

# Proca Equations of a Massive Vector Boson Field

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**Abstract.** Proca equation is a relativistic wave equation for a massive spin-1 particle. The weak interaction is transmitted by such kind of vector bosons. Also vector fields are used to describe spin-1 mesons (e.g.  $\rho$  and  $\omega$  mesons). After a brief biography, the paper presents an introduction into relativistic field theory, including Klein-Gordon, Dirac, and Maxwell fields, allowing to understand this scientific achievement and some consequences for the theory of strong interactions as well as for Maxwell-Proca and Einstein-Proca theories. The modern approach of the nonzero photon mass and the superluminal radiation field are also mentioned.

## 1 Introduction and Short Biography

A. Proca, one of the greatest physicists of 20th century, was born in Bucharest on October 16, 1897. Biographical details can be found in a book edited by his son [1], where his publications are reproduced; see also [2] the web site [3] with many links and W. Pauli's biography [4]. He passed away in the same year with A. Einstein. His accomplishments in theoretical physics are following the developments of prominent physicists including Maxwell, Einstein and Dirac. As a young student he started to study and gave public talks on the Einstein's theory of relativity. Then his PhD thesis was dedicated to the Dirac's relativistic theory of electrons. Proca equation is a relativistic wave equation for a massive spin-1 particle. Some of the other relativistic wave eqs. are: Klein-Gordon eq. describing a massless or massive spin-0 particle; Dirac equation for a massive spin-1/2 particle; Maxwell eqs. for a massless spin-1 particle, etc. In field theory, the Proca action describes a massive spin-1 field of mass  $m$  in Minkowski spacetime. The field involved is a real vector field. Maxwell eqs. and Proca eqs. may be found in many textbooks [5] as important examples of relativistically invariant formulation of the field equations for a free field; see also [6, 7]. There are many publications mentioning even in the title the *Einstein-Proca* (e.g. [8–11]), *Proca* [12, 13], or *Maxwell-Chern-Simons-Proca* [14] theory.

Shortly after finishing in 1915 the *Gheorghe Lazar* high school in Bucharest Proca had to interrupt his studies because the first world war started. After the war (1918–22) he was student at the Polytechnical School (PS) specialized in Electromechanics. Next years he was employed by the Electrical Society, Câmpina, and in the same time he has also been assistant professor of Electricity, PS Bucharest. In 1923

he moved to France because he felt he “have something to say in Physics”. In two years brilliantly passed the exams and was graduated by the Science Faculty, Sorbonne University, Paris. As an experimentalist at the *Institut du Radium* he was very much appreciated by Marie Curie. Shortly she realized he is very interested by the theory and encouraged him to join the newly founded *Institut Henri Poincaré*. In 1930 Proca received the French citizenship and married the Romanian Marie Berthe Manolesco. He started to work as a *Boursier de Recherches* at his PhD thesis under L. de Broglie’s supervision. The thesis was defended in 1933 in front of a famous commission: Jean Perrin, L. Brillouin, and L. de Broglie. In 1934 he was one year with E. Schrödinger in Berlin and few months with N. Bohr in Copenhagen, where he met Heisenberg and Gamow.

From 1936 to 1941 he developed the theory of the massive vector (spin 1) boson fields governing the weak interaction and the motion of the spin-1 mesons; *Proca equations* refer to this kind of fields, and are frequently used in Field Theories. Prestigious scientists like Yukawa, Wentzel, Taketani, Sakata, Kemmer, Heitler, Fröhlich and Bhabha, promptly reacted in favour of his equations in 1938. W. Pauli [15, 16] mentioned Proca’s theory in his Nobel lecture. As a particular sign of his world-wide recognition one can mention the invitation to attend in 1939 the *Solvay Congress*.

During the second world war he was for a short time Chief Engineer of the French Radiobroadcasting Company. Then in 1943 he moved to Portugal where he gave Lectures at University of Portô. In 1943–45 was in United Kingdom invited by the Royal Society and British Admiralty to join the war effort.

After the war in 1946 he started in Paris the *Proca Seminar* with many prestigious invited speakers from France and abroad. This seminar contributed very much to the education of young French scientists willing to work in the field of particle physics. Unfortunately Proca’s attempts in 1949 and 1950 to get a chair of Physics at the Sorbonne University and Collège de France failed for obscure reasons. Nevertheless in 1950 he accepted to organize with P. Auger the Theoretical Physics Colloquium of CNRS and in 1951 to be the French delegate at the General Meeting of International Union of Physics. Starting with 1953 Proca had to fight with a laryngeal cancer until December 13, 1955 when he passed away.

## 2 Particles and Fields

The projection of angular momentum of a particle can be denoted by  $l_z \hbar$ , with  $l_z$  an integer  $-l \leq l_z \leq l$ . The intrinsic angular momentum,  $s$ , is called spin. According to the spin values the particles obey to the Fermi-Dirac or Bose-Einstein statistics. A fermion [ $s = (n + 1/2)\hbar$ ,  $n$  integer] is called spinor if  $s = \hbar/2$ , e.g. leptons ( $e, \nu, \mu, \tau$ ) or spinor-vector if  $s = (3/2)\hbar$ . A boson ( $s = n\hbar$ ) can be scalar ( $s = 0$ ), vector ( $s = \hbar$ , e.g.  $\omega, \rho$  mesons,  $m = 0$  photons, weak massive  $W^\pm, Z^0$  bosons,  $m = 0$  gluons), or tensor ( $s = 2\hbar$ , e.g. graviton). *Vector bosons play a central role as they are mediators of three (of the four) fundamental interactions: electromagnetic, weak, and strong. Tensor bosons are assumed to mediate the gravitation* (see the Table 1). A particle is a localized entity. A field

Table 1. The four fundamental interactions.

FORCE	RANGE	TRANSMITTED BY BOSONS
gravitational	long	graviton, massless, spin 2
electromagnetic	long	photon ( $\gamma$ ), massless, spin 1
weak interaction	short	$W^\pm, Z^0$ , heavy, spin 1
strong interaction	short	gluons ( $g$ ), massless, spin 1

is an assignment of a quantity to every point of space. Einstein’s special theory of relativity allows for the existence of scalar, vector, tensor, fields. Examples of relativistic fields: Klein-Gordon ( $s = 0$ ); Dirac ( $s = 1/2$ ); Proca ( $s = 1, m \neq 0$ ); Maxwell ( $s = 1, m = 0$ ); Rarita-Schwinger ( $s = 3/2$ ), and Gravitation ( $s = 2$ ). In particle physics, quantum field theories, cosmology and quantum gravity, the *natural units*,  $\hbar = c = 1$ , are used. The units of length, time and mass are expressed in GeV: 1 meter =  $5.07 \times 10^{15}$  GeV<sup>-1</sup>; 1 second =  $1.52 \times 10^{24}$  GeV<sup>-1</sup> and 1 kg =  $5.61 \times 10^{26}$  GeV. The Newton’s gravitational constant is given in terms of Plank’s mass  $G = M_{Pl}^{-2}$ , where  $M_{Pl} = 1.22 \times 10^{19}$  GeV. In particle physics gravity becomes important when energies or masses approach  $M_{Pl}$ .

*Geometrical units*,  $c = G = 1$ , are used in classical general relativity, and every quantity is expressed in units of length, e.g.  $\hbar = L_{Pl}^2$  where the Planck length  $L_{Pl} = 1.6 \times 10^{-33}$  cm. In gravitation (general relativity) quantum effects become important at length scales approaching  $L_{Pl}$ . We shall use in the following the natural units.

A contravariant and covariant four vector has four components

$$a^\mu = (a^0; a^1, a^2, a^3); \quad a_\mu = (a_0; a_1, a_2, a_3) \tag{1}$$

e.g.  $x^\mu = (t; x, y, z) = (x^0; x^1, x^2, x^3), p^\mu = (E; \vec{p})$ .

The metric tensor (covariant components)

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \tag{2}$$

For the Lorentz metric the contravariant form  $g^{\mu\nu} = g_{\mu\nu}$ . One can change a contravariant 4-vector into a covariant one by using the relationship  $a_\mu = g_{\mu\nu} a^\nu$ . One assumes the Einstein’s convention: summ over repeated indices.

The scalar product is  $a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - \vec{a} \cdot \vec{b}$ . The derivatives  $\partial^\mu = (\partial^0; -\nabla)$ ;  $\partial_\mu = (\partial^0; \nabla)$   $\partial_\mu \partial^\mu = \partial_0^2 - \nabla^2 = \partial^\mu \partial_\mu = \square$ .  $\nabla^2$  is the Laplacian and  $\square = \left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right)$  is the d’Alembertian.

For a conservative force,  $\vec{F}$ , there is a potential  $V$  such that  $\vec{F} = -\nabla V$ . The Lagrangian in terms of the generalized coordinates  $q_i$  and velocities  $\dot{q}_i$  is given by  $L = T - V$ , where  $T$  is the kinetic energy. According to the Hamilton’s principle, the classical action developed during the time interval  $t_2 - t_1$

$$S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t) dt \quad (3)$$

has a stationary value ( $\delta S = 0$ ) for the dynamical path of motion. The Euler-Lagrange eq.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 \quad (4)$$

is obtained by performing a calculus of variations which leads to the Newton's second law in classical mechanics of a point particle.

In both special and general relativity one seeks covariant eqs. in which space and time are given equal status. The above defined action is not covariant. In field theory we replace the  $q_i$  by a field  $\varphi(x)$  where  $x \equiv (t, \vec{x})$ . A covariant form of action involves the Lagrangian density,  $\mathcal{L} = \mathcal{L}(\varphi, \partial_\mu \varphi)$ , which is a functional integrated over all space-time

$$\mathcal{S}[\varphi_i] = \int \mathcal{L}[\varphi_i, \partial_\mu \varphi] d^4x \quad (5)$$

The Lagrangian is the spatial integral of the density.

The least action principle leads to the Euler-Lagrange equation

$$\frac{\delta}{\delta \varphi} \mathcal{S} = -\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) + \frac{\partial \mathcal{L}}{\partial \varphi} = 0 \quad (6)$$

The quantum mechanics can describe a system with a fixed number of particles in terms of a many-body wave function. The relativistic quantum field theory with creation and annihilation operators was developed in order to include processes (e.g.  $n \rightarrow p + e + \bar{\nu}_e$  or  $e^+ e^- \rightarrow 2\gamma$ ) in which the number of particles is not conserved, and to describe the conversion of mass into energy and vice versa.

## 2.1 Klein-Gordon Field

For a massive ( $m \neq 0$ ) scalar (spin 0) and neutral (charge zero) field, the Lagrangian density (function of the fields  $\phi$  and their  $x, y, z, t$  derivatives) is

$$\mathcal{L} = (1/2)[(\partial_\mu \phi)(\partial^\mu \phi) - m^2 \phi^2] \quad (7)$$

The Euler-Lagrange formula requires

$$(\square + m^2)\phi = 0 \quad (8)$$

i.e. the Klein-Gordon equation. It was quantized by Pauli and Weisskopf in 1934. Relativistic wave equations are invariant under Lorentz transformations, expressing the invariance of the element of 4-vector length,  $ds^2 = dt^2 - (dx^2 + dy^2 + dz^2)$ . The Klein-Gordon equation was historically rejected as a fundamental quantum equation because it predicted negative probability density.

For free particles with an internal degree of freedom (e.g. electric charge) the real valued field is replaced by complex fields  $\phi^* \neq \phi$ , hence nonhermitean field operators  $\hat{\phi}^\dagger \neq \hat{\phi}$ .

## 2.2 Dirac Field

Dirac was looking for an equation linear in  $E$  or in  $\partial/\partial t$ . For a massive spinor (spin 1/2) field the Lagrangian density is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi ; \quad \bar{\psi} = \psi^*\gamma^0 \quad \text{Dirac adjoint} \quad (9)$$

where the four  $4 \times 4$  Dirac matrices  $\gamma^\mu$  ( $\mu = 0, 1, 2, 3$ ) satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \quad (10)$$

The corresponding equation of motion is the Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 ; \quad i(\partial_\mu\bar{\psi})\gamma^\mu + m\bar{\psi} = 0 \quad (11)$$

One set of  $4 \times 4$  matrices is

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad (12)$$

where one has  $2 \times 2$  matrices: identity  $I$ , zero and Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (13)$$

Quantization of the Dirac field is achieved by replacing the spinors by field operators and using the Jordan and Wigner quantization rules. Heisenberg's eq. of motion for the field operator  $\hat{\psi}(\vec{x}, t)$  reads

$$i\frac{\partial}{\partial t}\hat{\psi}(\vec{x}, t) = [\hat{\psi}(\vec{x}, t), \hat{H}] \quad (14)$$

There are both positive and negative eigenvalues in the energy spectrum. The later are problematic in view of Einstein's energy of a particle at rest  $E = mc^2$ . Dirac's way out of the negative energy catastrophe was to postulate a Fermi sea of antiparticles. This genial assumption was not taken seriously until the positron was discovered in 1932 by Anderson.

## 2.3 Maxwell Field

In classical field theory the differential form of Maxwell eqs. are given by Gauss's, Ampère's and Faraday's laws plus Maxwell's extensions. For homogeneous materials:

$$\nabla \cdot \vec{D} = \rho ; \quad \nabla \cdot \vec{B} = 0 ; \quad \vec{D} = \epsilon\vec{E} ; \quad \vec{B} = \mu\vec{H} \quad (15)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ; \quad \nabla \times \vec{B}/\mu = \vec{j} + \epsilon\frac{\partial \vec{E}}{\partial t} \quad (16)$$

Photons are assumed to be massless (forces of infinite range). Electrodynamics is gauge invariant ( $\varphi$  and  $\vec{A}$  are not unique): Maxwell eqs. do not change under gauge transformation [with  $\chi = \chi(\vec{r}, t)$  an arbitrary differentiable function]

$$\vec{A} \rightarrow \vec{A} + \nabla\chi(\vec{r}, t); \quad \varphi \rightarrow \varphi - \frac{\partial\chi}{\partial t} \quad (17)$$

The 4-potential and current density are  $A^\mu = (\varphi; \vec{A}) = (A^0; \vec{A})$ ,  $j^\mu = (\rho; \vec{j})$ . Scalar,  $\varphi$ , and vector,  $\vec{A}$ , potentials are introduced via  $\vec{E} = -\nabla\varphi - \partial\vec{A}/\partial t$ ,  $\vec{B} = \nabla \times \vec{A}$ . The four vector potential  $A^\mu = (\varphi, \vec{A})$ ;  $g_{\mu\nu}A^\mu A^\nu = \varphi^2 - \vec{A}^2$ . The antisymmetric field-strength tensor  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$  with components  $F^{0i} = \partial^0 A^i - \partial^i A^0 = -E^i$  and  $F^{ij} = \partial^i A^j - \partial^j A^i = -\epsilon^{ijk} B^k$ . The Levi-Civita symbol  $\epsilon^{ijk}$  is antisymmetric under exchange of any two indices. The Lagrangian density is

$$\mathcal{L} = (E^2 - B^2)/2 - \rho V + \vec{j}\vec{A} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu \quad (18)$$

The corresponding Euler-Lagrange eqs. are Maxwell's eqs. Kaluza and Klein attempted to unify the gravitation and electromagnetic theories by extending general relativity in 5 dimensions.

## 2.4 Proca (Massive Vector Boson) Field

Proca extended the Maxwell eqs. in quantum field theory. In 1934 for massive positively charged particles with spin there were two alternatives: Dirac eqs. having a spectrum of positive and negative energies, a positive charge and a finite spin ( $\pm E, +q, s \neq 0$ ) or Pauli-Weisskopf based on Klein-Gordon eqs. ( $\pm E, \pm q, s = 0$ ). Proca worked out new equations which would allow for positive and negative energies, both signs of the charge, and a finite spin ( $\pm E, \pm q, s \neq 0$ ). For a massive vector boson (spin 1) field the Proca equation [17]

$$\square A^\nu - \partial^\nu(\partial_\mu A^\mu) + m^2 A^\nu = j^\nu \quad (19)$$

is obtained as a Euler-Lagrange eq. emerging from the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu - j_\mu A^\mu \quad (20)$$

after expressing the field-strength tensor,  $F^{\mu\nu}$ , in terms of the four potential  $A^\mu$ . The Maxwell field is a massless ( $m = 0$ ) Proca field. In contrast to the Maxwell field the Lorentz condition is fulfilled by Proca field. In his Nobel lecture [15], W. Pauli noted: "The simplest cases of one-valued fields are the scalar field and a field consisting of a four-vector and an antisymmetric tensor like the potentials and field strengths in Maxwell's theory. While the scalar field is simply fulfilling the usual wave equation of the second order in which the term proportional to  $\mu^2$  has to be included, the other field has to fulfill equations due to Proca which are generalization of Maxwell's equations which become in the particular case  $\mu = 0$ ."

Assuming  $m \neq 0$  one has  $\partial_\nu A^\mu = (1/m^2)\partial_\nu j^\nu$ . If the source current is conserved ( $\partial_\nu j^\nu = 0$ ) or if there are no sources ( $j^\nu = 0$ ) it follows that  $\partial_\nu A^\nu = 0$ . The field eq. gets simplified  $(\square + m^2)A^\nu = 0$  for free particles, leading to four Klein-Gordon eqs. for projections.

After Yukawa's [18] hypothesis of a particle mediating the nuclear interaction this particle was initially called mesotron, or alternatively the Proca particle (see for example the ref. [19]).

### 3 Yukawa and the Strong Interaction (Color Force)

Hideki Yukawa (1907-1981) was born in Tokyo as a third son of Takuji Ogawa. Genyo Yukawa adopted<sup>1</sup> him when he married Sumiko Yukawa in 1932. Hideki won the Nobel Prize in 1949 *for his prediction (in 1934) of the existence of mesons on the basis of theoretical work on nuclear forces* [18]. His potential (of a Debye type)

$$g^2 \frac{e^{-\lambda r}}{r} \quad (21)$$

can explain the short range of the strong interaction. At that time there was no link between the quantum theory of fields and nuclear theory, except the Fermi's  $\beta$ -decay theory. When he was only 27 years old he predicted the existence of new particles now called pions. Yukawa calculated a mass  $\sim 200m_e$  ( $m_e$  is the electron mass). By analogy with photons mediating the electromagnetic interaction he assumed the nuclear forces, acting between nucleons, are mediated by such bosons. As he suggested, the study of cosmic rays gave the first experimental evidence of the new particles. Yukawa employed a scalar field equation. The right vector field was introduced by Proca (see e.g. [16]: "This case holds the center of current interest since Yukawa supposed the meson to have the spin 1 in order to explain the spin dependence of the force between proton and neutron. The theory for this case has been given by Proca"). At present we know that the strong interaction acting between quarks is mediated by gluons.

Yukawa was ahead of his time and found the key to the problem of nuclear forces. His paper was unnoticed until 1937 when Anderson announced his discovery in cosmic rays of a particle with a mass similar to that required by Yukawa's theory. Soon it was clear that Anderson's "mesotron" (now the muon or  $\mu$ -meson) did not possess the right properties; it is in fact a lepton. Cecil Powell (Nobel prize in 1950) discovered in 1947 the  $\pi$ -mesons (pions) in cosmic rays.

The name meson means middle-weight between electron and nucleon. Cosmic rays contained two intermediate mass particles: muon and pion. The muon is a lepton (a heavy counterpart to the electron) and not a meson (although it is still called  $\mu$ -meson for historical reasons), but the pion was a true meson of the kind predicted by Yukawa. Around 1970 there were many theories [20] attempting to explain the

<sup>1</sup> Adoption was a common practice in Japan in a family without son.

nuclear interaction. Presently according to quantum chromodynamics, the strong interaction, mediated by massless gluons, affects only quarks and antiquarks; it binds quarks to form hadrons (including proton and neutron). There are 8 types of gluons.

*The theory of the massive vector bosons with spin 1 was developed by A. Proca. Such bosons ( $W^\pm$  and  $Z^0$  bosons) are mediators of the weak interaction. Proca's equations are also used to describe spin 1 mesons, e.g.  $\rho$  and  $\omega$  mesons.*

#### 4 Nonzero Photon Mass and the Superluminal Radiation Field

The effects of a nonzero photon rest mass can be incorporated into electromagnetism through the Proca eqs. The massive electromagnetic field is described by Maxwell-Proca eqs.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \mu_\gamma^2 \varphi; \quad \nabla \cdot \vec{B} = 0 \quad (22)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \quad \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} - \mu_\gamma^2 \vec{A} \quad (23)$$

where  $\mu_\gamma^{-1} = \hbar/(m_\gamma c)$  is the Compton wave-length of a photon with mass  $m_\gamma$ . Implications of a massive photon: variation of  $c$ ; longitudinal electromagnetic radiation and gravitational deflection; possibility of charged black holes; the existence of magnetic monopoles [21]; modification of the standard model [22], etc. An upper limit for the photon rest mass [23] is  $m_\gamma \leq 1 \times 10^{-49} \text{ g} \equiv 6 \times 10^{-17} \text{ eV}$ .

The concept of a nonzero rest mass graviton may be defined [24] in two ways: phenomenologically, by introducing of a mass term in the linear Lagrangian density, as in Proca electrodynamics, and self-consistently, by solving Einstein's equations in the conformally flat case. The rest mass of the graviton was given in terms of the three fundamental constants: gravitational, Planck, and light velocity. The Einstein-Proca equations, describing a spin-1 massive vector field in general relativity, have been studied [8] in the static spherically-symmetric case. It was shown [25] that a special case of the metric-affine gauge theory of gravity is effectively equivalent to the coupled Einstein-Proca theory.

Einstein-Proca field eqs. are frequently discussed in connections with dark matter gravitational interactions [26]. At the level of string theories there are hints that non-Riemannian models, such as Einstein-Proca-Weyl theories [9] may be used to account for the dark matter. Other developments: Proca-Wightman field [27]; Maxwell-Chern-Simons-Proca model [28].

Superluminal (faster than light) particles, *tachyons*, with an imaginary mass of the order of  $m_e/238$ , can be described by a real Proca field with a negative mass square [29]. They could be generated in storage rings, jovian magnetosphere, and supernova remnants.

In conclusion, A. Proca lived in a period of great discoveries and development of quantum field theories to which he contributed in an essential way. After about eighty years of use, Proca equation of the vector boson fields remains one of the



basic relativistic wave equation. The weak interaction is transmitted by such kind of vector bosons. Also vector fields are used to describe spin-1 mesons such as  $\rho$  and  $\omega$  mesons.

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