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Relativistic Mean Field Theory with the Pion for Finite Nuclei

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Abstract.

We study the possible occurrence of finite pion mean field in finite nuclei in the relativistic mean field (RMF) theory. We calculate explicitly various N=Z closed-shell nuclei with finite pion mean field in the RMF theory with the standard parameter set and the pion-nucleon coupling in free space. The finite pion mean field is introduced by breaking the parity symmetry of intrinsic single-particle states. We demonstrate the actual occurrence and the property of the finite pion mean field.

1 Introduction

We are developing a mean field framework for nuclear structure which treats finite pion mean field on the same footing as other meson fields [1]. It is well-known that the pion is one of the most important mesons in nuclear physics. The pion mainly produces the tenor force between two nucleons, which plays significant roles in nuclear structure. The tensor force gives a large amount of attractive energy especially in light nuclei and, even in nuclear matter, it is responsible for about a half of attractive energy [2]. Shell model calculations show that one-third to a half of single-particle spin-orbit splittings are explained by the role of the tensor force [3]. It means that the pion is important for the shell structure of nuclei. Furthermore, in addition to conventional calculations, the recent more sophisticated calculations show that the interaction mediated by the pion gives a large part of attractive energy in light nuclei [4].

Although we know well that the pion is the essential ingredient of nuclear structure as mentioned above, many nuclear models, especially mean field models, do not treat the pion or the tenor force explicitly. The effect of the pion or tensor force is usually incorporated by renormalizing the other parts of the nuclear interactions as the central and the LS forces there. However, considering the important role of the pion, we believe that it is important to construct a mean field framework which includes the pion explicitly and to investigate the effect of the pion on a nuclear mean field.

The pion is a pseudoscalar and isovector meson. Because of these characters, the pion field becomes zero even if we include the pion term, when we assume the parity symmetry of a nuclear mean field as usually done in many mean field models. This is the reason why we do not treat the pion in the RMF theory. However, we can incorporate the effect of the pion into a nuclear mean field in a simple mean field framework by breaking the parity symmetry of a nuclear mean field. In a parity-mixed mean field, single-particle states are also parity-mixed states and a total wave function made from these single-particle states is not a parity-good state. However, it does not mean that the parity symmetry of a nucleus is broken but that the parity symmetry of the intrinsic state of the nucleus is broken. To obtain a parity-good total wave function we only need to perform the parity projection on the parity-mixed wave function. As a result, we can obtain a wave function of a nucleus with a parity-mixed intrinsic state, in which the effect of the pion is included in the nuclear mean field. We should mention here that this idea of parity-mixed single-particle states is not quite new. It was investigated by Bleuler and his collaborators in 1960's [5].

We formulate a mean field model based on the idea above and show in the recent study [1] that we can incorporate the finite pion mean field into the nuclear mean field by introducing such parity-mixed single-particle states. In this paper, we briefly explain our model and its application to N=Z magic nuclei. You can find the detailed discussion in our recent paper [1].

2 Relativistic Mean Field Theory with the Pion

We start with writing the relativistic meson-nucleon Lagrangian density, which naturally includes the pion term, π;

$$\begin{aligned}\mathcal{L} =&\bar{\psi}\big[i\gamma_\mu\partial^\mu - M - g_\pi\gamma_5\gamma_\mu\tau^a\partial^\mu\pi^a - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau^a\rho^{a\mu} \\ &- e\gamma_\mu\frac{(1-\tau_3)}{2}A^\mu\big]\psi + \frac{1}{2}\,\partial_\mu\pi^a\partial^\mu\pi^a - \frac{1}{2}m_\pi^2\pi^a\pi^a + \frac{1}{2}\,\partial_\mu\sigma\partial^\mu\sigma \\ &- \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &+ \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ ,\end{aligned} \tag{1}$$

where the field tensors H, G and F for the vector fields are defined through,

$$H_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \tag{2a}$$

$$G^a_{\mu\nu} = \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu - 2g_\rho \, \epsilon^{abc} \rho^b_\mu \rho^c_\nu, \tag{2b}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2c}$$

The pion term couples with the nucleon through the pseudovector coupling. We take here all the terms used in Ref. 6. Here, σ denotes the scalar meson, ω the vector meson and ρ the isovector-vector meson. The photon is denoted by A. It should be noted that the coupling of the spin and orbital angular momenta automatically appears in the relativistic formalism. From this Lagrangian with the Euler-Lagrange equation we obtain the Dirac equation for the nucleon wave function, which has the fields of the mesons and the photon as potential terms, and the Klein-Gordon equations for the fields of the mesons and the photon, which have nucleon densities as source terms. We solve them self-consistently.

For the single particle state we assume the form,

$$\psi_{njm} = \sum_\kappa \begin{pmatrix} iG_{nj\kappa}\mathcal{Y}_{j\kappa m} \\ F_{nj\kappa}\mathcal{Y}_{j\bar{\kappa} m} \end{pmatrix} = \begin{pmatrix} iG_{nj\kappa}\mathcal{Y}_{j\kappa m} + iG_{nj\bar{\kappa}}\mathcal{Y}_{j\bar{\kappa} m} \\ F_{nj\kappa}\mathcal{Y}_{j\bar{\kappa} m} + F_{nj\bar{\kappa}}\mathcal{Y}_{j\kappa m} \end{pmatrix} . \tag{3}$$

Here, $\mathcal{Y}_{j\kappa m}$ is the eigenfunction of the total angular momentum $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$ and $\mathcal{Y}_{j\bar{\kappa} m} = \boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}} \mathcal{Y}_{j\kappa m}$. Because $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}}$ is a 0^- operator, $\mathcal{Y}_{j\kappa m}$ and $\mathcal{Y}_{j\bar{\kappa} m}$ have the same total angular momentum j but different parities. We assume the spherical symmetry (jm are good quantum numbers) for the intrinsic state. G and F are the radial parts of the single particle wave function. The summation over κ means the parity mixing, where κ is $\kappa = -(j + 1/2)$ for $j = l + 1/2$ and $\kappa = j + 1/2$ for $j = l - 1/2$. This mixing of parities makes the pion field finite and, reversely, the finite pion field breaks the parity symmetry of a nuclear mean field. It is only the new point of our model compared to the conventional RMF theory. The single particle wave function (3) can be considered as a intrinsic single particle wave function in a parity-mixed intrinsic state. The physical state, which has a good parity, can be obtained by performing the parity projection on the parity-mixed intrinsic state.

3 Result

We show here the numerical results. We take the TM1 parameter set of Ref. 6 for all the parameters except for the pion-nucleon coupling. As for this we take the value of Bonn A potential [7], which corresponds to taking $g_\pi = f_\pi/m_\pi$ and $f_\pi \sim 1$. We stress here that we use all the terms given in the Lagrangian (1). This means that the saturation property is guaranteed and the bulk part of the nucleus tends to have the saturation density. Since we are especially interested in the occurrence of finite pion mean field and want to see its effect under the simplest

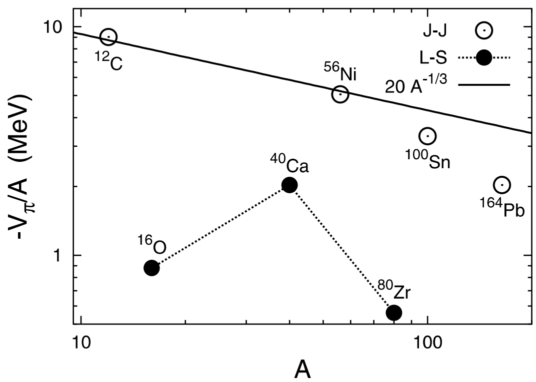

Figure 1. The pion energy per nucleon as a function of the mass number in the log-log plot. There are two groups; one is for the jj-closed shell nuclei denoted by open circle and the other is for the LS-closed shell nuclei denoted by closed circle. The pion energy per nucleon for the jj-closed shell nuclei decreases monotonically and follows more steeply than $A^{-1/3}$, which is shown by solid line.

condition, we neglect the Coulomb term. We calculate the N=Z closed-shell nuclei as ^{12}C, ^{16}O, ^{40}Ca, ^{56}Ni, ^{80}Zr, ^{100}Sn, and ^{164}Pb. The result shown here is the one without parity projection. The parity projection does not change the result so much [1].

We show the mass number dependence of the pion energy per nucleon in Figure 1. Here, we want to stress that the pion field becomes finite for all the nuclei calculated. It is not a trivial thing because kinetic energy becomes larger to make the pion field finite. To make the pion field finite, two parity states must be mixed in a single-particle state. It means that the single-particle state has a component of the major shell next to the original unperturbative state and this component has large kinetic energy. The energy gained by the pion-nucleon coupling is larger than the loss induced by the parity mixing and, as a result, the total energy gain is obtained as compared to the case of the no pion term.

We mention here that the kinetic energy and the sigma and omega energies per particle are almost constant of the mass number and hence they are volume-like [1]. We see, on the other hand, a peculiar behavior in the pion energy. The magnitudes of the pion energy are clearly separated into two groups. One group is large and the common feature is jj-closed shell nuclei; the magic number nuclei due to a larger spin-orbit partner (j-upper) being filled. The other group is small and they are LS-closed shell nuclei. The pion energy per nucleon for jj-closed shell nuclei decreases monotonically with the mass number. The rate of the decrease follow more strongly than $A^{-1/3}$. This means that the pion mean field energy behaves in proportion with the nuclear surface or even stronger than that. Hence, we use the word of surface pion condensation. Concerning the

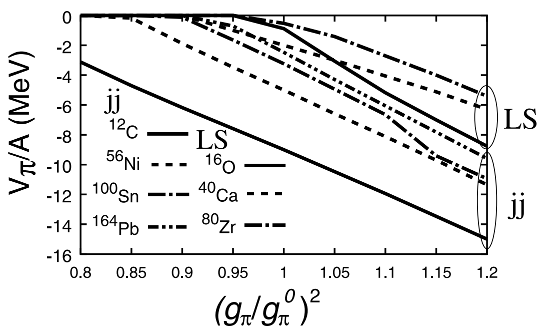

Figure 2. The pion energy per nucleon as a function of the pion-nucleon coupling constant square. (Here, g_π^0 corresponds to the value of the Bonn A potential.) In this systematic study, we have fixed the set of the filled single-particle states with the same spins as in the closed-shell configurations.

separation into two groups for the pion energy; LS-closed and jj-closed shell cases, we mention the possible reason in the paper [1].

We show the pion energy per nucleon for various nuclei as a function of the pion nucleon coupling constant (g_π) in Figure 2. We see the critical value of $g_\pi = g_\pi^{\rm cr}$ at which there arises the finite pion mean field. The critical values for various nuclei are distributed in the region of

$$0.92 g_\pi^0 \leq g_\pi^{\rm cr} \leq 0.97 g_\pi^0 \tag{4}$$

except for ^{12}C. This fact indicates that the symmetry breaking mean field is fragile in the sense that various effects not taken into account in the present study would influence the realization of the finite pion mean field in finite nuclei. We shall list up these effects in the summary.

4 Summary and Future Work

We have discussed the possible occurrence of the finite pion mean field in finite nuclei by introducing the pion field in the relativistic mean field (RMF) theory. We have extended the RMF theory by introducing the parity-mixed single-particle basis to accommodate the finite pion mean field. We have taken the TM1 parameter set in the RMF theory and introduced the pion field in the pseudovector coupling with the nucleon. With the use of the pion-nucleon coupling constant in free space, we have made calculations for N=Z closed-shell nuclei and demonstrated the actual occurrence of finite pion mean field. We have shown that the potential energy associated with the pion behaves as proportional to or even stronger than the nuclear surface. Hence, we name the on-set of the finite pion mean field as surface pion condensation. The large difference of the pion energies in jj-closed shell and LS-closed shell nuclei has been found and may be connected with the mechanism of a part of spin-orbit splitting as due to the tensor force discussed long time ago by Takagi *et al.* and Terasawa [3].

We would like to stress here that we are at the initial stage of our investigation on the role of the pion in finite nuclei. We have to do various studies in order to establish the mean field theory with the inclusion of the pion, pursued in this paper for medium and heavy nuclei. We have to definitely introduce the rho meson tensor coupling term, which acts against the finite pion mean field. We should include also the delta isobar pion coupling terms, which favor finite pion mean field. We have to study carefully the effect of the short range repulsions, so called g' term in the non-relativistic framework. We would like also to work out the exchange terms; the Fock terms in the relativistic many-body theory. One more important thing is that we need to expand a model space further by mixing proton and neutron components in a single-particle state to exploit the isovector character of the pion. It should enhance the finite pion mean field. We then have to work out the parameter search for the coupling constants in the Lagrangian. We shall end up with the reduction of the sigma and omega coupling constants, which enhances the effect of the pion mean field. We should further perform the study of the parity projection in the framework of the variation after projection (VAP). There are many studies to be done for the establishment of the occurrence of surface pion condensation. We are at the gate of exploring both theoretically and experimentally the phenomena caused by the essential meson, the pion, in nuclear physics.

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