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Rearrangement of the Experimental Data of Low Lying Collective Excited States

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Abstract.

The classification of low-lying excited states in even even deformed nuclei has been done. The available experimental data are represented as the energies parabolic distributed by number of collective excitations. With other words each excited state now is determined as the collective state with the corresponding number of bosons. In this paper we use the Interacting Vector Bosons Model to vindicate that the experimental data for low-lying excited states possessing not equal to zero spins can also be described with parabolic distribution function depending on the number of collective excitations building the corresponding state.

We represent the available experimental data in the form of the energies of the 0^+ excited states as distributed by positive integer parameter and determine this classification parameter in the way giving us information about collective structure peculiarities of these states.

In recent new representation of the experimental data of the low lying excited 0^+–states. has been applied using the distribution function.

$$E_n = An - Bn^2 + C \tag{1}$$

This form of the distribution function appears as the energy spectrum produced by model monopole Hamiltonian [1]

$$H = \alpha R_+^j R_-^j + \beta R_0^j R_0^j + \frac{\beta \Omega^j}{2} R_0^j \ , \tag{2}$$

which, written in terms of pure bosons $b, b^\dagger$ with $\left[b, b^\dagger\right] = 1,\ [b, b] = \left[b^\dagger, b^\dagger\right]$=0 has the form:

$$H = Ab^\dagger b - Bb^\dagger b b^\dagger b \ . \tag{3}$$

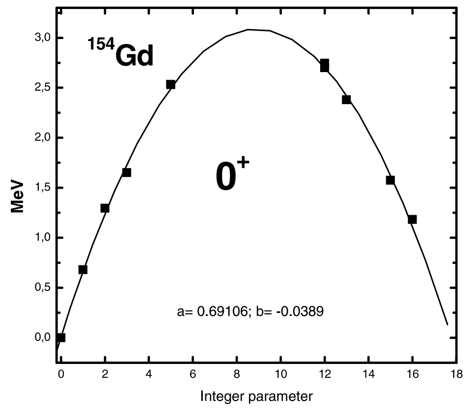

Figure 1. Experimental data for low lying excited 0^+ states in ^{154}Gd distributed by number of monopole bosons.

Some of the distributions of the experimental energies of the excited 0^+ states plotted using (1) are shown in Figure 1 for ^{154}Gd and ^{164}Er in Figure 2.

This parabolic distribution (1) reproduces with a great accuracy experimental values of low lying 0^+ excited states energies. Similarly, very nice agreement was obtained for all available experimental data of low lying 0^+ excited states in a large region of the even-even nuclei.

Of course it is straightforward now to see whether the low lying excited states having different from zero spin can be also represented in the same form

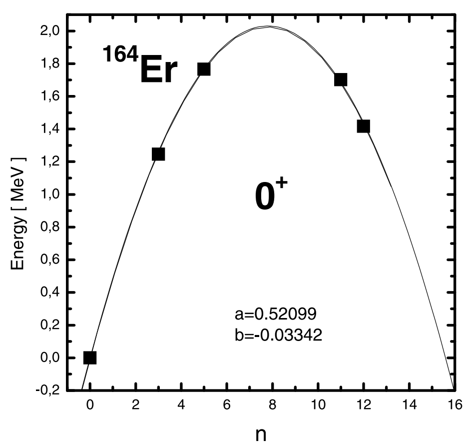

Figure 2. Experimental data for low lying excited 0^+ states in ^{164}Er distributed by number of monopole bosons.

of the energies distributed by parabolic type function and can we connect the new classification parameter with any measure of collectivity.

For this purpose let us shortly remind the Interacting Vector Boson Model (IVBM) [2], which is based on the introduction of two kinds of vector bosons (called p and n bosons), that "built up" the collective excitations in the nuclear system. The creation operators $u_m^+(\alpha)$ of these bosons are assumed to be $SO(3)$–vectors and they transform according to two independent fundamental representations $(1,0)$ of the group $SU(3)$. The annihilation operators $u_m(\alpha) = (u_m^+(\alpha))^\dagger$ transform according to the conjugate representations $(0,1)$. These bosons form a "pseudospin" doublet of the group $U(2)$ and differ in their "pseudospin" projection $\alpha = \pm\frac{1}{2}$. The introduction of this additional degree of freedom leads to the extension of the $SU(3)$ symmetry to $U(6)$ so that the two kind of bosons $u_m^+ \left(\alpha = \pm\frac{1}{2}\right)$ transform according to the fundamental representation $[1]_6$ of the group $U(6)$. The bilinear products of the creation and annihilation operators of the two vector bosons generate the noncompact symplectic group $Sp(12, R)$ [2]:

$$
\begin{aligned}
F_M^L(\alpha,\beta) &= \sum_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m^+(\beta) \,, \\
G_M^L(\alpha,\beta) &= \sum_{k,m} C_{1k1m}^{LM} u_k(\alpha) u_m(\beta) \,, \\
A_M^L(\alpha,\beta) &= \sum_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m(\beta) \,,
\end{aligned}
\tag{4}
$$

where C_{1k1m}^{LM} are the usual Clebsch–Gordon coefficients and L and M define the transformational properties of (4) under rotations.

We consider $Sp(12, R)$ to be the group of the dynamical symmetry of the model [2]. Hence the most general one- and two-body Hamiltonian can be expressed in terms of its generators. Using commutation relations between $F_M^L(\alpha,\beta)$ and $G_M^L(\alpha,\beta)$, the number of bosons preserving Hamiltonian can be expressed only in terms of operators $A_M^L(\alpha,\beta)$:

$$
\begin{aligned}
H = \sum_{\alpha,\beta} h_0(\alpha,\beta) A^0(\alpha,\beta) \\
+ \sum_{M,L} \sum_{\alpha\beta\gamma\delta} (-1)^M V^L(\alpha\beta;\gamma\delta) A_M^L(\alpha,\gamma) A_{-M}^L(\beta,\delta) \,,
\end{aligned}
\tag{5}
$$

where $h_0(\alpha,\beta)$ and $V^L(\alpha\beta;\gamma\delta)$ are phenomenological constants.

Being a noncompact group, the representations of $Sp(12, R)$ are of infinite dimension, which makes it rather difficult to diagonalize the most general

Hamiltonian. The operators $A^L_M(\alpha,\beta)$ generate the maximal compact subgroup of $Sp(12,R)$, namely the group $U(6)$:

$$Sp(12,R) \supset U(6)$$

So the even and odd unitary irreducible representations (UIR) of $Sp(12,R)$ split into a countless number of symmetric UIR of $U(6)$ of the type $[N,0,0,0,0,0]$=$[N]_6$, where N=0,2,4,... for the even one and N=1,3,5,... for the odd one [2]. Therefore the *complete* spectrum of the system can be calculated only trough the diagonalization of the Hamiltonian in the subspaces of *all* the UIR of $U(6)$, belonging to a given UIR of $Sp(12,R)$.

Let us consider the rotational limit [2] of the model defined by the chain:

$$U(6) \supset SU(3) \times U(2) \supset SO(3) \times U(1) \qquad (6)$$
$$[N] \quad\; (\lambda,\mu) \quad\; (N,T) \quad\; K\;\; L \qquad T_0 \qquad (7)$$

where the labels below the subgroups are the quantum numbers (7) corresponding to their irreducible representations. Their values are obtained by means of standard reduction rules and are given in [2]. In this limit the operators of the physical observables are the angular momentum operator

$$L_M = -\sqrt{2}\sum_{M,\alpha} A^1_M(\alpha,\alpha)$$

and the truncated ("Elliott") quadrupole operator

$$Q_M = \sqrt{6}\sum_{M,\alpha} A^2_M(\alpha,\alpha)\ ,$$

which define the algebra of $SU(3)$.

The "pseudospin" and number of bosons operators:

$$T_{+1} = \sqrt{\frac{3}{2}}A^0(p,n); \qquad T_{-1} = -\sqrt{\frac{3}{2}}A^0(n,p);$$
$$T_0 = -\sqrt{\frac{3}{2}}[A^0(p,p) - A^0(n,n)]; \qquad N = -\sqrt{3}[A^0(p,p) + A^0(n,n)]\ ,$$

define the algebra of $U(2)$.

Since the reduction from $U(6)$ to $SO(3)$ is carried out by the mutually complementary groups $SU(3)$ and $U(2)$, their quantum numbers are related in the following way:

$$T = \frac{\lambda}{2}, \quad N = 2\mu + \lambda \qquad (8)$$

Making use of the latter we can write the basis as

$$|[N]_6;\Big(\lambda,\mu = \frac{N}{2}\Big);K,L,M;T_0\rangle = |(N,T);K,L,M;T_0\rangle \qquad (9)$$

The ground state of the system is:

$$|0\rangle = |(0,0);0,0,0;0\rangle = \\ = |(N=0,T=0);K=0,L=0,M=0;T_0=0\rangle \quad (10)$$

which is the vacuum state for the $Sp(12,R)$ group.

Then the basis states [2] associated with the even irreducible representation of the $Sp(12,R)$ can be constructed by the application of powers of raising generators $F_M^L(\alpha,\beta)$ of the same group. The $SU(3)$ representations (λ,μ) are symmetric in respect to the sign of T_0.

Hence, in the framework of the discussed boson representation of the $Sp(12,R)$ algebra all possible irreducible representations of the group $SU(3)$ are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators N,T^2, and T_0. The equivalent use of the (λ,μ) labels facilitates the final reduction to the $SO(3)$ representations, which define the angular momentum L and its projection M. The multiplicity index K appearing in this reduction is related to the projection of L in the body fixed frame and is used with the parity to label the different bands in the energy spectra of the nuclei. The parity of the states is defined as π=$(-1)^T$. This allows us to describe both positive and negative bands.

The Hamiltonian, corresponding to this limit of IVBM is expressed in terms of the first and second order invariant operators of the different subgroups in the chain (6):

$$H = aN + \alpha_6 K_6 + \alpha_3 K_3 + \alpha_1 K_1 + \beta_3 \pi_3 \ , \quad (11)$$

where K_n are the quadratic invariant operators of the $U(n)$ – groups in (6), π_3 is the $SO(3)$ Casimir operator. As a result of the connections (8) the Casimir operators K_3 with eigenvalue $(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)$, is express in terms of the operators N and T:

$$K_3 = 2Q_2 + \frac{3}{4}L^2 = \frac{1}{2}N^2 + N + T^2$$

After some transformations the Hamiltonian (11) takes the following form

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 \pi_3 + \alpha_1 T_0^2 \ , \quad (12)$$

and is obviously diagonal in the basis (9) labeled by the quantum numbers of the subgroups of chosen chain (6). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12,R)$:

$$E((N,T);KLM;T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2 \ . \quad (13)$$

Using the (λ,μ) labels facilitates and choosing for instance $(\lambda,0)$ multiplet together with the reducing rules (8) after simple regrouping of the terms in (13)

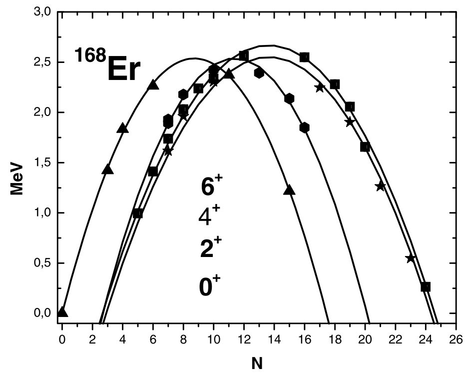

Figure 3. Experimental data for low lying excited states in ^{168}Er distributed by collective classification parameter $n = \lambda/4$.

we can write the energy spectrum corresponding to this $(\lambda, 0)$ multiplet as:

$$E(\lambda) = A\lambda - B\lambda^2 + C \quad (14)$$

here A, B and C are the combinations of free model parameters of (13) a, b, α_3, β_3 and α_1.

Hence choosing any permitted by (8) (λ, μ) multiplet we again may classify the low lying excited states energies in even even nuclei applying the parabolic type distribution function and considering label λ as a measure of collectivity of the corresponding excited states possessing different from 0 spins. In Figure 3 are shown some examples for the classification (consider also $n = \lambda/4$) of the energies of 2^+, 4^+, and 6^+ excited states in ^{168}Er isotopes.

The experimental energies with great accuracy follow the parabolic distribution function (14) and similar agreement can be obtained for all spectra in even even nuclei. All experimental data are taken from [3]. This work was partially supported by Bulgarian Science Committee under contract number Φ – 905.

References

[1] V. P. Garistov (2002) "Phenomenological Description of the Yrast Lines", *nucl-th/0201008*.

[2] A. Georgieva, P. Raychev, R. Roussev (1982) *J. Phys. G: Nucl. Phys.* **8** 1377-1389; A. Georgieva, P. Raychev, R. Roussev (1983) *J. Phys. G: Nucl. Phys.* **9** 521-534; V. P. Garistov, A. Georgieva, H. Ganev (2002) "On Simultaneous Description of the Positive and Negative Bands in the Interacting Vector Boson Model" *Algebraic Methods in Nuclear Theory*, collection of scientific papers edited by Anton N. Antonov.

[3] Mitsuo Sacai Atomic Data and Nuclear Data Tables 31 (1984) 399-432; **Level Retrieval Parameters** *http://iaeand.iaea.or.at/nudat/levform.html*.