Description of Parity-Doublet Splitting in Odd-A Nuclei

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Abstract. A collective model describing coherent quadrupole-octupole oscillations and rotations with a Coriolis coupling between the even-even core and the unpaired nucleon is applied to odd nuclei. The particle-core coupling provides a split parity-doublet structure of the spectrum. The formalism successfully reproduces the parity-doublet splitting in a wide range of odd-A nuclei. It provides model estimations for the angular momentum projection K on the intrinsic symmetry axis and the related intrinsic nuclear structure.

1 Introduction

The simultaneous manifestation of quadrupole and octupole degrees of freedom in atomic nuclei is associated with typical spectroscopic characteristics of nuclear collective motion [1]. In general the spectrum contains positive and negative parity levels, some of them being related with enhanced electric E1 and E3 transitions [2]. In even–even nuclei the even and odd angular momentum levels appear with positive and negative parities, respectively, due to the shape reflection asymmetry. In odd nuclei the structure of the spectrum is determined by the coupling between the reflection asymmetric even-even core and the motion of the unpaired particle. The combination of the core intrinsic parity with that of the particle to a “total intrinsic parity” provides a split parity doublet structure of the spectrum. The mutual disposition of the doublet counterparts up or down depends on the parity of the ground state as well as on the possible change in the intrinsic parity at some higher angular momenta. As in some cases, especially in heavy odd nuclei, the angular momentum of the ground state and/or its projection K are not unambiguously determined, the complicated structure of the spectrum represents a challenging subject for a study from both experimental and theoretical points of view. That is why various theoretical models, developed initially to explain the properties of quadrupole–octupole deformations in even-even nuclei, have been extended to describe the respective properties in odd nuclei [2, 3]. Recently a collective model for the quadrupole–octupole vibration–rotation motion in even-even nuclei has been proposed [4]. It was able to reproduce some basic characteristics as energy levels, parity shift and electric transition properties in nuclei with collective bands built on coupled quadrupole-octupole vibrations.

The purpose of the present work is to extend our model approach [4] to the case of odd nuclei and to apply it to collective spectra in the region of heavy odd nuclei. For this reason we consider the Coriolis coupling of the "soft" quadrupole–octupole oscillating core to the motion of the unpaired nucleon. The model scheme is developed respectively to take into account the total intrinsic parity of the system and to incorporate consequently the split parity doublet structure of the spectrum.

2 Quadrupole–Octupole Hamiltonian with Coriolis Interaction

We consider that the even–even core of an odd nucleus is allowed to oscillate with respect to the quadrupole β_2 and octupole β_3 axial deformation variables mixed through a centrifugal (rotation-vibration) interaction. The unpaired nucleon contributes to the collective motion of the total system through the Coriolis interaction. The collective Hamiltonian of the odd nucleus can then be taken in the form

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I) + h_{\text{coriol}}, \tag{1}$$

where

$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{\hat{I}^2 - {\hat{I}_z}^2}{2(d_2\beta_2^2 + d_3\beta_3^2)} \tag{2}$$

is the potential of quadrupole and octupole oscillations coupled through the collective angular momentum $\hat{I}$ and its third projection $\hat{I}_z$. B_2 and B_3 are the effective quadrupole and octupole mass parameters and C_2 and C_3 are the stiffness parameters for the respective oscillation modes. The last term in (1) represents the Coriolis interaction

$$h_{coriol} = -\frac{(\hat{I}_+\hat{j}_- + \hat{I}_-\hat{j}_+)}{2(d_2\beta_2^2 + d_3\beta_3^2)}, \tag{3}$$

where $\hat{I}_\pm = \hat{I}_x \pm i\hat{I}_y$ and $\hat{j}_\pm = \hat{j}_x \pm i\hat{j}_y$ are the spherical components of the total nuclear and the intrinsic (unpaired) particle angular momenta, respectively. After taking into account the action of the total angular momentum operators and the Coriolis term in the "particle+rotor" space , Eq. (3) can be superposed to the third term in Eq. (2). Then the terms $U(\beta_2,\beta_3,I)$ and h_{coriol} in Eq. (1) can be replaced by the potential

$$U(\beta_2,\beta_3,I,K,\pi a) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{X(I,K,\pi a)}{d_2\beta_2^2 + d_3\beta_3^2}, \tag{4}$$

where

$$X(I,K,\pi a) = \frac{1}{2}\left[d_0 + I(I+1) - K^2 + \pi a\delta_{K,\frac{1}{2}}(-1)^{I+1/2}\left(I+\frac{1}{2}\right)\right]. \tag{5}$$

The decoupling parameter a is defined between the unpaired particle states $a = \langle\chi_K|\hat{j}_+|\chi_{-K}\rangle$ (with $K = 1/2$). The sign of its contribution in the potential energy depends on the total intrinsic parity $\pi = \pm$ of the system (see below). The parameter d_0 characterizes the shape of the potential in the ground state.

The properties of the even core potential (2) have been studied in detail in [4]. The respective eigenvalue problem is solved by using polar variables $\beta_2 = \sqrt{d/d_2}\,\eta\cos\phi$, $\beta_3 = \sqrt{d/d_3}\,\eta\sin\phi$, with $d = (d_2 + d_3)/2$. By assuming a coherent interplay between the quadrupole and octupole modes the following correlation between the stiffness, inertia and mass parameters are imposed $d_2/(dC_2) = d_3/dC_3 = 1/C$, $d_2/dB_2 = d_3/dB_3 = 1/B$. As a result the potential reads

$$U_{I,K,\pi a}(\eta) = \frac{1}{2}C\eta^2 + \frac{X(I,K,\pi a)}{d\eta^2}, \tag{6}$$

and the model Hamiltonian obtains a simple form in the polar variables

$$H_{qo} = -\frac{\hbar^2}{2B}\left[\frac{\partial^2}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial}{\partial\eta} + \frac{1}{\eta^2}\frac{\partial^2}{\partial\phi^2}\right] + U_{I,K,\pi a}(\eta). \tag{7}$$

After separating the variables in the Schrödinger equation one has

$$\frac{\partial^2}{\partial\eta^2}\psi(\eta) + \frac{1}{\eta}\frac{\partial}{\partial\eta}\psi(\eta) + \frac{2B}{\hbar^2}\left[E - \frac{\hbar^2}{2B}\frac{k^2}{\eta^2} - U_{I,K,\pi a}(\eta)\right]\psi(\eta) = 0; \tag{8}$$

$$\frac{\partial^2}{\partial\phi^2}\varphi(\phi) + k^2\varphi(\phi) = 0, \tag{9}$$

where k is the separation quantum number. Eq. (8) is solved analytically for the potential (6) providing the energy spectrum

$$E_{n,k}(I,K,\pi a) = \hbar\omega\left[2n + 1 + \sqrt{k^2 + bX(I,K,\pi a)}\right], \tag{10}$$

where $\omega = \sqrt{C/B}$, $n = 0,\ 1,\ 2,\ \ldots$ and $b = \dfrac{2B}{\hbar^2 d}$. The respective eigenfunctions $\psi(\eta)$ are obtained in terms of the Laguerre polynomials, and their explicit form is given in [4]. Eq. (9) in the variable ϕ is solved under the boundary condition $\varphi(-\pi/2) = \varphi(\pi/2) = 0$, which provides two different solutions with positive, $\pi_\varphi = (+)$, and negative $\pi_\varphi = (-)$ parity, respectively

$$\varphi^+(\phi) = \sqrt{2/\pi}\cos(k\phi)\,, \qquad k = 1,\ 3,\ 5,\ \ldots\,; \tag{11}$$

$$\varphi^-(\phi) = \sqrt{2/\pi}\sin(k\phi)\,, \qquad k = 2,\ 4,\ 6,\ \ldots\,. \tag{12}$$

If the lowest energy of the motion in the variable ϕ is considered, one has $k = k_+ = 1$ for φ^+ and $k = k_- = 2$ for φ^-. The total wave function has the form

$$\Psi \sim \psi(\eta)\varphi^{\pm}(\phi)\sqrt{\frac{2I+1}{16\pi^2}}\left(D^I_{K\,M}(\theta)\chi_K \pm (-1)^{I+K}D^I_{-K\,M}(\theta)\chi_{-K}\right). \tag{13}$$

The total intrinsic parity is determined as

$$\pi = \pi_\varphi \cdot \pi_\chi , \tag{14}$$

where π_φ is the parity of the even-core oscillation function $\varphi^{\pm}(\phi)$ and π_χ is the parity of the unpaired particle function χ_K.

3 Description of Parity-Doublet Spectra

In the strong coupling limit between the core and the unpaired nucleon the spectrum has a parity doublet structure $I^{(\pi=\pm)} = I_0^{\pm},\ (I_0+1)^{\pm},\ (I_0+2)^{\pm},\ (I_0+3)^{\pm},\ \dots,$ where I_0 is the spin of the ground state. The parity of the wave function $\varphi^{\pm}(\phi)$ is determined by (14) as $\pi_\varphi = \pi \cdot \pi_\chi$. As a result the parity doublets are split with respect to the quantum number k in Eq. (10). The possible ways of splitting are shown in the table below.

π_χ	I^π	π_φ	k	shift
$(+)$	I^+	$(+)$	1	down
	I^-	$(-)$	2	up
$(-)$	I^+	$(-)$	2	up
	I^-	$(+)$	1	down

It is seen that the direction in which the positive and negative parity counterparts of the doublet are shifted to each other depends on the parity of the unpaired nucleon state. For the lowest energy part of the spectrum we may consider that the parity of the unpaired nucleon coincides with that of the ground state. Thus when the ground state parity is positive, the negative counterparts of the doublet are shifted up with respect to the positive ones, while for a negative-parity ground state the opposite situation is realized. At some higher angular momentum the intrinsic nucleon parity can be changed due to an alignment process in the core [5]. Then the unpaired nucleon parity π_χ changes in sign. The parity π_φ of the core oscillation function also changes due to the relation (14). As a result the sign of the parity splitting of the states with $I^{\pm}$ is changed according to the scheme in the above table. This effect can be easily identified in the structure of the experimental spectra. In the present model the change in the intrinsic nucleon parity at given angular momentum is taken into account phenomenologically by switching the rule for the quantum number k between the first ($\pi_\chi = +$) and the second ($\pi_\chi = -$) rows in the above table. This change can be taken into account microscopically through the deformed shell model consideration of the intrinsic single particle structure [5].

The above split parity doublet structure is observed in a wide range of heavy odd nuclei for which the present model scheme can take a place. We have applied the model to the parity-doublet spectra of the nuclei ^{151}Nd, ^{151,153}Pm, ^{153,155}Sm,

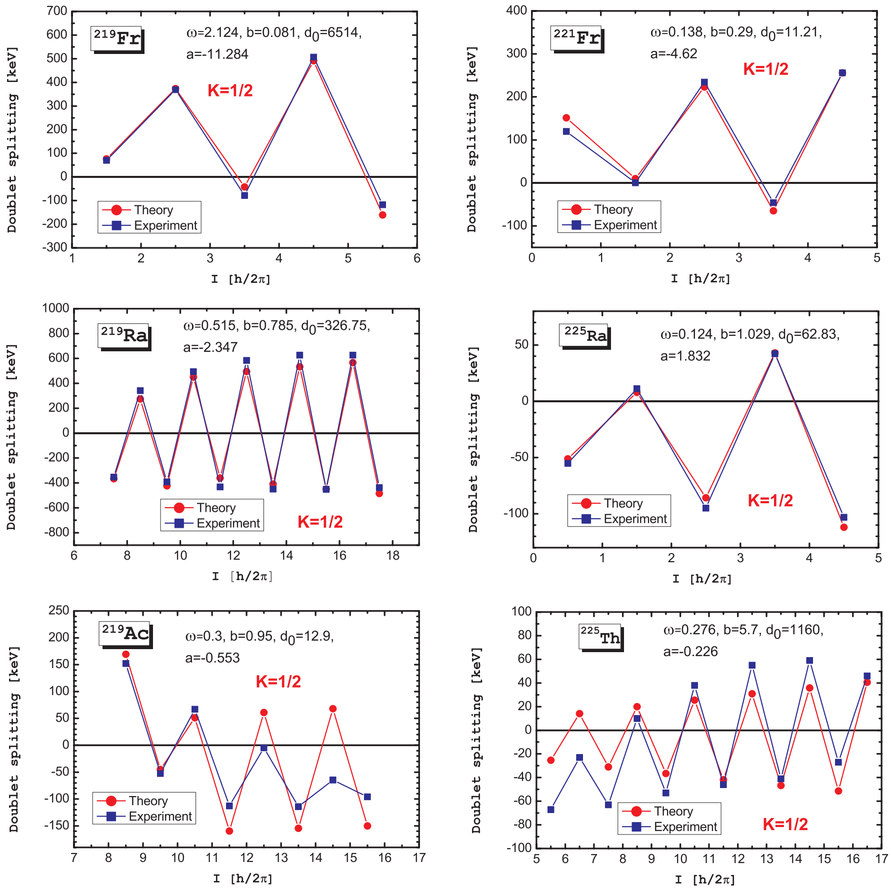

Figure **1.** Experimental and theoretical parity doublet splitting in ^{219,221}Fr, ^{219,225}Ra, ^{219}Ac and ^{225}Th.

^{157,159}Gd, $^{159-165}$Dy, $^{219-225}$Fr, $^{219-227}$Ra, $^{219,223-227}$Ac, ^{223,225,229,231}Th, $^{233-237}$U, $^{237-243}$Pu, $^{239-245}$Am, ^{245,247}Cm and ^{247,249}Bk. The theoretical energy levels are obtained by taking $\widetilde{E}_{0,k}(I) = E_{0,k}(I) - E_{0,k}(I_0)$ from Eq. (10), with I_0 being the ground-state angular momentum. The model parameters ω, b, d_0 and the decoupling parameters a (for the cases of $K = 1/2$) have been adjusted with respect to the experimental data. The third angular momentum projection K was taken as suggested in the experimental references, while for nuclei without available suggestions we have assumed $K = I_0$. In all considered nuclei the calculations successfully reproduce the energy levels with both positive and negative parities. As an example, in Tables 1–3 the theoretical and the experimental levels of the nuclei $^{219-225}$Fr, $^{219-227}$Ra, $^{219-227}$Ac, ^{223,225}Th, ^{237}U and ^{239}Pu are compared. We see in Table 1 that the level structures in ^{219}Fr (with $I_0 = (9/2)^-$ and $K = 1/2$) and ^{221}Fr (with $I_0 = (5/2)^-$ and $K = 1/2$), which are strongly perturbed by the Cori-

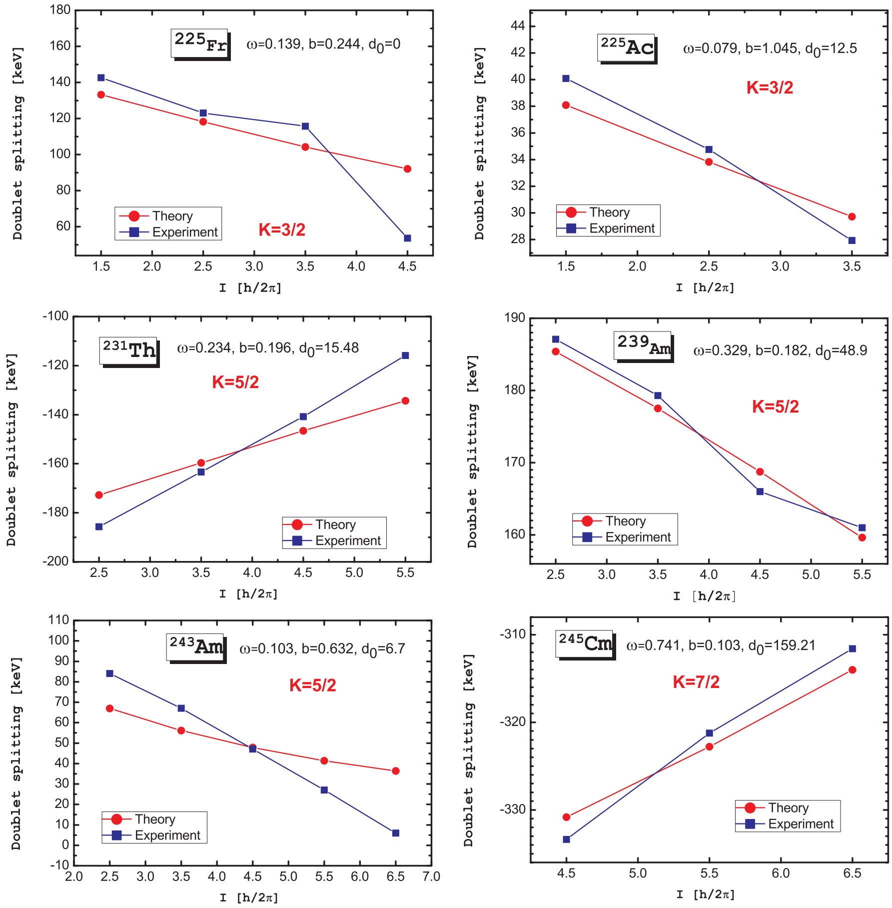

Figure **2.** The same as Figure 1, but for the nuclei ^{225}Fr, ^{225}Ac, ^{231}Th, ^{239,243}Am and ^{245}Cm.

olis interaction, are reproduced quite accurately. In Tables 2 and 3 the quality of the model description in $^{219-227}$Ac and in the longer level sequences in ^{223,225}Th, ^{237}U and ^{239}Pu is illustrated. In the cases of gaps in the sequences of experimental data the calculations provide respective model predictions.

The obtained numerical results allow an analysis of the parity doublet splitting, given by the quantity $\Delta E(I^{\pm}) = E(I^{+}) - E(I^{-})$, as a function of the angular momentum. In Figure 1 it is illustrated that the model calculations reproduce the staggering behavior of $\Delta E(I^{\pm})$ induced by the Coriolis interaction in the spectra of ^{219,221}Fr, ^{219,225}Ra, ^{219}Ac and ^{225}Th with $K = 1/2$. In Figure 2 we observe a smooth decrease of $\Delta E(I^{\pm})$ as a function of I for the nuclei ^{225}Fr, ^{225}Ac, ^{231}Th, ^{239,243}Am and ^{245}Cm where $K \neq 1/2$. In Figure 3 it is illustrated that the applied formalism provides a model estimation for the possible values of the quantum number K in ^{223}Ra. It is seen that the initially assumed value $K = 3/2$, Figure 3(a), does

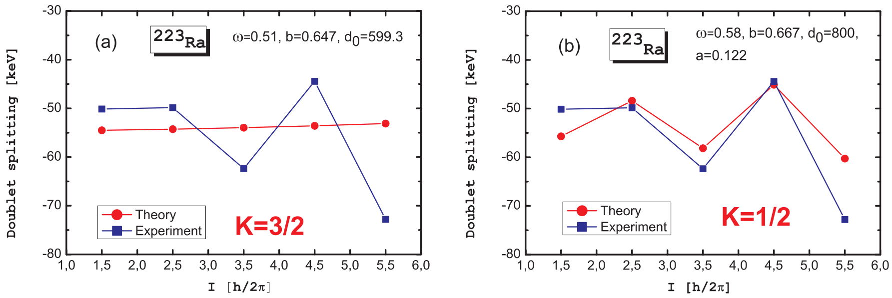

Figure **3.** Experimental and theoretical doublet-splitting in ^{223}Ra with (a) $K = 3/2$ and (b) $K = 1/2$.

Table **1.** Theoretical and experimental energy levels (in keV) of the positive and negative parity bands in $^{219-225}$Fr and $^{219-227}$Ra. The respective parameter values (in MeV), and the root mean square (RMS) deviations (in keV) are given in the first column. Data from [6].

Nucl/K params	I	$\pi=(+)$ th	exp	$\pi=(-)$ th	exp	Nucl/K params	I	$\pi=(+)$ th	exp	$\pi=(-)$ th	exp
^{219}Fr	1/2	310.6		55.9	81.0	^{223}Fr	3/2	139.4	134.5	0	0
$K=1/2$	3/2	229.6	210.4	153.1	139.8	$K=3/2$	5/2	171.2	172.0	32.8	12.9
$\omega=2.123$	5/2	390.7	384.3	17.4	15.0	$\omega=0.623$	7/2	215.3	219.5	78.4	82.1
$b=0.081$	7/2	201.8	191.3	244.1	269.2	$b=0.279$	9/2	271.3		136.2	
$d_0=6514$	9/2	491.5	506.5	0	0	$d_0=303$	11/2	338.8		205.8	
$a=-11.28$	11/2	195.0	216.0	355.8	333.5	RMS=8.7	13/2	417.2		286.6	
RMS=16.9	13/2	419.4		199.5							
^{221}Fr	1/2	160.4	145.8	8.9	26.0	^{225}Fr	3/2	133.2	142.6	0	0
$K=1/2$	3/2	114.4	99.9	104.1	99.6	$K=3/2$	5/2	153.2	151.6	35.0	28.5
$\omega=0.138$	5/2	223.1	234.5	0	0	$\omega=0.139$	7/2	179.2	198.2	75.0	82.5
$b=0.29$	7/2	126.7	150.0	191.5	195.8	$b=0.244$	9/2	210.0	181.6	117.9	128.1
$d_0=11.2$	9/2	300.5	294.8	44.5	38.5	$d_0=0$	11/2	244.3		162.5	
$a=-4.62$	11/2	170.9		286.0		RMS=13.5	13/2	281.4		208.1	
RMS=12.3	13/2	386.8		119.9							
^{219}Ra	7/2	0	0	228.5							
$K=1/2$	9/2	259.8		124.7		^{223}Ra	3/2	0	0	54.5	50.1
$\omega=0.515$	11/2	133.4		434.5		$K=3/2$	5/2	29.4	29.9	83.7	79.7
$b=0.785$	13/2	497.5	475.2	289.1		$\omega=0.510$	7/2	70.4	61.4	124.4	123.8
$d_0=326$	15/2	331.1	251.1	697.6	604.1	$b=0.647$	9/2	122.7	130.1	176.3	174.6
$a=-2.3$	17/2	789.2	853.5	514.5	512.4	$d_0=599$	11/2	186.2	174.6	239.3	247.4
RMS=37.1	19/2	587.1	546.0	1011.0	937.7	RMS=5.9	13/2	260.5		313.1	316.0
	21/2	1184.5	1245.9	735.0	751.3						
	23/2	953.2	893.2	1313.4	1325.0	^{225}Ra					
	25/2	1561.5	1638.2	1066.2	1053.3		1/2	0	0	51.1	55.2
	27/2	1303.4	1288.3	1711.3	1738.2	$K=1/2$	3/2	45.9	42.8	38.0	31.6
	29/2	1973.3	2038.8	1439.4	1411.3	$\omega=0.124$	5/2	23.8	25.4	109.5	120.4
	31/2	1692.3	1701.6	2141.4	2152.9	$b=1.029$	7/2	123.8	111.6	80.9	69.4
	33/2	2414.9	2460.2	1848.4	1833.2	$d_0=62$	9/2	87.5	100.5	199.4	203.5
	35/2	2114.4	2130.3	2598.8	2568.1	$a=1.83$	11/2	229.4	226.9	158.7	
	37/2	2881.8		2287.9	2289.7	RMS=7.8	13/2	182.7		312.6	
	39/2	2564.8	2580.5	3079.3	3003.6						
	41/2	3370.2		2753.3	2767.9	^{227}Ra					
	43/2	3039.	3045.7	3579.5			3/2	0	0	87.5	90
	45/2	3876.7		3240.5	3272.7	$K=3/2$	5/2	29.1	25.8	104.7	101.9
	47/2	3533.6	3522.6	4096.3		$\omega=0.093$	7/2	61.4	64.1	126.6	
	49/2	4398.9		3746.4	3793.4	$b=0.32$	9/2	95.4		152.0	
	51/2	4045.6		4627.3		$d_0=0.$	11/2	130.3		180.0	
	53/2	4934.3		4268.2	4345.8	RMS=19.8	13/2	165.7	186	209.862	139

Table **2.** The same as in Table 1 but for the nuclei $^{219-227}$Ac, ^{223}Th [6] and ^{225}Th (data from [7]).

Nucl/K		$\pi = (+)$		$\pi = (-)$		Nucl/K		$\pi = (+)$		$\pi = (-)$	
params	I	th	exp	th	exp	params	I	th	exp	th	exp
^{219}Ac,	9/2	188.0		0.	0.	^{225}Ac,	3/2	38.1	40.1	0.	0.
$K = 1/2,$	11/2	259.2		263.7	341.0	$K = 3/2$	5/2	67.3	64.7	33.5	29.9
$\omega = 0.3$	13/2	521.9	576.8	343.6	355.2	$\omega = 0.079$	7/2	103.9	105.1	74.2	77.1
$b = 0.950$	15/2	601.0		629.9	657.6	$b = 1.045$	9/2	145.8	145.0	119.6	
$d_0 = 12$	17/2	883.9	866.6	714.4	714.6	$d_0 = 12.54$	11/2	191.3		168.2	
$a = -0.56$	19/2	967.4	965.3	1012.6	1017.7	RMS=2.2	13/2	239.4		218.8	
RMS=38.2	21/2	1207.7	1183.0	1156.4	1116.0						
	23/2	1295.6	1301.0	1455.0	1413.7	^{227}Ac,	3/2	25.0	27.4	0.000	0.
	25/2	1602.9	1547.0	1542.0	1551.7	$K = 3/2,$	5/2	52.1	46.4	27.398	30.0
	27/2	1692.1	1698.8	1846.3	1813.0	$\omega = 0.191$	7/2	89.4	84.5	65.195	74.1
	29/2	2002.8	1959.3	1934.7	2023.8	$b = 1.318$	9/2	112.9	109.9	136.622	126.9
	31/2	2092.8	2149.3	2242.9	2245.2	$d_0 = 195$	11/2	169.8	187.3	193.021	198.7
						RMS=10.7	13/2	235.4	210.8	257.995	271.3
^{223}Ac,	5/2	72.2	64.6	0.	0.						
$K = 5/2$	7/2	110.5	110.1	38.8	42.4						
$\omega = 0.522$	9/2	159.4	167.5	88.3	90.7						
$b = 0.459$	11/2	218.5		148.1	141.0						
$d_0 = 499$	13/2	287.7		218.1							
RMS=5.3											
^{223}Th						^{225}Th	3/2	0	0	13.4	
$K = 5/2,$	5/2	0	0	36.5		$K = 1/2$	5/2	41.8	31	39.7	
$\omega = 0.446$	7/2	53.8	51.3	90.1		$\omega = 0.276$	7/2	78.7	68	98.1	
$b = 1.265$	9/2	122.5	118.9	158.5	180.5	$b = 5.7$	9/2	153.5	135	145.3	
$d_0 = 525.$	11/2	205.7	212.3	241.3	243.0	$d_0 = 1159.$	11/2	211.0	187	236.3	254
RMS=14.1	13/2	303.0	320.0	338.2	324.1	$a = -0.226$	13/2	318.0	303	303.9	326
	15/2	448.6	428.7	413.8	412.4	RMS=13.2	15/2	395.6	370	426.6	433
	17/2	572.0	569.6	537.7	547.3		17/2	533.8	530	513.9	520
	19/2	707.9	706.0	674.2	657.0		19/2	630.7	615	667.3	668
	21/2	855.8	858.1	822.6	838.1		21/2	799.0	807	773.5	769
	23/2	1015.0	1021.6	982.4	962.1		23/2	914.3	911	956.1	957
	25/2	1185.0	1185.4	1152.9	1179.4		25/2	1111.1	1127	1080.3	1072
	27/2	1365.1	1370.6	1333.7	1313.8		27/2	1243.9	1250	1290.6	1291
	29/2	1554.9	1551.7	1524.1	1558.4		29/2	1467.6	1485	1431.8	1426
	31/2	1753.7	1757.0	1723.6	1702.5		31/2	1617.0	1631	1668.3	1658
	33/2	1961.1	1953.0	1931.6			33/2	1866.0	1870	1825.4	1824
							35/2	2030.7	2047	2086.3	2057

not support the staggering behavior of the parity splitting observed in experimental data. Figure 3(b) illustrates that if a $K = 1/2$ value is assumed the staggering behavior of $\Delta E(I^{\pm})$ is reproduced. In such a way the presence of the staggering effect indicates a strong contribution of an intrinsic $K = 1/2$ configuration in the Coriolis coupling interaction.

4 Conclusion

In conclusion, the model scheme based on two-dimensional coherent quadrupole-octupole oscillations with a Coriolis coupling to a single particle suggests a collective mechanism for the appearance of split parity-doublet structures in odd-A nuclei. The model reproduces split parity-doublet spectra in a wide range of nuclei with a good accuracy and provides predictions for further not yet observed energy

Table **3.** The same as in Table 1 but for the nuclei ^{237}U and ^{239}Pu (data from [6]).

Nucl/K		$\pi = (+)$		$\pi = (-)$		Nucl/K		$\pi = (+)$		$\pi = (-)$	
params	I	th	exp	th	exp	params	I	th	exp	th	exp
^{237}U	1/2	0	0	375.2		^{239}Pu	1/2	0	0	349.2	469.8
$K = 1/2$,	3/2	20.3	11.4	393.2		$K = 1/2$	3/2	12.6	7.9	374.0	492.1
$\omega = 1.46$	5/2	51.5	56.3	425.4		$\omega = 1.58$	5/2	55.3	57.3	394.2	505.6
$b = 0.102$	7/2	98.4	82.9	467.1		$b = 0.107$	7/2	84.4	75.7	451.9	556.0
$d_0 = 618$	9/2	154.0	162.3	524.6		$d_0 = 791$	9/2	160.7	163.8	488.0	583.0
$a = 0.038$	11/2	226.8	204.0	589.2		$a = -0.3$	11/2	206.0	193.5	577.7	661.2
RMS=30	13/2	305.8	317.3	671.0		RMS=63	13/2	314.8	318.5	629.1	698.7
	15/2	403.1	375.1	757.7	846.4		15/2	375.6	359.2	749.8	806.4
	17/2	504.2	518.2	862.4	930.0		17/2	515.4	519.5	816.0	857.5
	19/2	624.6	592.0	969.8	1027.5		19/2	590.9	570.9	966.0	992.5
	21/2	746.2	762.8	1096.1	1131.0		21/2	760.0	764.7	1046.2	1058.1
	23/2	888.0	853.0	1222.8	1250.7		23/2	849.4	828.0	1224.0	1219.4
	25/2	1028.5	1048.7	1369.1	1376.1		25/2	1045.9	1053.1	1317.3	1300.9
	27/2	1189.7	1155.1	1513.4	1515.7		27/2	1148.1	1127.8	1521.0	1487.4
	29/2	1347.5	1372.2	1678.0	1662.3		29/2	1369.9	1381.5	1626.4	1584.9
	31/2	1526.3	1494.1	1838.5	1821.8		31/2	1484.0	1467.8	1854.1	1795.4
	33/2	1699.7	1729.2	2019.8	1987.7		33/2	1729.1	1748.5	1970.7	1908.9
	35/2	1894.4	1868.2	2194.9	2166.5		35/2	1854.0	1847.0	2220.4	2143.4
	37/2	2081.7	2117.2	2391.2	2349.7		37/2	2120.3	2152.2	2347.4	2272.0
	39/2	2290.6	2272.2	2579.5	2547.5		39/2	2255.2	2263.0	2617.2	2529.4
	41/2	2490.5	2530.1	2789.3	2746.7		41/2	2540.7	2590.1	2753.6	2672.0
	43/2	2712.1	2702.5	2989.4	2960.5		43/2	2684.6	2714.0	3041.8	2951.4
	45/2	2922.9	2963.8	3211.2	3174.7		45/2	2987.6	3060.1	3186.7	3108.0
	47/2	3155.8	3154.5	3422.0	3401.5		47/2	3139.5	3198.0	3491.6	3407.0
	49/2	3376.5	3415.8	3654.6	3630.0		49/2	3458.3	3559.1	3644.2	3578.0
	51/2	3619.5	3625.5	3874.9	3865.0		51/2	3617.5	3713.0	3964.2	3895.0
	53/2	3848.9	3886.8	4117.1	4105.0		53/2	3950.5	4087.1	4123.8	4080.0
	55/2	4100.7	4115.0	4345.8	4344.0		55/2	4116.2	4256.0	4457.4	4413.0
	57/2	4338.0	4377.0	4596.7	4597.0						
	59/2	4597.7		4832.9	4835.0						

levels. It describes the fine staggering behavior of the doublet-splitting as a function of the angular momentum and provides respective estimations for the intrinsic K-configurations associated with the collective energy bands of odd-A nuclei.

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