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Transition from Nuclear to Quark Matter

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Abstract.

We describe the complex phase diagram of Quantum Chromodynamics (QCD) which, apart from the high-temperature unconfined quark-gluon plasma (QGP) and the ordinary low-temperature confined hadronic gas, contains different possible colour superconducting phases at high pressure and not too high temperature which may be more stable than nuclear matter.

Using asymptotic freedom which is valid both at high temperature and at large pressure, it is possible to see that the bubbles of QGP which are remnants of the primordial confinement transition when sufficiently compressed and quark matter inside neutron stars have relevant strangeness and may become colour superconducting being possible sources of strangelets.

1 QGP – Hadronic Phase Transition

At a temperature $T \sim 100$ MeV, intuitive arguments indicate that the almost free quarks and antiquarks confine into baryons and mesons, accompanied by the breaking of the approximate $SU(2)$ chiral symmetry of flavours u and d.

The phase diagram must be drawn in the plane $T-$ baryonic chemical potential μ.

In the deconfined quark gas if $\mu \ll T$ the entropy density $s \sim T^3$ and the baryonic number density $n_B \simeq n_u - n_{\overline{u}} \sim T^2\mu$, so that the value $\frac{n_B}{s} \sim 10^{-10}$ needed by primordial nucleosynthesis requires $\mu \sim 10^{-2}$ eV at the confinement transition.

Looking at the chiral symmetry breaking, the hadronic phase is characterized by the order parameter

$$\langle \overline{q}^i_{L\alpha} q^{\alpha j}_R \rangle = \sigma\delta^{ij} + \pi.\tau^{ij} \ , \tag{1}$$

where $\alpha = 1, 2, 3$ are colour indices and $i = 1, 2$ flavour ones.

The chiral symmetry breaking corresponds [1] to $O(4) \to O(3)$ where $\varphi = (\sigma, \pi)$ chooses the σ direction and π become Goldstone bosons.

Considering $m_{u,d} = 0$, the chiral transition is of second order for values of μ from 0 to a value corresponding to a tricritical point beyond which it becomes of first order. For physical values $m_{u,d} \neq 0$ the second order line disappears leading to a cross-over between the two phases and the tricritical point transforms into an end-point for the first order line.

But it has been shown [2] that the confinement transition is more than the chiral-breaking one and that the appropriate order parameter is the condensate of magnetic charges whose numerical study indicates a first order line for all values of $m_{u,d}$.

If one includes also the quark s as light the chiral symmetry is $SU(3)$, the order parameter analogous to Eq. (1) gives an octet of Goldstone bosons and the chiral transition is of first order even for an equal mass of the 3 quarks different from zero.

2 Colour Superconductor – Nuclear Matter – Hadron Transition

In the other extrteme of the phase diagram, $i.e.$ very high pressure or μ and relatively low temperature, one expects quark matter but attractive 1-gluon exchange between two quarks if they are in the coupled antisymmetric $\overline{3}$ of colour may produce Cooper pairs.

For $\mu \gg m_s$ the 3 flavours are paired being favoured the antisymmetric total spin zero state so that the condensate will be

$$\langle q^{\alpha}_{Li}\left(\mathbf{p}\right) q^{\beta}_{Lj}\left(-\mathbf{p}\right)\rangle = \Delta\left(\mathbf{p}^2\right) \varepsilon^{\alpha\beta A} \varepsilon_{ijA} \ , \tag{2}$$

where the chirality of quarks is the same because the momenta are equal and opposite, the flavour index $i = 1, 2, 3$ and the sum over A gives a link between colour and flavour. A similar condensate arises for R chirality.

The symmetry of QCD, with the approximation $m_q = 0$,

$$G = SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$$

is broken by the condensate of Eq.(2) to the so-called colour-flavour-locked (CFL) symmetry

$$G_{CFL} = SU(3)_{c+L+R} \times Z_2 \ .$$

The breaking of the local $SU(3)_c$ gives 8 massive gluons, analogously to the massive photons of electric superconductivity.

The breaking of global $SU(3)_L \times SU(3)_R$ produces 8 Goldstone bosons interpreted as the scalar meson octet.

The breaking of baryonic $U(1)_B$ makes the medium superfluid because the energy of excitations will be larger than the gap Δ. This is the Landau condition [3] for superfluidity because the excitations that decrease the energy of a fluid with velocity υ must have a rest frame dispersion relation $\varepsilon\left(\mathbf{p}^2\right)$ such that $\upsilon > \varepsilon/\left|\mathbf{p}\right|$ which is not satisfied for velocities below $\min\left(\varepsilon/\left|\mathbf{p}\right|\right) \neq 0$ in the gapped case.

The electromagnetic $U(1)_{em}$ is obviously broken by the CFL condensate but a modified charge is conserved

$$\widetilde{Q} = Q + \frac{1}{\sqrt{3}}T_8 \ , \tag{3}$$

with the ordinary charge $Q = diag\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$ in u, d, s and the broken colour generator $T_8 = \frac{1}{\sqrt{3}} diag\left(-2, 1, 1\right)$ in r, g, b.

It is seen that $\widetilde{Q} = 0$ for all Cooper pairs so that the $\widetilde{Q}-$photon will be massless. It corresponds to the field potential

$$A_\mu^{\widetilde{Q}} = \frac{gA_\mu + \frac{e}{\sqrt{3}}G_\mu^8}{\sqrt{g^2 + \frac{e^2}{3}}} \ , \tag{4}$$

where the electromagnetic potential A_μ and the colour one G_μ^8 couple to fermions through the covariant derivative $\partial_\mu + ieA_\mu + igT_8G_\mu^8$. Therefore, electron couples with $A_\mu^{\widetilde{Q}}$ with charge

$$\widetilde{e} = \frac{eg}{\sqrt{g^2 + \frac{e^2}{3}}} \ , \tag{5}$$

and quarks of i and α with

$$\widetilde{Q} = \frac{g\left(\frac{2}{3}e, -\frac{1}{3}e, -\frac{1}{3}e\right) + e\left(-\frac{2}{3}g, \frac{1}{3}g, \frac{1}{3}g\right)}{\sqrt{g^2 + \frac{e^2}{3}}} \ , \tag{6}$$

so that the possible values are $\widetilde{Q} = 0, \pm\widetilde{e}$.

There is a correspondence between the excitations in hypernuclear matter and CFL one:

baryons $<->$ quarks
scalar mesons $(\pi...)$ $<->$ scalar mesons (Goldstone bosons)
vector mesons $(\varrho...)$ $<->$ massive gluons

so that in principle there is no transition between hadronic and CFL phases.

Since hypernuclear matter is strongly coupled $g \gg e$ whereas CFL is weakly coupled due to asymptotic freedom, increasing $\mu \left(\widetilde{e}, A_\mu^{\widetilde{Q}}\right)$ start from (e, A_μ).

The above scheme, which is valid also for non-vanishing masses $m_s = m_{u,d}$, changes in the realistic case $m_s \gg m_{u,d} \neq 0$.

On one hand, there may be a transition from ordinary nuclear matter of p, n to hypernuclear matter.

Moreover there is a rich list of possible colour superconducting phases at intermediate μ before reaching the CFL one.

Considering pairs of only light u, d quarks, the two-superconducting $(2SC)$ phase would be formed characterized by the condensate

$$\left\langle q_{Li}^{\alpha}\left(\mathbf{p}\right) q_{Lj}^{\beta}\left(-\mathbf{p}\right)\right\rangle = \Delta\left(\mathbf{p}^2\right)\varepsilon^{\alpha\beta 3}\varepsilon_{ij3}\ . \tag{7}$$

Here the colour symmetry would be partially broken $SU(3)_c \rightarrow SU(2)_c$ producing 5 massive gluons, whereas the approximate symmetry $SU(2)_L \times SU(2)_R$ would remain intact. Baryonic $U(1)_B$ is broken but $B + \dfrac{1}{6} diag$ $(1, 1, -2)$ is not, so that $2SC$ phase is not superfluid.

Starting from ordinary nuclear matter and increasing μ there might be a transition to $2SC$ to restore chiral symmetry and then another one to CFL to break it again. But it is possible that $2SC$ is never stable.

Since for nondegenerate flavours μ are different, at not extremely high pressure it may be convenient to pair quarks of different flavours close to their respective Fermi surface and therefore with non-vanishing total momentum. This condensate breaks translational and rotational invariance giving crystals of colour [4].

Finally, it would be possible that pairs of quarks of the same flavour are formed, paying therefore the price of having total spin 1 in order to keep anti-symmetry of the state due to attractive $\overline{3}$ colour [5].

3 Strange Quark Matter

If confinement transition is of first order, it was carried out by bubbles of hadronic phase. When it was almost completed, due to universe expansion there were remnants of the high T phase subsequently compressed by the negative pressure corresponding to the energy density $\sim \Lambda_{QCD}^4$ of false vacuum inside them, with $\Lambda_{QCD} = 200$ MeV.

With the zero-order QCD approximation of asymptotic freedom, the number and energy densities and pressure of one-flavour quarks with 2 spin projections and 3 colours inside these remnant bubbles will be

$$\frac{N}{V} = \frac{3}{\pi^2}\int_m^\infty d\varepsilon\varepsilon\sqrt{\varepsilon^2 - m^2}\frac{1}{\exp\left[\frac{1}{T}(\varepsilon-\mu)\right]+1} \tag{8a}$$

$$\frac{E}{V} = \frac{3}{\pi^2}\int_m^\infty d\varepsilon\varepsilon^2\sqrt{\varepsilon^2 - m^2}\frac{1}{\exp\left[\frac{1}{T}(\varepsilon-\mu)\right]+1} \tag{8b}$$

$$p = \frac{1}{\pi^2}\int_m^\infty d\varepsilon \quad \left(\varepsilon^2 - m^2\right)^{\frac{3}{2}} \quad \frac{1}{\exp\left[\frac{1}{T}(\varepsilon-\mu)\right]+1}. \tag{8c}$$

The initial minimum radius of these bubbles [6] coming from equality of velocities of coalescence of low T phase and universe expansion is $R \sim 1$ cm.

Assuming no evaporation of hadrons and charge neutrality of the bubble without electrons and surface effects, for each radius the baryonic density n_B determines μ_u and a relation between μ_d and μ_s. Minimization of energy gives all chemical potentials and the pressure due to quarks and antiquarks of the 3 flavours can be calculated [7].

The evolution of the radius of the bubble, considering positive velocity υ during contraction, will be given by

$$F = \frac{d}{dt}\left(\frac{M}{\sqrt{1-\upsilon^2}}\upsilon\right) + \frac{dP_\nu}{dt}\ , \tag{9}$$

where the compression force F is due to the difference of false vacuum pressure and that of quarks and antiquarks, the rest mass M comes from 3 flavours quarks and antiquarks contributions Eq. (8,b) together with false vacuum energy, and the neutrino momentum P_ν seen from lab frame corresponds to $q\overline{q}$ annihilation which allows maintaining the thermodynamical distributions.

When $R \sim 10^{-3}$ cm the increase of chemical potential makes the internal pressure surmount that due to false vacuum so that the velocity of contraction decreases and stops for $R \sim 10^{-4}$ cm when $\mu \sim 10T$. Since the time for the bubble contraction is $\sim 10^{-10}$ sec, much smaller than the age of universe at confinement $\sim 10^{-5}$ sec, it is consistent to take for the process constant $T = 150$ MeV.

To see whether the compressed bubble may become colour superconducting one has to evaluate the critical temperature for the transition to ordinary QGP which, if it is of second order, is $T_c \simeq 0.57\Delta_0$ in terms of the gap at zero temperature. If the transition of loss of colour superconductivity is of first order, the estimation [8] is that the critical temperature is 10% higher.

The calculation of Δ_0 is not simple because in the Nambu-Jona Lasinio approximation of 4 fermion interaction with coupling $G \simeq 1/GeV^2$

$$\Delta_0 \simeq \mu \exp\left[-\frac{\pi^2}{2G\mu^2}\right] , \tag{10}$$

which would allow to reach $T_c \gtrsim 150$ MeV for the very high values of μ corresponding to the final stage of the bubble.

But the evaluation with perturbative QCD with gluon-quark coupling g gives

$$\Delta_0 = b\frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) , \tag{11}$$

where the coefficient is primarily $b = 512\pi^4 \left(\frac{2}{N_C}\right)^{5/2}$, $N_f = 3$ is the number of dynamical flavours for large values of μ and the coupling runs as $\alpha_s = \frac{g^2}{4\pi} \simeq \frac{0.7}{\ln\left(\frac{\mu}{\Lambda_{QCD}}\right)}$. Additional corrections to b may decrease it by a factor of 5 or increase it by 20.

For $\mu \sim 1$ GeV Eq. (11) gives a value of Δ_0 similar to that of Eq. (10). What is clear from Eq. (11) is that due to asymptotic freedom of QCD, Δ_0 will finally decrease for extremely large μ. Therefore more trustful calculations should be done to determine whether primordial bubbles could have become colour superconducting.

Another possible source of the colour superconducting phase is the inner part of neutron stars with low T and high pressure. In these conditions Eq. (8) may be approximated by

$$\frac{N}{V}z \simeq \frac{1}{\pi^2}\left(\mu^2 - m^2\right)^{3/2} \tag{12a}$$

$$p \simeq \frac{m^4}{4\pi^2}\left[\left(\frac{\mu^2}{m^2} - 1\right)^{1/2} \frac{\mu}{m}\left(\frac{\mu^2}{m^2} - \frac{5}{2}\right) + \frac{3}{2}\sinh^{-1}\left(\frac{\mu^2}{m^2} - 1\right)^{1/2}\right] . \tag{12b}$$

We compare ordinary quark matter from neutron deconfinement with that of equal number of 3 flavours.

In the former case it must be $\frac{N_d}{V} = 2\frac{N_u}{V}$ so that for $m_{u,d} \simeq 0$ from Eq. (12a)

$$\mu_d = 2^{1/3}\mu_u , \tag{13}$$

and the energy of 3 quarks at Fermi surface will be $\mu_u\left(1 + 2 \cdot 2^{1/3}\right)$.

In the latter case $\widetilde{\mu}$ will be the same for u and d whereas to have $\dfrac{N_s}{V} = \dfrac{N_d}{V}$ it must be

$$\mu_s = \sqrt{\widetilde{\mu}^2 + m_s^2}\,, \tag{14}$$

and the energy of 3 quarks will be $\mu_s + 2\widetilde{\mu}$.

Using Eq. (12b) to relate $\widetilde{\mu}$ to μ_u with the condition of equal pressure one sees that, with $m_s = 150$ MeV, for $\widetilde{\mu} > 200$ MeV the energy of strnge quark matter is smaller than the ordinary one. This shows the tendency to have strange quark matter at high pressure and perhaps indicates that CFL may be more stable than $2SC$ also at not extreme values of μ.

But both for what occurs inside neutron stars as in primordial bubbles, to see if matter may become colour superconducting one has to inspect the free energy for varying number of particles

$$\frac{\Omega}{V} = \frac{E}{V} - T\frac{S}{V} - \mu\frac{N}{V} = -p\,. \tag{15}$$

For the paired case at $T = 0$ one has to sum the expression for the unpaired case but with the restriction of equal Fermi momentum for all flavours, in order to have the same number of quarks, plus a contribution coming from the gap [9], $i.e.$ for uds

$$\left.\frac{\Omega}{V}\right|_{paired}^{(T=0)} = \left.\frac{\Omega}{V}\right|_{unpair}^{(T=0)} \left(p_F^u = p_F^d = p_F^s\right) - \frac{3}{\pi^2}\Delta_0^2\mu^2\,. \tag{16}$$

The condition of equal Fermi momenta increases Ω_{unpair} compared to the same expression for free quarks, but the difference is compensated by the last term of Eq. (16) for $\mu > \dfrac{m_s^2}{4\Delta_0} \sim 500$ MeV according to the above expressions for Δ_0. This pressure should be reached inside neutron stars even though there are controversies about that [10].

For $T > 0$, necessary for primordial bubbles, Ω_{unpair} must be taken with chemical potentials such that the densities of quarks of different flavours are equal [11] together with a gap contribution which vanishes for a continuous transition [3] at $T = T_c$, *i.e.*

$$\left.\frac{\Omega}{V}\right|_{paired} = \left.\frac{\Omega}{V}\right|_{unpair}(n_u = n_d = n_s) - \frac{18\mu^2 T_c^2}{7\varsigma(3)}\left(1 - \frac{T}{T_c}\right)^2\,. \tag{17}$$

Also here the first term of RHS is larger than the free quark expression minimizing energy but the difference for primordial bubbles is very small because $\mu \gg m_s$ so that for T slightly below T_c the CFL will be favoured.

4 Astrophysical Conclusions

It seems theoretically evident that for $\mu \gtrsim 500$ MeV some form of colour superconducting phase should be achieved with a transition to QGP for temperatures above $T_c \sim 50$ MeV or even higher for μ well above 1 GeV.

Heavy ions collisions should not reach too high μ so that this is a region whose phenomenological study requires astrophysical observations.

Possible origin of CFL phase may be sufficiently high pressure inside neutron stars or cosmological first-order confinement transition leading to extremely compressed bubbles of quark matter.

Collision of neutron stars or cooling of bubbles may give rise at present to lumps of strange matter which if keep atomic number $A \gtrsim 1000$ would have [12] a mass per baryon less than 940 MeV and be stable. These are the strangelets that could reach the high altitude observatories like that of Chacaltaya.

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