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Ground-γ Band Mixing and Odd-Even Staggering beyond the Well Deformed Nuclei

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Abstract.

We propose an extended phenomenological approach for description and analysis of the ground-γ band mixing interaction in collective spectra beyond the regions of well deformed nuclei. It is based on the band coupling mechanism developed in the Vector Boson Model with SU(3) dynamical symmetry. In this framework we describe the staggering effects observed in the γ bands of Mo, Ru and Pd nuclei. The systematic behaviour of the staggering magnitude together with some general characteristics of the spectrum such as the mutual disposition of the bands, and the collective band structures is explained as the manifestation of respective changes in dynamical symmetry of the system. We suggest that the used approach could provide a relevant test of newly obtained experimental data and their interpretation in terms of the exact symmetry limits as SU(3), O(6) and U(5) as well as in terms of some phase transition symmetries as E(5) and X(5).

1 Introduction

The low lying excited states of even-even nuclei are usually described in a geometrical approach as the levels corresponding to harmonic vibrations, rotations of deformed shapes or unstable shape rotations [1]. These three geometrical models have been associated with the symmetry limits of the Interacting Boson Model (IBM) [2], in which the low-lying excited states are classified according to the irreducible representations of three chains of subgroups of the group U(6), labelled as U(5), SU(3) and O(6). These symmetries are considered as the stable limits of collectivity in nuclear structure. However, most of nuclei have a transitional behaviour taking place in regions between the above mentioned symmetries. Recently it has been suggested that two additional symmetries, E(5) [3] and X(5) [4], might correspond to U(5)-O(6) and U(5)-SU(3) phase transition, respectively. It has been shown that some nuclei as ^{134}Ba [5] and ^{152}Sm [6] may undergo such phase transitions in the shape.

The odd-even staggering effect observed in the γ bands is among the most sensitive phenomena carrying information about the symmetry changes mentioned above. It is well pronounced in nuclear regions characterized by the U(5) and O(6) limits of IBM and relatively weaker in nuclei near the SU(3) limit. In the latter case the staggering effect can be reproduced through the γ-β band mixing interaction. In the U(5) and O(6) limits the gsb and γ bands are grouped into the same multiplets and the effect is explained on the basis of the gsb-γ band mixing interaction. A detailed theoretical study of the ground-γ band coupling mechanism has been implemented in the framework of the Vector Boson Model with SU(3) dynamical symmetry [7, 8]. It suggests a relevant model interpretation of the ground-γ band mixing interaction and explains the related odd-even staggering effects in terms of the SU(3) multiplets inherent for the underlying algebraic scheme.

Further, the schematic E(5) symmetry spectrum suggest a 4_1^+, 2_2^+ and 6_1^+, 4_2^+, 3_1^+ levels degeneracy. So, in this case neither the typical γ-band structure nor the staggering effects can be reproduced.

In the above way the presence or the lack of the staggering effect as well as its magnitude and form might give a specific information for the appearance or absence of particular symmetry characteristics of the spectrum.

From experimental point of view essential progress in the collective nuclear structure study has been made through the use of a new generation of multidetector γ-ray arrays, such as Eurogam [9], Euroball [10] and Gammasphere [11], which provides opportunity to investigate the prompt γ-rays emitted from fission fragments. In the last decade this technique has been applied in a number of experiments on both spontaneous and induced fission. In such a way excited states in neutron-rich nuclei $^{100-108}$Mo [12–15], $^{104-114}$Ru [16, 17] and $^{108-118}$Pd [18–20] have been populated.

These new data provide a rather detailed test for the symmetries mentioned above. For example, the addition or subtraction of few neutrons in Mo isotopic chain leads to rapid changes from U(5) to SU(3) symmetry. This is illustrated in Figure 1 where the excitation energy ratios $R_4 = E(4)/E(2)$ and $R_6 = E(6)/E(2)$ are plotted as functions of the neutron number. In these terms ^{100}Mo exhibits nearly E(5) behaviour, while ^{102}Mo appears to be a good example for an O(6) nucleus. A X(5) phase transition between U(5) and SU(3) collectivity might take place in ^{104,106,108}Mo isotopes. These symmetry changes are less pronounced in Ru isotopic chain. The R_4 and R_6 ratios of ^{102}Ru are similar to that of an E(5) nucleus, while ^{104}Ru shows nearly O(6) behaviour. Recent E(5)

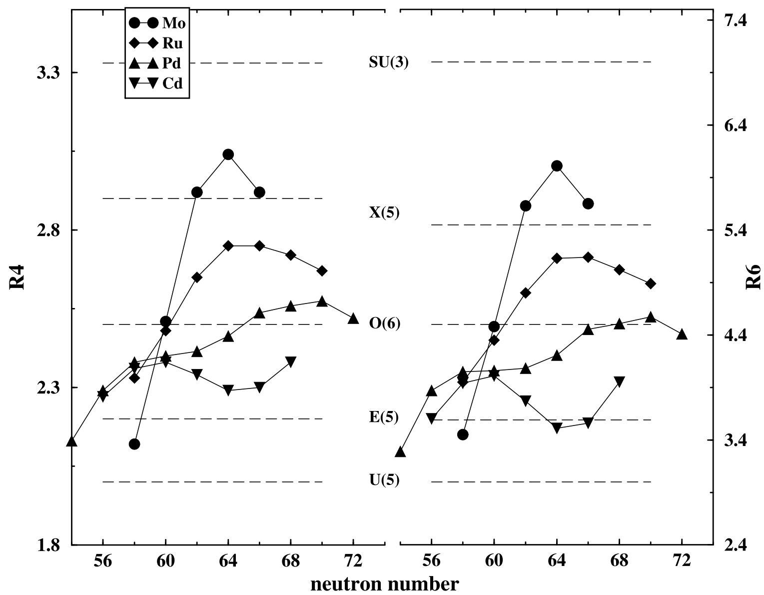

Figure 1. The excitation energy ratios R_4 and R_6 (see text) in several Mo, Ru, Pd and Cd isotopes are plotted as function of neutron number.

symmetry calculations predict U(5)-O(6) phase transition in that nucleus [21]. With the increasing neutron number Ru isotopes become better X(5) nuclei and an U(5)-SU(3) phase transition may occur in ^{108,110}Ru. The nuclei ^{100,102}Pd have nearly E(5) behaviour, while the Pd isotopes with A$\geq$58 have been usually given as a good example for O(6) nuclei. The isotopes ^{104,112,114}Cd have nearly E(5) behaviour.

The systematic analysis of the new experimental data suggests that the low lying collective states of the ground band and the γ-band interact in a way similar to what is observed in rotational SU(3) nuclei in the framework of the Vector Boson Model (VBM) [7]. This circumstance suggests a possible scenario of a transition between rotational and vibrational collective spectra in which the same ground-γ band coupling mechanism persists while the collective band structure is changed. That is way the low lying ground and γ-band states can be a subject of interest in the examination of the new "transition" symmetries. We only remark that while in the rotational SU(3) region the term γ-band is well defined in association with a rotation built on a γ-vibrational state, in nuclei towards transitional and vibrational regions this geometrical meaning is not clear anymore. On the above basis we propose that it will be reasonable to apply an extended model in which the mixing scheme inherent for the SU(3) dynamical symmetry of VBM is modified by introducing of an appropriate phenomenological factor of the level spacing. This makes possible to reproduce the changes in the band structure between rotational, transitional and vibrational modes.

So, the purpose of the present work is to develop and apply an appropriate formalism for analysis and interpretation of the ground-γ band interaction in a wide range of collective nuclei beyond the exact symmetry limits of collectivity. We apply it to nuclei in Z$\sim$ 50 region, providing a detailed model interpretation of the experimental ground and γ-band levels and their interaction together with the respective odd-even staggering phenomena observed there. As it will be shown bellow, the obtained results give a relevant systematics of the inter-band interaction strengths and systematics of the R_4 excitation energy ratios in dependence on a specific phenomenological factor.

In Section 2 we present a formalism for an unique description of the gsb - γ-band interaction. In Section 3 model descriptions for the energy levels and the corresponding staggering patterns of nuclei in the region $40 < Z < 50$ are presented. A detailed analysis of the systematic changes in nuclear cllectivity is presented also there. In Section 4 concluding remarks are given.

2 Ground-γ-Band Mixing Formalism and Odd-Even Staggering

The odd-even staggering effect represents a relative displacement of the even angular momentum levels of the γ-band with respect to the odd levels. It has been explained in the framework of the Vector Boson Model trough the interaction of the even γ-band levels with their counterparts in the gsb [8].

The basic assumption of the VBM is that the low lying collective states of deformed even-even nuclei can be reproduced through the use of vector bosons, whose creation operators are O(3) vectors. The angular momentum operator $\mathbf{L}$ as well as the quadrupole operator $\mathbf{Q}$ are constructed by these vector bosons. The VBM Hamiltonian is constructed as linear combination of three O(3) scalars from the enveloping algebra of SU(3): $\mathbf{L}^2$, $\mathbf{L}.\mathbf{Q}.\mathbf{L}$ and $\mathbf{A}^\dagger\mathbf{A}$ [7], where the second and the third terms are third and fourth order effective interactions, reducing the SU(3) symmetry to the rotational group O(3). In the framework of VBM the gsb and γ - bands belong to the same split SU(3) multiplet labelled by the quantum numbers (λ, μ). In this framework the model provides a relevant way to study the interaction between these two bands. In the simplest case of multiplets of the type $(\lambda, 2)$ the even counterparts of both bands are mixed trough a secular equation of the form

$$\begin{vmatrix} V^g_{SU(3)} - E & V_{12} \\ V_{21} & V^\gamma_{SU(3)} - E \end{vmatrix} = 0 \tag{1}$$

where V_{12}, V_{21} $V^g_{SU(3)}$ and $V^\gamma_{SU(3)}$ are the matrix elements of the model Hamiltonian in the used SU(3) basis states, known as the basis of Bargmann-Moshinsky [22, 23]. For a given angular momentum there are two solutions (corresponding

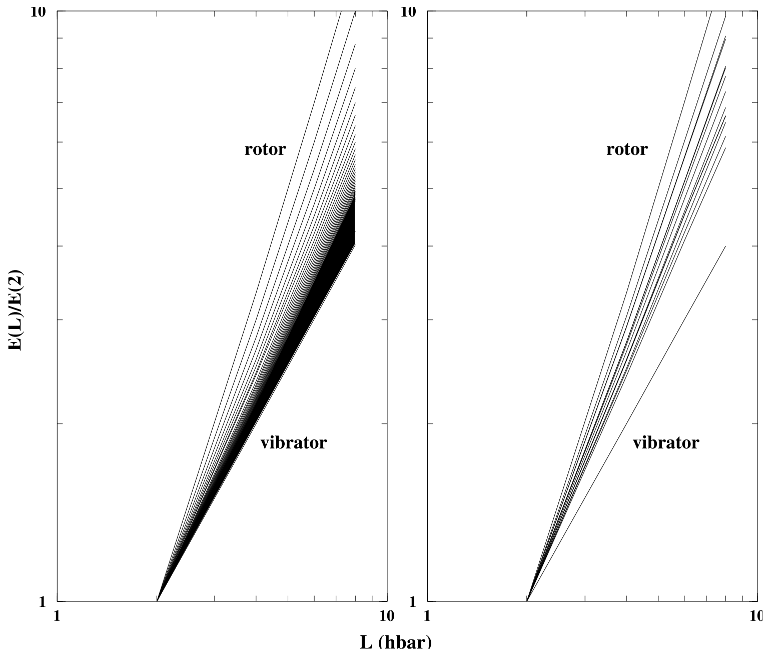

Figure 2. The excitation energy ratio $E(L)/E(2)$ is given vs angular momentum for the: (a) energy term $E(L) = L(L+n)$ with $n = 1, 2, 3, \ldots$; (b) experimental gsb levels of Mo, Ru, Pd nuclei.

to the each band) containing terms of the form $L(L+1)$ [8]. This formalism has been applied in rare-earth and actinide regions giving a successful description of the odd-even staggering effect in the γ-bands.

However, the Pd and Ru nuclei are far from the SU(3) symmetry regions, so that the original VBM formalism cannot be applied directly. Therefore, in order to develop a unique framework for description of the staggering effect on the basis of the gsb-γ - band interaction we have to modify properly the rotational term $L(L+1)$ with a generalized expression capable to generate linear dependence of the energy levels $\sim L$. In such a way the changes of the spectrum from SU(3) to U(5) direction can be taken into account. For this reason we introduce a relevant phenomenological parameter of collectivity, which is denoted here as 'n' and provides a modified angular momentum dependence of the energy in the form $L(L+n)$. For $n \sim 1$ it gives the level spacing of a good rotor while for large values of n it gives a level spacing close to that of a vibrator as it is demonstrated in Figure 2. For intermediate n-values this term gives a transitional spectrum. As it will be demonstrated below the most deformed nucleus in $Z \sim 50$ region, ^{106}Mo, is described by $L(L+3)$, while the nucleus ^{100}Mo nearest to U(5) limit by $L(L+26)$.

So, we propose a modified Hamiltonian in which the linear L-dependence is generated by a diagonal term $\widetilde{V}$ providing the following form of the secular Eq. (1):

$$\begin{vmatrix} V_d^g - E & V_{12} \\ V_{21} & V_d^\gamma - E \end{vmatrix} = 0 \,, \tag{2}$$

where

$$V_d^{g,\gamma} = V_{SU(3)}^{g,\gamma} + \widetilde{V} \,. \tag{3}$$

Here the off-diagonal elements remain the same, which express the assumption that the same gsb-γ band coupling mechanism persists while the collective band structure is changed.

In this way Eq. (2) provides a modified expression for the gsb and γ-band level energies:

$$\begin{aligned} E_{\pm} =& 1/2\{(V_{SU(3)}^g + V_{SU(3)}^\gamma + 2\widetilde{V}) \\ & \pm \sqrt{(V_{SU(3)}^g - V_{SU(3)}^\gamma)^2 + 4V_{12}V_{21}}\} \end{aligned} \tag{4}$$

in which the energy splitting term (the square root) inherent for VBM remains the same. After applying the respective VBM terms in Eq. (4), we obtain it in the form:

$$\begin{aligned} E^g(L_{even}) &= B + (A - BC)L(L+n) \\ &\quad - |B|\sqrt{1 + aL(L+1) + bL^2(L+1)^2} \\ E^\gamma(L_{even}) &= B + (A - BC)L(L+n) \\ &\quad + |B|\sqrt{1 + aL(L+1) + bL^2(L+1)^2} \\ E^\gamma(L_{odd}) &= 2B + AL(L+n) \end{aligned} \tag{5}$$

In the framework of VBM the model parameters A, B, C, a and b are functions of the Hamiltonian parameters [8] and on the SU(3) quantum numbers (λ, μ). However, in the present extension of this formalism the exact form of the term generating energy dependence of the type $L(L+n)$ is not known. Therefore, there is no direct relation between the parameters in Eq. (5) and the quantum numbers of the group SU(3). Hence, we consider them as free model parameter that have to be adjusted to the experimental data.

We have to remark that a phenomenological parameter of collectivity similar to "n" has been used in a Sp(4,R) classification scheme with respect to the low-lying ground band states of even-even nuclei [24–26]. It has been shown that such an approach successfully reproduces the changes in the ground band structure from rotational to vibrational collective regions.

In Eq. (5) the even levels of the gsb interact with their γ band counterparts through the square root term. Namely this term causes the relative shift between the odd and even states in the γ-band, i.e. it generates an odd-even staggering effect.

As a relevant characteristics of the staggering effect we consider the following three-point formula [27]

$$\delta E(L) = E(L) - \frac{(L+1)E(L-1) + LE(L+1)}{2L+1} \,, \tag{6}$$

where $E(L)$ denotes the energy of the level with angular momentum L. Obviously, for the simple rigid rotor energy $L(L=1)$ one has $\delta E = 0$. Thus any energy dependence with $\delta E \neq 0$ will correspond to respective deviation from the regular rotor behaviour of the system. As it will be seen in the next section, this characteristics carries detailed information for the evolution of collectivity in different nuclear regions.

3 Numerical Results and Discussion

In order to reproduce the ground and γ-band energy levels we fit the model parameters A, B, C, a and b with respect to the corresponding experimental data by using a χ^2 minimization procedure. The phenomenological parameter n is determined so as to minimize the root mean square (RMS) deviation

$$\sigma_E = \sqrt{\frac{1}{N_B} \Sigma_{L,\nu} [E_\nu^{th}(L) - E_\nu^{exp}(L)]^2} \,, \tag{7}$$

where N_B is the number of the levels used in the fitting procedure and $\nu = g, \gamma$ for the gsb and γ-band respectively. We restrict our calculations up to the back-banding region. That is why the two bands are considered up to angular momentum $L = 8\hbar$.

The procedure has been applied to the nuclei ^{100,102,104,106,108}Mo, $^{104-112}$Ru and for $^{108-116}$Pd. As a result the respective gsb and γ-band energy levels have been reproduced quite accurately. This is demonstrated in Tables 1, 2, and 3, where the obtained theoretical descriptions are compared with the experimental data.

Figure 3 represents the dependence of the model parameters a and b on the phenomenological parameter n. A correlation between the model parameters a and b and the phenomenological parameter n is well established up to $n = 12$ where a saturation is observed. It could be deduced that the distortion is due to the symmetry changes.

The parameters A and $A - BC$ have the meaning of the moment of inertia parameter $\hbar^2/2\Im$ for the odd-spin sequence and for the even-spin gsb and γ-band sequences, respectively. Figure 4 represents the dependence of the model parameters A and $A - BC$ on the phenomenological parameter n.

In Figure 7 the corresponding theoretical and experimental staggering plots are shown. It is seen that the staggering effect in these nuclei is described successfully, with the respective phase and amplitude characteristics of the stagger-

Table 1. Theoretical end experimental gsb and γ energy levels in (keV) for Mo nuclei. The fitting parameters and the RMS factors Eq. (7) are also given.

	L	E_{gsb}^{th}	E_{gsb}^{exp}	E_{γ}^{th}	E_{γ}^{exp}
^{100}Mo $n = 26$ [12]	2	534	536	1103	1064
$\sigma = 31.65$	3			1540	1607
A= 11.5468	4	1136	1136	1759	1771
B= 267.8468	5			2325	2288
C= 0.0064	6	1828	1847		
a= 0.0229					
b= -0.0003	8	2637	2626		
^{102}Mo $n = 7$ [13]	2	314	296	876	848
$\sigma = 19.62$	3			1210	1246
A= 22.5967	4	755	743	1387	1398
B= 266.0827	5			1888	1870
C= 0.0162	6	1322	1328		
a= 0.0186	7				
b= 0.0001	8	2015	2019		
^{104}Mo $n = 5$ [14]	2	223	192	835	812
$\sigma = 15.29$	3			1009	1028
A= 17.3603	4	573	561	1216	1215
B= 296.3152	5			1461	1475
C= 0.0025	6	1065	1080	1720	1724
a= 0.0122	7			2051	2037
b= -0.0002	8	1723	1722	2325	2326
^{106}Mo $n = 3$ [15]	2	188	172	722	710
$\sigma = 8.07$	3			876	885
A= 19.2189	4	528	523	1068	1068
B= 264.9240	5			1299	1307
C= 0.0006	6	1026	1033	1561	1563
a= 0.0031	7			1875	1868
b= -0.0001	8	1689	1688	2194	2194
^{108}Mo $n = 3$ [14]	2	202	193	611	586
$\sigma = 11.77$	3			776	783
A= 20.1844	4	564	564	970	978
B= 206.3506	5			1220	1232
C= 0.0008	6	1084	1091	1491	1508
a= -0.0028	7			1826	1817
b= 0.0001	8	1755	1752	2180	2170

ing patterns being reproduced accurately.

The following observations and comments can be made now. In ^{108,110,112}Pd isotopes the staggering effect is well pronounced, in spite of the small number of data available. In ^{114,116}Pd the staggering amplitude is strongly

Table 2. The same as in Table 1 but for Ru isotopes.

	L	E^{th}_{gsb}	E^{exp}_{gsb}	E^{th}_{γ}	E^{exp}_{γ}
^{104}Ru $n = 15$ [16]	2	405	358	920	893
$\sigma = 23.16$	3			1216	1242
A= 14.1863	4	898	888	1508	1502
B= 224.9008	5			1868	1872
C= 0.0058	6	1521	1556	2173	2196
a= 0.0560	7			2634	2623
b= -0.0007	8	2331	2320	2855	2847
^{106}Ru $n = 7$ [16]	2	302	270	834	792
$\sigma = 22.81$	3			1067	1091
A= 18.2663	4	732	715	1295	1307
B= 259.4464	5			1615	1642
C= 0.0043	6	1288	1297	1905	1908
a= 0.0083	7			2309	2285
b= 0.0000	8	1970	1975		
^{108}Ru $n = 6$ [17]	2	269	242	741	708
$\sigma = 17.83$	3			954	974
A= 18.6971	4	671	665	1180	1183
B= 224.6123	5			1478	1497
C= 0.0052	6	1223	1241	1749	1762
a= 0.0189	7			2151	2133
b= -0.0002	8	1948	1942	2426	2421
^{110}Ru $n = 5$ [17]	2	265	241	646	613
$\sigma = 17.01$	3			849	860
A= 19.7289	4	677	663	1074	1085
B= 187.6927	5			1362	1376
C= 0.0033	6	1236	1239	1662	1685
a= 0.0050	7			2033	2021
b= 0.0000	8	1938	1944	2412	2398
^{112}Ru $n = 5$ [17]	2	251	237	554	524
$\sigma = 15.52$	3			736	748
A= 18.6310	4	641	645	975	981
B= 144.3896	5			1220	1236
C= 0.0013	6	1173	1190	1550	1571
a= 0.0171	7			1854	1841
b= 0.0000	8	1850	1840	2275	2264

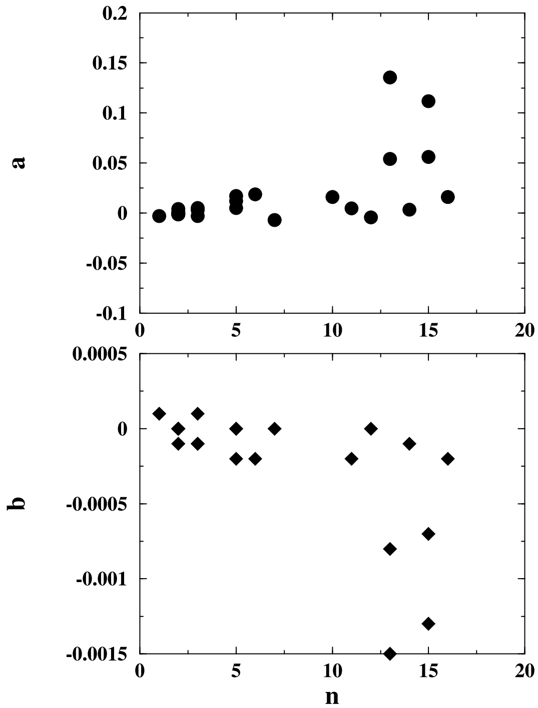

Figure 3. Model parameter a and b vs. phenomenological parameter n.

suppressed, and moreover in ^{116}Pd the oscillations are almost reduced. Ru isotopes with N=60, 62 and 64 exhibit staggering patterns with almost equal amplitudes, weaker than the amplitudes observed in ^{108,110,112}Pd isotopes. In ^{110}Ru the staggering effect is again strongly suppressed. In ^{104,106,108}Mo isotopes the observed staggering amplitudes are generally smaller compared to Pd and Ru nuclei. On the above basis we deduce that for the considered nuclei the increasing neutron number and decreasing number of protons lead to a systematic suppression of the odd-even staggering effect in the γ-bands. In such a way a region of a relatively better formed rotation structure in these bands is outlined.

For comparison, in rare-earth and actinide nuclei a gradual decrease of the staggering effect is observed towards the mid-shell regions [8]. It is explained with the respective better pronounced (less perturbed) rotational band structures there. In the framework of VBM this is related with a decrease in the gsb-γ band interaction strength. Here, on the same basis we can interpret the suppressed staggering effect as the result of a decreasing interaction strength between the gsb and γ-band.

Now we will consider another important characteristics of the interacting

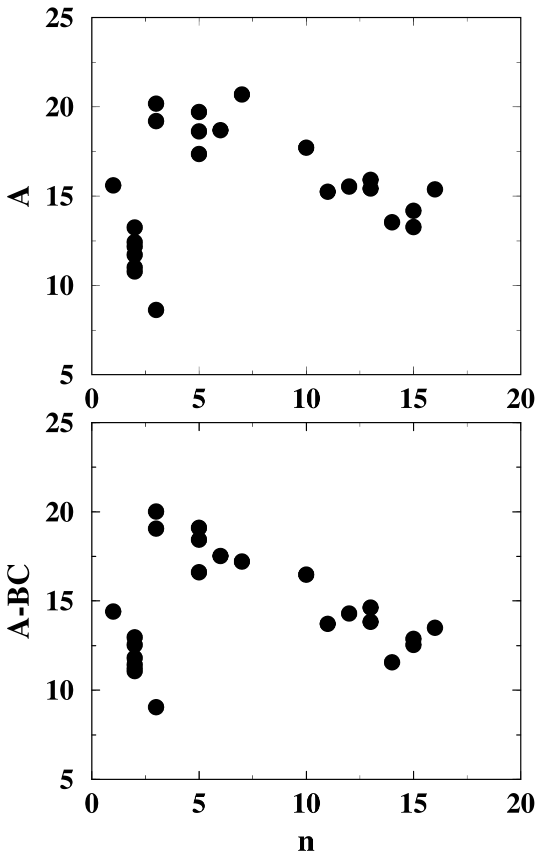

Figure 4. Model parameter A and $A - BC$ vs. phenomenological parameter n.

ground and γ bands

$$\Delta E_2 = \frac{E(2_\gamma) - E(2_g)}{E(2_g)} \,, \tag{8}$$

which describes their mutual disposition. In terms of VBM, Eq. (8) corresponds to the splitting of a SU(3) state with even angular momentum L into the levels of the gsb and the γ-band, i.e. it characterizes the splitting of the SU(3) multiplet. It has been shown that in well deformed nuclei the larger splitting is associated with smaller band mixing interaction which is the case observed in the mid-shell regions of rare-earth and the actinide nuclei [28]. Moreover, it has been demonstrated that the extremely large magnitude of the splitting could be related to a situation of completely separated (noninteracting) bands, known from group theoretical point of view as an SU(3) contraction process. Similar analysis can be done for the nuclear region under study. However, now we should have in mind that away from the exact SU(3) region the term ground-γ multiplet has not the same clear group theoretical meaning, so that the term "splitting" should simply refer to the mutual disposition of the bands.

Table 3. The same as in Table 1 but for Pd nuclei.

	L	E^{th}_{gsb}	E^{exp}_{gsb}	E^{th}_{γ}	E^{exp}_{γ}
^{108}Pd $n = 16$ [18]	2	475	434	959	931
$\sigma = 28.46$	3			1341	1335
A= 15.3942	4	1051	1048	1570	1625
B= 231.7060	5			2080	2083
C= 0.0082	6	1741	1772	2283	2259
a= 0.0162	7				
b= -0.0002	8	2562	2549		
^{110}Pd $n = 12$ [18]	2	403	374	861	814
$\sigma = 28.37$	3			1163	1212
A= 15.5428	4	922	921	1369	1398
B= 231.8628	5			1785	1759
C= 0.0054	6	1557	1574	1992	1987
a= -0.0041	7				
b= 0.0000	8	2302	2296		
^{112}Pd $n = 13$ [19]	2	391	348	781	736
$\sigma = 27.08$	3			1084	1096
A= 15.4425	4	884	883	1339	1362
B= 171.2245	5			1732	1759
C= 0.0094	6	1512	1550	1983	2002
a= 0.0540	7			2504	2483
b= -0.0008	8	2336	2318	2652	2638
^{114}Pd $n = 15$ [20]	2	389	333	736	695
$\sigma = 28.68$	3			989	1012
A= 13.2847	4	864	852	1314	1321
B= 135.9780	5			1600	1631
C= 0.0055	6	1462	1501	1970	1984
a= 0.1118	7			2318	2290
b= -0.0013	8	2227	2216	2660	2655
^{116}Pd $n = 13$ [20]	2	394	341	762	738
$\sigma = 22.75$	3			1042	1067
A= 15.9262	4	889	878	1379	1374
B= 138.6619	5			1711	1719
C= 0.0093	6	1526	1559	2088	2101
a= 0.1356	7			2507	2493
b= -0.0015	8	2352	2344	2843	2840

The lowest splitting ratio, ΔE_2, carries an information about the systematic changes in the mutual band disposition and respective band-mixing interaction in dependence on the place of the nucleus in a given region. It has been shown [7] that in the rare-earth region this value is within the limits $7 < \Delta E_2 < 18$, while in the actinides it is between $13 < \Delta E_2 < 25$.

The experimental and theoretical values of ΔE_2, obtained for nuclei in the region $40 < Z < 50$ are given in Table 4. We see that in ^{104,106,108}Mo ΔE_2 varies between 2 and 3. For $^{104,\ldots,112}$Ru it is between 1 and 2, while in Pd nuclei ΔE_2 is about 1.1-1.2. We remark that in the considered nuclear region the relative displacement of the ground and the γ band is essentially smaller compared with the rare-earth and actinide regions, which is naturally a prerequisite for their essentially stronger mutual perturbation. Here ΔE_2 overally decreases from Mo to Pd nuclei as an indication for the respective increasing of the band mixing interaction.

In Figure 5 we present the systematic relation between the excitation ratio R_4 and the fitted values of the phenomenological model parameter n. It demonstrates the way in which the collective properties of nuclei under study deviate from the SU(3) symmetry. We see that starting from the SU(3) rotational region with values near 3.33, R_4 decreases with the increase of n until reaching the transition region near $R_4 = 2.5$. After that, in the region ($10 \leq n \leq 30$) we observe an overall saturation around values typical for the ground state bands with a structure between transitional and vibrational. There R_4 values appear

Table 4. Experimental and theoretical excitation ratios and band disposition factors (see the text) in Mo, Ru and Pd nuclei. The neutron number N is also given.

	A	ΔE_2^{exp}	ΔE_2^{th}	R_3^{exp}	R_3^{th}	R_4^{exp}	R_4^{th}	R_6^{exp}	R_6^{th}
Mo:	100	0.99	1.07	1.30	1.50	2.12	2.13	3.45	3.42
	102	1.86	1.79	1.38	1.53	2.51	2.40	4.49	4.21
	104	3.23	2.74	1.87	2.19	2.92	2.57	5.63	4.78
	106	3.13	2.84	2.05	2.25	3.04	2.81	6.01	5.46
	108	2.04	2.02	1.99	2.18	2.92	2.79	5.65	5.37
Ru:	104	1.49	1.27	1.74	1.99	2.48	2.22	4.35	3.76
	106	1.93	1.76	1.72	1.98	2.65	2.42	4.80	4.26
	108	1.93	1.75	1.79	2.06	2.75	2.49	5.13	4.55
	110	1.54	1.44	1.91	2.11	2.75	2.55	5.14	4.66
	112	1.21	1.21	2.04	2.31	2.72	2.55	5.02	4.67
Pd:	108	1.15	1.02	1.72	1.50	2.41	2.21	4.08	3.67
	110	1.18	1.14	1.47	1.68	2.46	2.29	4.21	3.86
	112	1.11	1.00	1.74	1.84	2.54	2.26	4.45	3.87
	114	1.09	0.89	1.97	2.28	2.56	2.22	4.51	3.76
	116	1.16	0.93	1.93	2.20	2.57	2.26	4.57	3.87

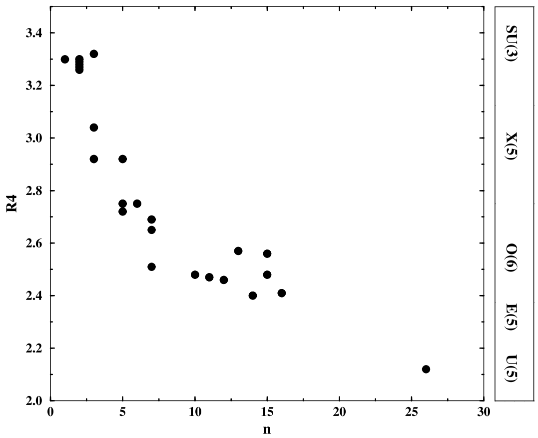

Figure 5. The excitation energy ratio R_4 is given vs. the parameter n for Mo, Ru and Pd. Also, results for Xe, Ba, Os and Pt nuclei are included for better statistics.

to be split as a function of the parameter n. The above result can be explained as follows. While R_4 is a characteristics of the lowest ground state band structure the phenomenological parameter n carries additional information about the structure of the ground band and the gsb-γ bands interaction as well. Having this in mind we can conclude that the well pronounced correlation between R_4 and n observed in the region of near-SU(3) nuclei may be considered as the result of a gradual evolution of collectivity in the two bands. Although in the case of near-SU(3) nuclei the gsb and γ-band essentially interact, their mutual perturbation is still not strong enough and conserves the individual characteristics of the two bands. As a result R_4 remains a good characteristics of collectivity not only for the gsb structure, but also for the entire low-lying spectrum in the given nucleus. In the region $10 \leq n \leq 30$ (see Figure 5) the situation is different. According to our analysis, there the mutual perturbation of the two bands should be quite strong. Therefore, the information that the gsb characteristics R_4 could carry for the γ-band structure will be essentially limited, so it will not characterize anymore the complicated ground - γ band configuration, and the overall collective properties of the given nucleus. For example it could happen that the gsb is characterized by near rotational R_4 value with the γ-band carrying some characteristics of a close to vibrational structure. On the other side the phenomenological parameter n is continuously capable of taking into account the common collective characteristics of the gsb and γ band structures in the different nuclei. This circumstance can explain the observed saturation of R_4 as a function of n in the region $10 \leq n \leq 30$.

Similar analysis could be done with respect to the energy ratio

$$R_3 = \frac{E(4_2^+ \to 2_2^+)}{E(3_1^+ \to 2_2^+)}\,, \tag{9}$$

which characterizes the γ-band collective structure. (It takes the limiting values 2.33 and 2.00 for the pure rotational and vibrational spectra respectively.) By considering its systematic behaviour in the studied nuclei (see Table 4) one can easily deduce that towards the near-SU(3) region (Mo nuclei) R_3 is correlated with "n" similarly to R_4, while away from SU(3) the correlation is more complicated.

The so far obtained results allow us to reveal the specific signatures of the changing nuclear collectivity in terms of the symmetry limits of IBM and the new "phase" transition limits. Thus the saturation of R_4 with respect to n, observed in Figure 5 (the region $R_4 = 2.5$) can be interpreted as a manifestation of the γ-softness structure inherent for the O(6) symmetry. Also, the gradual decrease in the staggering effect observed in Figure 7 can be interpreted as the general result of the change from SU(3) to O(6) symmetry. The nuclei with near X(5) collective structure are characterized by ΔE_2 values between 2 and 3, indicating a weak interband interaction strength. With the approaching of the O(6) limit the ratio ΔE_2 decreases to values near 1.1 with respective increase in the bandmixing interaction. For the region of E(5) nuclei and nuclei near U(5) symmetry the analysis of our results indicate the trend of sharply increasing interaction strength which may also be considered as the hallmark of a completely rearranged structure of collective spectrum.

In Figure 6 the values of the phenomenological parameter of collectivity "n" obtained for Mo, Ru and Pd nuclei are plotted as functions of the neutron num-

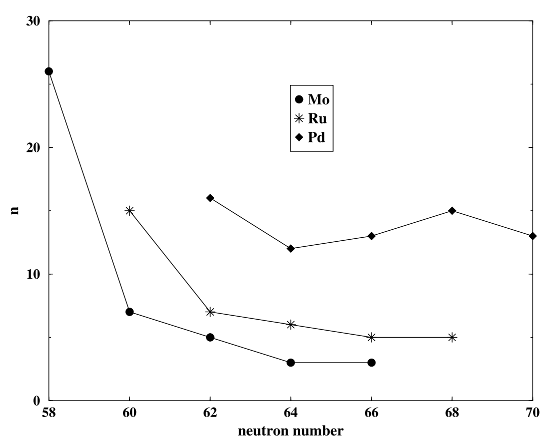

Figure 6. The model parameter n vs. neutron number.

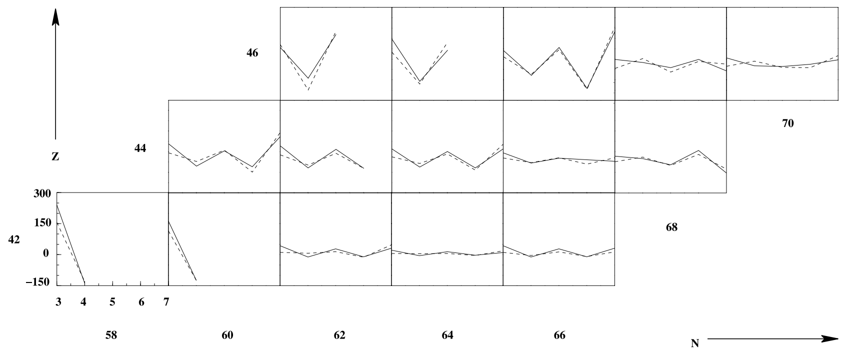

Figure 7. Theoretical (dashed lines) and experimental (solid lines) odd-even staggering plots (in keV) for some Mo, Ru and Pd nuclei.

ber. It is clearly seen that from Mo to Pd nuclei the respective curves are systematically shifted up, demonstrating the overall move towards the vibrational collective mode. In addition, in any particular group of isotopes we observe a decrease in “n” with the increase of the neutron number N towards the mid-shell regions. This result confirms the physical significance of the parameter “n” as a characteristics of the changing collectivity.

4 Conclusion

In the present paper we propose an extended model formalism for description of the low-lying ground and γ-band collective states and their band mixing interactions in a wide range of nuclei starting from the SU(3) rotation regions and reaching nuclei near the almost vibrational region.

On this basis we obtain model description and implement detailed analysis of the ground – γ-band structure in the region of Mo, Ru and Pd isotopes. As a result we reproduce accurately not only the levels up to the back-bending region but also the respective experimentally observed odd-even staggering effect in the gamma bands.

The analysis of the results obtained allow us to draw the following main conclusions.

In the considered region $40 <$Z< 50 the γ – gsb interaction strength increases away from the SU(3) symmetry with the approaching of the transitional and vibrational collective regions. It conserves the same band coupling mechanism typical for the VBM scheme although in this region the overall structure of the spectrum is changed in consistence with the multiplet characteristics inherent for the O(6) and U(5) limits of IBM. The systematic analysis of the mutual ground γ-band disposition (characterized by the quantity ΔE_L) provides detailed quantitative characteristics of the different regions of collectivity in Pd, Ru and Mo isotopes. The respective changes in the structure of the spectrum reflect in the respective fine odd-even staggering patterns observed there.

The implemented theoretical analysis clearly outlines the evolution of collectivity along the considered isotopic chains. The obtained values of the model characteristics n provide a detailed information about the way of deviation from the SU(3) symmetry, indicating a good consistence with the gsb ratio of collectivity R_4, when $3.33 \geq R_4 \geq 2.6$, and saturation of R_4 near 2.5. Thus it provides a specific map of collectivity for the ground – γ-band structure covering the regions with nearly-SU(3), X(5), O(6), E(5) and nearly-U(5) symmetries. We suggest that the proposed approach could provide a relevant tool for further analysis of available and newly obtained experimental data and their interpretation in terms of the above exact and transition symmetries.

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