ can be constructed by the application of powers of raising generators $F_M^L(\alpha,\beta)$ of the same group. Each raising operator will increase the number of bosons N by two. The $Sp(12,R)$ classification scheme for the $SU(3)$ boson representations for even values of the number of bosons N is shown on Table 1. Each row (fixed N) of the table corresponds to a given irreducible representation of the $U(6)$. Then the possible values for the pseudospin are $T=\frac{N}{2},\frac{N}{2}-1,...0$ and are given in the column next to the respective value of N. Thus when N and T are fixed, $2T+1$ equivalent representations of the group $SU(3)$ arise. Each of them is labeled by the eigenvalues of the operator T_0 : $-T,-T+1,...,T$, defining the columns of Table 1. The $SU(3)$ representations (λ,μ) are symmetric in respect to the sign of T_0.

Hence, in the framework of the discussed boson representation of the $Sp(12,R)$ algebra all possible irreducible representations of the group $SU(3)$ are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators N,T^2, and T_0. The equivalent use of the (λ,μ) labels facilitates the final reduction to the $SO(3)$ representations, which define the angular momentum L and its projection M. The multiplicity index K appearing in this reduction is related to the projection of L. In the body fixed frame and is used with the parity to label the different bands in the energy spectra of the nuclei. The parity of the states is defined as $\pi = (-1)^T$. This allows us to describe both positive and negative bands.

The Hamiltonian, corresponding to this limit of IVBM is expressed in terms of the first and second order invariant operators of the different subgroups in the chain (3):

$$H = aN + \alpha_6 K_6 + \alpha_3 K_3 + \alpha_1 K_1 + \beta_3 \pi_3, \tag{8}$$

where K_n are the quadratic invariant operators of the $U(n)$ - groups in (3):, π_3 is the $SO(3)$ Casimir operator. As a result of the connections (5) the Casimir operators K_3 with eigenvalue $(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$, is express in terms of the operators N and T:

$$K_3 = 2Q_2 + \frac{3}{4}L^2 = \frac{1}{2}N^2 + N + T^2$$

After some transformations the Hamiltonian (8) takes the following form

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 \pi_3 + \alpha_1 T_0^2, \tag{9}$$

and is obviously diagonal in the basis (6) labeled by the quantum numbers of the subgroups of chosen chain (3). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12, R)$:

$$E((N,T); KLM; T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2 \tag{10}$$

Table 1.

N	(T)\ T_3	...	4	3	2	1	0
0	0						$(0,0)$
2	1 0					$(2,0)$	$(2,0)$ $(0,1)$
4	2 1 0				$(4,0)$	$(4,0)$ $(2,1)$	$(4,0)$ $(2,1)$ $(0,2)$
6	3 2 1 0			$(6,0)$	$(6,0)$ $(4,1)$	$(6,0)$ $(4,1)$ $(2,2)$	$(6,0)$ $(4,1)$ $(2,2)$ $(0,3)$
8	4 3 2 1 0		$(8,0)$	$(8,0)$ $(6,1)$	$(8,0)$ $(6,1)$ $(4,2)$	$(8,0)$ $(6,1)$ $(4,2)$ $(2,3)$	$(8,0)$ $(6,1)$ $(4,2)$ $(2,3)$ $(0,4)$
...		...	...	...	...	...	...

3 Application of IVBM for the Description of the Ground State and Octupole Bands Energies

In the present application of the IVBM we extend the model [5]for the description of the first excited even and odd parity bands in order to reproduce the energies of states with much higher angular momenta. We apply our model on the even- even deformed nuclei, which exhibit next to the ground band, a low lying negative parity band traditionally considered as octupole band [1]. We first identify these experimentally observed bands with the sequences of basis states for the even representation of $Sp(12, R)$ given on Table 1. We choose the $SU(3)$ - multiplet $(0, \mu = \frac{N}{2})$ for description of the ground band $(K^{\pi} = 0^{+})$, whereas for the octupole one $(K^{\pi} = 0^{-})$ - the $SU(3)$ - multiplet $(2, \mu = \frac{N}{2} - 1)$. In terms of (N, T) this choice corresponds to $(N = 2\mu, T = 0)$ for the positive and $(N = 2\mu + 2, T = 1)$ - the negative parity band respectively.

Our new approach is based on the fact that the energies (10) are increasing with the increase of N. This allows the consideration of these bands as yrast bands, in the sense, that we take into account the states with a given L, which minimize the energy values with respect to N. Hence their minimal values are obtained at the values of the angular momentum $L = N/2$ for the ground band and $L = N/2 - 1$ for the octupole band respectively. So for the description of the ground band our choice corresponds to the sequence of states with different number of bosons $N = 0, 4, 8, ...$. and pseudospin $T = 0$ in the column $T_0 = 0$ of Table 1. In analogy for description of the negative parity band we choose the set of states from the same column $T_0 = 0$ of the Table 1, corresponding to quantum numbers $N = 8, 12, ...$ and $T = 1$ respectively.

According to the above chosen correspondence between the basis states and the experimental ones from the ground and octupole bands, the last term in the energy formula (10) becomes zero. The parameters a, b, α_3, and β_3 are phenomenological model parameters, which are evaluated by a fit to the experimental data. Their values obtained for some even-even deformed nuclei belonging to light actinides and rare earth region are given in Table 2. In the second column are given the numbers of the experimental states used in the fitting procedure.

As examples of the quality of the fit the comparison between the theoretical and experimental spectra for the ground and octupole bands of the nuclei $Ra^{224}$and Yb^{168} are presented in the Figure 1.

As a result from the assumption that the lowest positive and negative bands can be considered as yrast ones relations between number of bosons N and the angular momentum L are obtained: $L = \frac{N}{2}, \mu = \frac{N}{2}$ for the ground band and $L = \frac{N}{2} - 1, \mu = \frac{N}{2} - 1$ - for the octupole band respectively. Making use of the

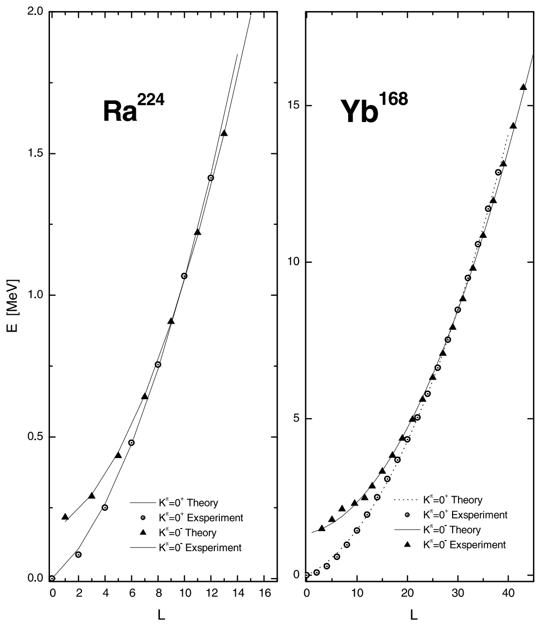

Figure 1. Energy spectra.

latter, the energies are rewritten in the following form:

$$E(L) = \beta L(L+1) + (\gamma + \eta)L + \xi, \tag{11}$$

where the new parameters β, γ, η, and ξ are related to the ones in (10) by

$$\beta = 4b + \beta_3, \quad \gamma = 2a - 4b, \quad \eta = 8b, \quad \xi = 2a + 4b + 2\alpha_3. \tag{12}$$

If we apply (11) in the fitting procedure the values of β and γ are determined from the positive band, and η and ξ from the negative one respectively. In this way the first band is only described trough parameters β and γ, whereas for describing of the second band -we add η and ξ. The values of these parameters determine the behavior of these two bands in respect to each other. In some cases the two bands are almost parallel. The distance between them is dependent on the parameter ξ. When they are very close they interact trough the $L-$dependent interaction with a strength $\gamma + \eta$.

Analyzing (11), we can see that eigenstates of the first positive and negative bands consists of rotational and vibrational modes, which are represented by the classical terms $L(L+1)$ and L respectively. For the considered nuclei the parameter η (12)is negative (see Table 2) , which means that the negative

Table 2.

Nucleus	n_s	a	b	α_3	β_3
Ra^{224}	13	0.0119	-0.0022	0.0789	0.0155
Ra^{226}	18	0.0269	-0.0005	0.0226	0.0060
Th^{222}	26	0.0558	0.0000	-0.0557	0.0030
Th^{224}	18	0.0242	-0.0011	0.0362	0.0100
Th^{226}	20	0.0194	-0.0009	0.0522	0.0094
Th^{228}	18	0.0092	-0.0020	0.1470	0.0138
Th^{232}	29	0.0155	-0.0021	0.3244	0.0128
U^{234}	19	0.0124	-0.0010	0.3608	0.0085
U^{236}	25	0.0154	-0.0010	0.2846	0.0086
U^{238}	27	0.0142	-0.0016	0.2851	0.0110
Yb^{168}	41	0.0235	-0.0056	0.6512	0.0295
Sm^{152}	15	0.0194	-0.0045	0.4290	0.0274

parity band is less vibrational than the positive one. Up to very high angular momentum both the positive and negative bands can not be considered as pure rotational ones.

In this mass region in the collective rotational spectra of deformed even-even nuclei some fine structure effects as back-banding and staggering behavior are observed . Odd-even staggering patterns between ground and octupole bands have been investigated very recently [8]. A more detailed test of the model is the application on the energies $E(L)$ of the staggering function defined as:

$$StagE(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) \\ + \Delta E(L+2) + \Delta E(L-2), \quad (13)$$

where $\Delta E(L) = E(L+1) - E(L)$. The function (13) is a finite difference of fourth order in respect to $\Delta E(L)$ or of fifth order in respect to energy $E(L)$ and is characteristic for the deviation of the bands energies from from the rigid rotor behavior. In our case this deviation is present in the two bands, trough the L dependent terms in (11). In Figure 2 we illustrate both the calculated and experimental staggering patterns. We obtain a good qualitative and quantitative agreement. We also reproduce the "beat"patterns of the staggering between the two bands.

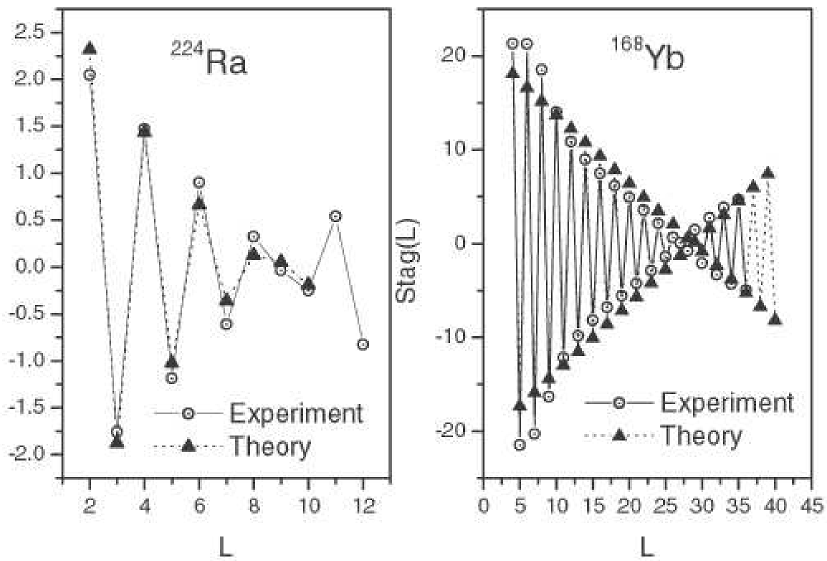

Figure 2. Staggering patterns.

4 Conclusions

We have applied the IVBM for the description of the ground and octupole bands in some even-even rare earth and actinide nuclei up to very high spins. The successful reproduction of the experimental energies and of their odd-even staggering was achieved as a result of their consideration as yrast energies in respect to the number of phonon excitation N that build the collective states. The introduction of this notion was possible, as we extended the IVBM to its symplectic dynamical symmetry , which allows the change of the number of bosons that are the building blocks of the model Hamiltonian. Through the algebraic properties of the Hamiltonian the physical meaning of the introduced phenomenological parameters is clarified. In the rotational limit of the model in addition to the rotational character of the considered bands a vibrational mode is appearing, which introduces also some interaction between the two bands. The correct reproduction of the experimental staggering patterns is obtained as a result of the introduced interaction between the positive and negative parity bands. The obtained physically meaningful results are also simple and easy for application and permit the application of the model to larger class of nuclei The rich multiplet substructure of the symplectic dynamical symmetry permits the consideration of many other collective effects.

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