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C^{12} Shape Isomers in the Chiral Field Solitons Approach

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Abstract.

The variational approach to the problem of seeking axially symmetric solitons with B=12 is presented. The numerically obtained local minima of the skyrmion mass functional and baryon charge distributions are pointing to the possible existence of shape isomers in C^{12} spectra in the the framework of the original Skyrme model. Theoretical analysis reveals the exclusiveness of each individual state manifested in the structure of the solitons from the given topological sector B=12.

1 Introduction

Relativistic quantum field theory works well for the point-like particles such as electrons, but conceptual and technical problems arise for particles such as nucleons (protons and neutrons) which have a spatial extent. Chiral field soliton model (Skyrme model) is the model which leads to the localizable solitons with finite sizes. The topological chiral solitons (skyrmions) are classical configurations of chiral fields incorporated in unitary matrix $SU(2)\otimes SU(2)$ or $SU(3)$ and characterized by topological, or winding number identified with baryon number B. The classical energy (mass) of these configurations is found usually by minimization of energy functional depending on chiral fields. As any extended object skyrmions possess also other characteristics like moments of inertia, mean square radii of mass and baryon number distribution, etc. In according to the basic statement of the founder of the model topological solitons of $SU(2)\otimes SU(2)$ chirally-symmetric model of the pseudo-scalar fields can reproduce all baryon properties and their interactions. This model (the Skyrme model) was the most elementary generalization of the nonlinear σ - model, having stable, soliton-like solutions with an integer topological charge. This model give us the instrument to describe extended objects like proton. The concept of extended objects makes inessential the distinction between an "elementary" particle (for example, nucleon) and the bound system of such particles (atomic nucleus). For the theory of extended objects a unity of methods and approaches, used at the description of the structure of baryons and their systems is characteristic. So the model can also be used to study more complicated objects like multibaryons and nuclei.

Now chiral soliton approach, starting with a number basic principle incorporated in Lagangian [1] provides realistic description of baryon and baryonic systems and as a model for the strong interactions of hadrons was very successful in describing nucleons as quantum states of the chiral soliton in original and generalized Skyrme model [2].

The baryon and baryonic systems in this approach are presented as quantized solitonic solutions of equation of motion, characterizes by the winding number B. The chiral field configurations of the lowest energy possess different topological properties. The shape of the mass and B-number density distributions are different for different topological sectors. It is a sphere for B=1 hedgehoge, torus for B=2 [3], and more complicated configurations for higher B. The Skyrme model gives us very unusual instrument to study new physics especially in the light nuclei region. In this region traditional one-nucleon degrees of freedom are possibly not so important as solitonic ones because nucleon sizes are comparable to the nuclear radiuses [4, 5].

There is no analytic solutions for the Skyrme model equations of motion. We still have to use variational approaches. The most popular in between them is the so called rational map ansatz [6] leading to the a number of the solution with discrete space symmetries and topological charges corresponding to light nuclei atomic numbers up to 22. They are very like to fullerene structures more usual for the larger molecular scale [7]. In any way such solutions are like pure numerical solutions obtained in [8] for topological charges 2, 3, 4, 5 and 6.

The variational approach to the problem of seeking axially symmetric solitons with B=12 is presented in [9] The numerically obtained local minima of the skyrmion mass functional and baryon charge distributions are pointing to the possible existence of shape isomers in C^{12} spectra in the the framework of the original Skyrme model.

In a recent paper [10] was reported on an exotic strangeness $|S|$=1 baryon states observed as a sharp resonance at 1.54 GeV in photoproduction from neutrons. The configuration of this finding would give strong support to topological soliton model [11]for a description of baryons in the non-perturbative regime of QCD. Higher multiplets containing states carrying exotic quantum numbers arise naturally in the SU(3) version of the model. These states called exotic because, within quark models, such states cannot be built of only 3 valence quarks.

In soliton model there is nothing exotic about these states, they just come as members of the next higher multiplets. The quantization of zero modes of chiral solitons allows to obtain the spectrum of states with different values of quantum numbers: spin,isospin, strangeness, etc. [12–15]. This approach allows for quite reasonable description of various properties of baryons, nucleons and hyperons, therefore, it is of interest to consider predictions of the models of this kind for baryonic systems with B>1.

Electromagnetic nucleon formfactors can be described quite well within Skyrme soliton model in wide interval of momentum transfers [16, 17] reasonable agreement with data takes place for deuteron and 2N - system [8], therefore, one can expect reasonable predictions for systems with greater baryon numbers.

Here we try to search soliton with axially symmetric baryon charge distribution. Quantization procedure for the states with baryon number equal to 2, 3 and 4 was worked out in [18–22] without vibrations have been taken into account and including the breathing mode [23, 24]. The [14, 25] describe quantization rules for axially symmetric soliton we are considering here.

The variational ansatz we use here was proposed independently in [26–28]. The ansatz being very simple, gives the possibility to do analytical analysis of a part of the nuclear problem.

In this paper we present the results of our variational calculations of the classical soliton structure with baryon charge B=12 in the framework of the original $SU(2)$ Skyrme model. After the quantization procedure some of these solitons could be identified with shape isomers of C^{12}.

2 Ansatz for the Static Solutions

We follow our papers [29, 30] with some modifications. In variational form of the chiral field U:

$$U(\vec{r}) = \cos F(r) + i(\vec{\tau}\cdot\vec{N})\sin F(r)\ . \qquad (1)$$

we use the next general assumption about the configuration of the isotopic vector field $\vec{N}$ for axially symmetric soliton:

$$\vec{N} = \{\cos(\Phi(\phi,\theta))\cdot\sin(T(\theta)), \sin(\Phi(\phi,\theta))\cdot\sin(T(\theta)), \cos(T(\theta))\}\ . \qquad (2)$$

In Eq. (2) $\Phi(\phi), T(\theta)$ are some arbitrary functions of angles (θ,ϕ) of the vector $\vec{r}$ in the spherical coordinate system.

3 Mass Functional and Solutions for Static Equations

After some algebra (1), (2) and the Lagrangian density $\mathcal{L}$ for the stationary solution

$$\mathcal{L} = \frac{F_\pi^2}{16} \cdot Tr(L_k L_k) + \frac{1}{32e^2} \cdot Tr\Big[L_k, L_i\Big]^2 , \tag{3}$$

expressed through the left currants $L_k = U^+ \partial_k U$ lead to the expression

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 , \tag{4}$$

where

$$\mathcal{L}_2 = -\frac{F_\pi^2}{8} \left\{ \left(\frac{\partial F}{\partial x}\right)^2 + \left[\left(\frac{\sin T}{\sin\theta} \frac{\partial \Phi}{\partial \phi}\right)^2 + \left(\frac{\partial T}{\partial \theta}\right)^2 + \sin^2 T \left(\frac{\partial \Phi}{\partial \theta}\right)^2 \right] \frac{\sin^2 F}{r^2} \right\}$$

and

$$\mathcal{L}_4 = -\frac{1}{2e^2} \cdot \frac{\sin^2 F}{r^2} \cdot \left\{ \frac{\sin^2 T}{\sin^2\theta} \left(\frac{\partial T}{\partial \theta}\right)^2 \left(\frac{\partial \Phi}{\partial \phi}\right)^2 \cdot \frac{\sin^2 F}{r^2} \right.$$
$$\left. + \left[\frac{\sin^2 T}{\sin^2\theta} \left(\frac{\partial \Phi}{\partial \phi}\right)^2 + \left(\frac{\partial T}{\partial \theta}\right)^2 + \sin^2 T \left(\frac{\partial \Phi}{\partial \theta}\right)^2 \right] \left(\frac{\partial F}{\partial x}\right)^2 \right\} \tag{5}$$

The variation of the functional L=$\int \mathcal{L} d\vec{r}$ with respect to Φ leads to an equation which has a solution of the type

$$\Phi(\phi) = k(\theta) \cdot \phi + \text{const} \tag{6}$$

with a constrain:

$$\frac{\partial}{\partial \theta} \left[\sin^2 T(\theta) \cdot \sin\theta \cdot \frac{\partial k(\theta)}{\partial \theta}\right] = 0 . \tag{7}$$

It is easily seen from Eq. [29] that function $k(\theta)$ may be piecewise constant function (step function) in general case:

$$\Phi(\theta, \phi) = \begin{cases} k^{(1)}\phi + \rho^{(1)}, & \text{for} \quad 0 \le \theta < \theta_1 , \\ k^{(2)}\phi + \rho^{(2)}, & \text{for} \quad \theta_1 \le \theta < \theta_2 , \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ k^{(l)}\phi + \rho^{(l)}, & \text{for} \quad \theta_{l-1} \le \theta < \pi . \end{cases}$$

Moreover $k(\theta)$ must be integer in any region $\theta_m \le \theta \le \theta_{m+1}$, where θ_m, θ_{m+1} are successive points of discontinuity. The positions of these points are determined by the condition

$$T(\theta_m) = m \cdot \pi , \tag{8}$$

with integer m, as follows from Eq. (7).

Now we have the following expression for the mass of the soliton

$$M = \gamma \cdot \Big[a \cdot A + b \cdot B + C\Big] \ , \tag{9}$$

where $\gamma = \pi \cdot F_\pi / e$ and $x = F_\pi \cdot e \cdot r$ and the a, b and A, B, C are the following integrals:

$$a = \int\limits_0^\pi \Big[k^2 \frac{\sin^2 T}{\sin^2 \theta} + (T')^2\Big] \sin\theta d\theta, \ \ b = k^2 \int\limits_0^\pi \frac{\sin^2 T}{\sin^2 \theta}(T')^2 \sin\theta d\theta \ , \tag{10}$$

$$A = \int\limits_0^\infty \sin^2 F \Big[\frac{1}{4} + (F')^2\Big] dx, \ B = \int\limits_0^\infty \frac{\sin^4 F}{x^2} dx, \ C = \frac{1}{2}\int\limits_0^\infty (F'x)^2 dx \ . \tag{11}$$

Here we use the symbol prime to denote the following derivatives

$$\Phi' = \frac{\partial \Phi}{\partial \phi}; \ \ T' = \frac{\partial T}{\partial \theta}; \ \ F' = \frac{\partial F}{\partial r} \tag{12}$$

We consider the configurations with finite masses. The only configurations which obey the finiteness of mass condition are the configurations with $F(0) = n \cdot \pi$ where n-is some integer number. Without loss of generality we take $F(\infty) = 0$. As it was shown in [29] $T(\theta)$ has the following behaviour near the boundary of the domain of its definition

$$T(\theta) \to \theta^k, \ for \ \theta \to 0 \ ; \ \ T(\theta) \to \pi \cdot l - (\pi - \theta)^k, \ for \ \theta \to \pi \ . \tag{13}$$

Here l is an integer number. Thus we have the following estimation for the number of discontinuity points d:

$$0 \leq d \leq l - 1 \ . \tag{14}$$

Now all solutions $U_{l\{k_i,n_i\}}$ are classified by a set of integer numbers l, k_0,..., k_{l-1} and n_0,..., n_{l-1}. The functions $F(x)$ and $T(\theta)$ have to obey the equations (14,15) from [29] in arbitrary space region with given number k.

4 Baryon Charge Distribution and the Soliton Structure

Now consider more carefully the structure of solitons. For that purpose let us calculate the baryon charge density

$$J_0^B(\vec{r}) = -\frac{1}{24\pi^2} \cdot \epsilon_{0\mu\nu\rho} Tr(L_\mu L_\nu L_\rho) \ . \tag{15}$$

The straightforward calculation gives

$$J_0^B(r,\theta) = -\frac{1}{2\pi^2} \cdot \frac{\sin^2 F}{r^2} \cdot \frac{dF}{dr} \cdot \frac{\sin T}{\sin\theta} \cdot \frac{dT}{d\theta} \cdot \frac{d\Phi}{d\phi} \ . \tag{16}$$

Eq. (16) immediately results in the expression for the corresponding topological charge

$$B = -\sum_{m=0}^{l-1} (-1)^m \cdot n_m \cdot k_m \ . \tag{17}$$

In [29] we have investigated toroidal multiskyrmion configurations with baryon numbers $B = 1$, 2, 3, 4, 5 and more complicated nontoroidal (including antiskyrmions ($\bar{S}$)) configurations.

It is obvious that setting $k_m<0$ for even m, $k_m>0$ for odd m and $n_m>0$ for all m, we obtain configuration with positive baryon charge. In the general case for obtaining configuration with positive baryon charge we must require that

$$\begin{aligned} n_m \cdot k_m &> 0 \qquad \text{for odd } l \ , \\ n_m \cdot k_m &< 0 \qquad \text{for even } l \ . \end{aligned}$$

5 The Masses and Baryon Charge Distributions

Here we reproduce the mass values and baryon charge distributions corresponding to the obtained local minima of the energy functional for the Skyrme field. We have to point out that we discuss multiskyrmion configurations we search for not only classically stable configurations (The decay in two or more skyrmions is forbidden energetically). Nonstable configurations are also in our attention because they may become stable after the quantization procedure [29] or pion field Casimir energy would taken into account in full quantum description of the considered solitons.

We restrict ourself to configuration with a symmetric distribution of energy (mass) density in the (x, y) plane. This mean that from the class of all solutions considered, characterized by the numbers $l, \{k_m, n_m\}|_1^l$, we choose only the solution satisfying the condition

$$\begin{aligned} k_i &= k_{l+1-i}, \quad n_i = n_{l+1-i} \qquad \text{for odd } l \ , \\ k_i &= -k_{l+1-i}, \quad n_i = -n_{l+1-i} \quad \text{for even } l \ . \end{aligned} \tag{18}$$

In Table 1 we present the masses of shape isomers of C^{12}. The calculated soliton masses are given in $(\pi F_\pi/e)$ units.

From Table 1 one see that in calculated part of spectrum all configurations have very different structures. Presence of such isomers could probably be seen in high energy ion-ion scattering experiments.

Table 1. C^{12} soliton mass spectrum.

Configuration	Mass $(\pi F_\pi/e)$
2{3.2-3.2}	157.6778
2{6.1-6.1}	137.9763
3{2.2-2.2-2.2}	172.3576
3{2.2-4.1-2.2}	161.6704
3{4.1-2.2-4.1}	145.1425
3{4.1-4.1-4.1}	134.4552

The reason we are looking for possible not pure toroidal solitons is that there is a number of solutions with smaller masses among them. For example a configuration composed from three toroidal multibaryons 3{4.1-4.1-4.1} has a smaller mass. Now this state can not decay into 12 classical skyrmions with B=1 ($M_{1\{1.1\}}$=11.60608$\pi F_\pi/e$) or into three toroidal skyrmion with B=4 ($M_{1\{4.1\}}$=47.67478$\pi F_\pi/e$). We point such a skyrmion as classically stable configuration. The configuration 3{2.2-2.2-2.2} do not obey the condition of classical stability.

For calculated axially symmetric configurations we present baryon density distributions integrated on $d\Omega = \sin(\theta)d\phi d\theta$ in Figure 1. Here we use dimensionless coordinates $x = F_\pi er$.

Formfactors correspondig to the calculated densities are presented on Figure 2 for the fixed values of F_π=109.45 MeV and e=4.138. They demonstrate the very characteristic behaviour of the formfactors for the distributions with the hole in the central region.

In according to our calculations the solitons from the same topological sector can have strongly different masses corresponding to their different structures. We also have to point out that a number of the states have shell like structure.

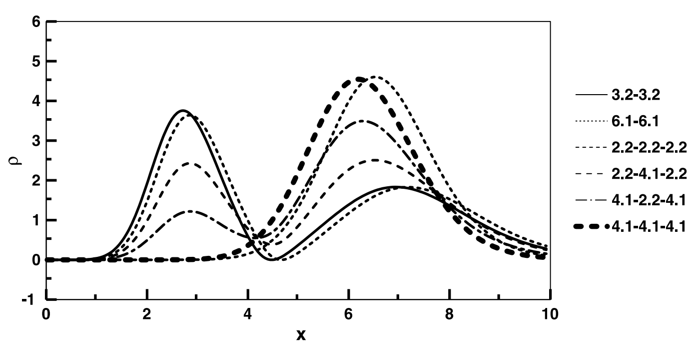

Figure 1. Baryon density of C^{12} shape isomers.

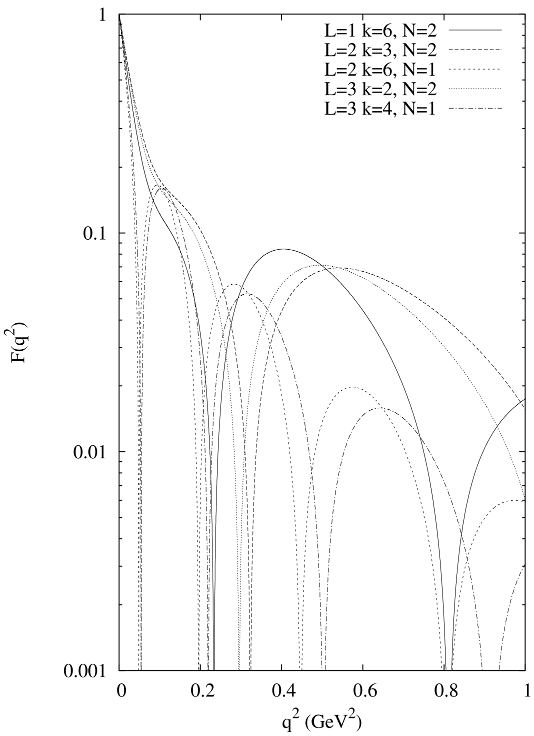

Figure 2. Formfactors of C^{12} shape isomers. Constants: F_π=109.45 MeV, e=4.138.

There are states which has $n_i \neq 1$. Such shape isomer can give different specific contribution to physics processes in light nuclei.

6 Conclusions

The axially symmetric solitons with baryon number B=12 have been investigated in the framework of the very general assumption about the form of the solution of the Skyrme model equations. The obtained solitons could be seen in nuclear reactions as isomer contributions in reactions involving C^{12}. Such isomers correspond to different form of baryon density distribution. We have to point out that used ansatz leads to stable solitons with B=12 and shell like structure of the baryon density distribution.

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References

[1] T. H. R. Skyrme (1962) *Nucl.Phys.* **31** 556.

[2] V. A. Nikolaev, V. Yu. Novozhilov, O. G. Tkachev (1994) *IL Nuovo Cimento 107(A)* **12** 2673.

[3] V. B. Kopeliovich, B. E. Stern (1987) *JETP Lett.* **45** 203.

[4] V. A. Nikolaev, O. G. Tkachev (2001) *Proceedings of the Twentieth International Workshop on Nuclear Theory, June 2001, Rila, Bulgaria* (Notre Dame Indiana) 167.

[5] V. A. Nikolaev, O. G. Tkachev (2001) *hep-ph/0109192* v**1**.

[6] C. J. Houghton, N. S. Manton and P. M. Sutcliffe, Nucl.Phys. **B510**, 507 (1998).

[7] Rychard A. Battye, Paul M. Sutcliffe (2001) *hep-th/0012215* v.**3**.

[8] E. Braaten, S. Townsend, L. Carson (1990) *Phys.Lett. B* **235** 147.

[9] V. A. Nikolaev, Yu. V. Chubov, O. G. Tkachev (2002) *hep-ph/0212014*.

[10] T. Nakano *et al.* (2003) hep-ex/0301020.

[11] H. Walliser and Kopeliovich (2003) *hep-ph/0304058*.

[12] H. Weigel (1998) *Eur. Phys. J. A* **2** 391.

[13] V. B. Kopeliovich, B. Schwesinger and B. E. Stern (1992) *Nucl. Phys. A* **549** 485.

[14] V. A. Nikolaev, R. M. Nikolaeva, O. G. Tkachev, E. Sorace, M. Tarlini, I. V. Musatov (1996) *Physics of Atomic Nuclei* **59** 2099.

[15] E. Guadanini (1984) *Nucl. Phys. B* **236** 35.

[16] G.Holzwarth (2000) *Nucl. Phys. A* **666** 24.

[17] K. E. Lassila, E. N. Magar, V. A. Nikolaev, V. Yu. Novozhilov, O. G. Tkachev (2000) *hep-ph/0005160*.

[18] H. Weigel, B. Schwesinger, G. Holzwarth (1986) *Phys. Lett. B* **168** 556.

[19] V. B. Kopeliovich, B. E. Stern (1987) *Pisma v ZhETF* **45** 165.

[20] J. J. M. Verbaarshot (1987) *Phys.Lett.* **B195** 235.

[21] N. S. Manton (1987) *Phys.Lett.* **B192** 177.

[22] I. R. Kozhevnikov, Yu. P. Rybakov, M. B. Fomin (1989) *T.M.F.* **75** 353.

[23] V. A. Nikolaev (1989) in *Particles and Nuclei* **vol. 20 N2**, p. 403 (Moscow, Atomizdat).

[24] V. A. Nikolaev, O. G. Tkachev (1989) *JINR Rapid Communications* **N1[34]-89** p. 28 Dubna; (1989) **N4[37]-89** p. 18 Dubna.

[25] V. A. Nikolaev, R. M. Nikolaeva, O. G. Tkachev (1992) *Sov. J. Part. Nucl.* **23(2)** 239.

[26] V. A. Nikolaev, O. G. Tkachev *JINR Preprint E4-89-56, Dubna (1989)*; TRIUMF (FEW BODY XII), TRI-89-2, p. F25, Vancouver, (1989) .

[27] T. Kurihara, H. Kanada, T. Otofuji, S. Saito (1989) *Progr. of Theor. Phys.* **81** p. 858.

[28] E. Sorace, M. Tarlini (1989) *Phys.Lett.* **B232** 154.

[29] V. A. Nikolaev, R. M. Nikolaeva, O. G. Tkachev (1992) *J.Phys.G: Nucl.Part.Phys* **18** 1149.

[30] V. A. Nikolaev, R. M. Nikolaeva, O. G. Tkachev (1993) *Journal of Nucl. Phys* **56(7)** 173.