Possible chirality in the doubly-odd $^{198}$Tl nucleus

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Abstract. Excited states in the odd-odd $^{198}$Tl nucleus were populated in the $^{197}$Au($\alpha$,3n) reaction at a beam energy of 40 MeV. These states were then studied using $\gamma$-ray spectroscopy carried out at iThemba LABS, and electron-$\gamma$ spectroscopy performed at Orsay. The level scheme of $^{198}$Tl was extended and a weak side band feeding into the yrast $\pi h_9/2 \otimes i_{13/2}$ band was observed. These two bands look somewhat similar to the pairs of bands in several A $\sim$ 130 nuclei, which are considered as possible candidates for chiral doublets. The two bands in $^{198}$Tl have relative excitation energy of about 500 keV, and definite difference in the staggering pattern. However their other properties, including alignments, moments of inertia, and B(M1)/B(E2) transition probabilities are very similar. The interpretations of the pair of bands in terms of a chiral doublet is discussed. It is possible that the displacement in excitation energy and the different staggering patterns occur as a result of the moderate nuclear quadrupole deformation of $^{198}$Tl. Alternative interpretations are also investigated.

1 Introduction

Chiral systems are known in many different fields of physics, but are not yet confirmed beyond doubt for nuclear matter. Thus, the possibility that nuclear chirality is generating the doublet bands observed in some nuclei in the A $\sim$ 130 and 100 mass regions is of great interest. For an ideal chiral system two degenerate rotational bands are expected with identical properties. The difficulty in proving (or disproving) nuclear chirality is caused by difficulties in interpreting differences in some of the properties of the observed partner bands (e.g. often the bands do not have the same staggering pattern, or they are not degenerate and do not reach energy degeneracy, etc.). It is not often clear if the observed differences rule out a chiral interpretation for the partner bands, or if they are a consequence of other nuclear properties. Thus, it is important to obtain extensive experimental data for nuclei that may show chirality, and preferably for different nuclear configurations and in different mass regions. In this contribution we report on the observation of a pair of bands in $^{198}$Tl, which are associated with a particle-hole configuration suitable for a chiral system. These are the first such bands to be observed in the Tl isotopes. We also include a discussion on possible fingerprints for chirality and investigate different interpretations of the observed pair of bands in $^{198}$Tl.
A structure is called chiral if its image in a mirror plane cannot be brought to coincide with itself. A possible way to construct a chiral system in a nucleus is by coupling three angular momenta aligned in mutually perpendicular directions with total angular momentum directed "out of plane", i.e. out of the three planes constructed by the other two angular momenta [1]. Actually as long as the three angular momenta have components along the three major nuclear axes the total angular momentum is directed out of plane, and the system is thus chiral. The symmetries of such a system imply a pair of two degenerate bands, each band with a spin sequence of $\Delta I = 1$, and no signature splitting [1, 2]. Both bands should be degenerate in excitation energy or rather reach degeneracy at higher spins [1], and have the same properties, i.e. quasi-particle alignments, moments of inertia, B(M1) and B(E2) transition probabilities. One way of constructing a chiral system for a deformed rotating nucleus is by coupling the angular momentum of a particle (aligned along the short nuclear axis), the angular momentum of a hole (aligned along the long nuclear axis) and the collective rotation angular momentum (if it is aligned along the third nuclear axis) [1]. In order to have the rotational angular momentum perpendicular to the particle and hole angular momenta, one needs to assume a stable triaxial shape. In that case a rotation around the intermediate nuclear axis becomes favourable, because the moment of inertia along this axis, $I_\text{i}$, becomes larger than those along the short ($I_\text{s}$) and long ($I_\text{l}$) axes. For maximal non-axiality with $\gamma = 30^\circ$, and for an irrotational flow moments of inertia, $I_\text{i} = 4I_\text{s} = 4I_\text{l}$.

At rotational frequencies for which the Coriolis and centrifugal effects become important, staggering in the excitation energies of odd- and even-spin levels may occur. It is expected that since the energy due to Coriolis and centrifugal interactions is proportional to the scalar product of the total angular momentum $\vec{I}$ and the single particle angular momentum $\vec{j}$, $E_C = \vec{I} \cdot \vec{j}/(2\mathcal{I}) = (\vec{R} \cdot \vec{j} + \vec{j} \cdot \vec{j})/(2\mathcal{I})$, the resulting odd-even staggering in the level energies will be reduced for an aplanar system in comparison with a planar one. If the moment of inertia of the nucleus $\mathcal{I}$ does not change as a function of the spin $I$, the staggering should also be independent of $I$.

Reaching energy degeneracy by the chiral partner bands is considered an important fingerprint of chiral symmetry. However it should be noted that energy degeneracy is not necessarily a sufficient proof for nuclear chirality, because it can occur for different reasons. For instance the partner bands in $^{134}\text{Pr}$ (regarded at the time as the best candidate for a chiral symmetry system), do reach energy degeneracy at higher excitation energy, but their quasi-particle alignments [3, 4], and B(M1) and B(E2) transition probabilities [5] are different, which rules out the static chirality scenario. A good candidate for a chiral system was found in $^{128}\text{Cs}$, where the partner bands have similar quasi-particle alignments, and B(M1) and B(E2) transition probabilities [6], although they do not reach energy degeneracy even at high spins.

Further studies on the energy degeneracy as a fingerprint of chiral systems were performed using particle plus rotor calculations describing a quasi-proton and a quasi-neutron coupled to a triaxial core [7]. It was shown that the relative excitation energy of the side band depends on the proton and neutron Fermi levels within their respective shells. Energy degeneracy was reached only if the quasi-proton and
quasi-neutron occupy orbitals at the bottom and the top of the high-j shells. When the neutron Fermi level was moved inside the high-j shell a large energy displacement between the partner bands was found [7].

Energy degeneracy is also strongly dependent on the nuclear shape. Calculations of the excitation energy as a function of the quadrupole deformation $\beta_2$ and non-axiality $\gamma$ were performed using phenomenological core-particle-hole coupling model [8–10]. Within the same theoretical framework the expectation value of the orientation operator $\sigma = (\vec{j}_\pi \times \vec{j}_\nu) \cdot \vec{R}$, was calculated [8, 9]. Classically the expectation value of this operator (when normalized) is equal to 0 for planar vectors, and to 1 for mutually perpendicular vectors. Any value greater than 0 corresponds to a chiral system, i.e. a system with total angular momentum directed out of plane. These calculations showed that for large deformations and for maximum non-axiality ($\gamma=30^\circ$) energy degeneracy of the partner bands is reached, and the expectation value of the orientation operator has its highest value. It should be noted that for medium quadrupole deformation, or not so large non-axiality, substantial energy displacement occurs. In particular for $\beta_2 \leq 0.2$ energy displacement as large as 500 keV can occur and also considerable energy staggering, much stronger for the yrast band, develops [8, 9]. Nevertheless the expectation value of the orientation operator is about 0.3, showing thus an aplanar orientation of the total angular momentum [8, 9]. Therefore it seems that for moderate nuclear deformations aplanar angular momentum orientation (and thus a chiral system) is associated with energy displacement of the side band and energy staggering, in particular for the yrast band.

Energy displacement between the partner bands may also occur as a result of "chiral vibrations", i.e. tunneling between the left- and right-handed chiral systems as suggested in [10].

A characteristic staggering in the inband and out-of-band B(M1)/B(E2) ratios was proposed as another fingerprint of chirality [11]. This staggering was derived for a chiral system when both the odd proton and neutron occupy $h_{11/2}$ orbitals and for maximum non-axiality of the nuclear shape with $\gamma=30^\circ$. Recent calculations within a framework of the quasi-particle-plus-rotor model showed that the staggering in the in-band and out-of-band B(M1) probabilities depends strongly on the nuclear deformation and on the Fermi level, and is not a feature of all chiral systems [7].

Thus, chirality seems to be associated with the following features: (i) energy degeneracy and no staggering in the partner bands in well deformed nuclei with stable $\gamma = 30^\circ$ deformation and for particle and hole Fermi levels positioned at the bottom and top of the high-j shells. For moderately deformed nuclei or for not so large non-axiality or for Fermi levels positioned inside the high-j shell, energy displacement occurs and staggering in the level energies, particularly for the yrast band, can be expected. (ii) Similarity of the properties of the partner bands in terms of quasiparticle alignments, moments of inertia, B(M1) and B(E2) transitions probabilities, etc. for particle-hole configurations with Fermi levels at the bottom and top of the high-j shells. (iii) A characteristic staggering in the in-band and out-of-band B(M1) transition probabilities for specific cases. (iv) Reduced staggering in the levels energies in comparison with a "planar" system and independence of the staggering S(I)
on the nuclear spin I for constant moment of inertia. (v) It is important to stress that a non-axiality of the nuclear shape is a crucial requirement for a chiral system.

2 Experiments and data analysis

The excited $^{198}$Tl nuclei were produced with the same $^{197}$Au($\alpha$,3n)$^{198}$Tl reaction at a beam energy of 40 MeV in two complementary experiments. The first experiment was performed at the tandem accelerator laboratory at Orsay, and it was aimed at electron-$\gamma$ spectroscopy measurements. We used the electron spectrometer of Orsay [12], consisting of two magnetic lenses directing the internal conversion electrons towards two Si(Li) detectors. The two lenses were positioned at 90° and at 180° with respect to the beam direction, respectively. Due to problems with the focusing of the beam through the 180° lens, only the spectrometer at 90° was used. The magnetic field swept to focus electrons with different energies to the Si(Li) detectors. Eight large Ge detectors were positioned in the hemisphere opposite to the 90° lens. Seven BaF$_2$ detectors were also used as a filter. The target was a 0.2 mg/cm$^2$ thin self supporting foil of $^{197}$Au. Events were collected by two different data acquisition systems. The first one saved events on tape when two $\gamma$ rays or an electron and a $\gamma$ ray were detected in coincidence with the beam burst and a BaF$_2$ signal. The second one created direct electron and gamma spectra if the emitted electrons and gamma rays were in coincidence with the beam burst and a BaF$_2$ signal.

The data analysis involved a search for low-energy transitions in the electron-$\gamma$ and $\gamma$-$\gamma$ matrices and measurements of the internal conversion coefficients using mostly the direct electron- and $\gamma$-spectra. Multipolarities were then assigned to many new transitions.

The second experiment was performed at iThemba LABS, South Africa. A thick, 13mg/cm$^2$, foil of $^{197}$Au was used and the $\gamma$ rays were detected with the AFRODITE array [13,14], consisting of 8 clover and 6 LEPS detectors. The trigger condition required a coincidence of at least two clovers.

The $\gamma$ coincidences were studied using a $\gamma$-$\gamma$ matrix. In order to deduce multipolarities of the new transitions DCO ratios and for several transitions linear polarization anisotropies were measured.

3 Results

Electron and $\gamma$ spectra gated on the 391 keV $\gamma$ ray, (which de-excites the 8$^-$ band head level of the yrast band), are plotted in Fig. 1.

In conversion electron spectroscopy supression of delta electrons is crucial when searching for low-energy transitions. A new 72 keV transition was identified through its L, M and N internal conversion lines (as seen in Fig. 1) and placed at the bottom of the yrast band. Two new bands and several other transitions were also added to the known level scheme of $^{198}$Tl [15]. Links between the new bands and the yrast band proved that no other low-energy transitions belong to the yrast band. This
confirmed that the yrast band shows signature inversion. Based on DCO ratio, linear polarization and internal conversion measurements spins and parities were assigned to most of the new levels. A partial level scheme showing the extended yrast band and the new side band is plotted in Fig. 2.

4 Discussion

The new side band has the same spin and parity as the yrast band, and relative excitation energy of about 500 keV with respect to the yrast band (see Fig. 3). The configuration of the yrast band involves a high-K proton and a low-K neutron, $\pi h_{9/2} \otimes \nu h_{11/2}/2$. It seems no other two-particle configuration can fit the spin and parity of the side band. The intense links between the two bands also suggest similarities in their single-particle configurations. This nucleon configuration is favourable for a chiral system, because the proton particle occupies the lowest $h_{9/2}$ orbital (with $\Omega = 9/2$), while the neutron hole is fully aligned along the rotational axis. The measured quasi-particle alignments, kinematic moments of inertia and $B(M1)/B(E2)$ transition probability ratios are very similar for these two bands (shown in Fig. 4) and also support possible chirality. However, signature inversion
Figure 2. Partial level scheme showing the pair of bands in $^{198}$Tl is observed in the yrast band, while no signature splitting is shown in the side band (see Fig. 3). The difference in the staggering patterns and also the large displacement in the energies of the partner bands are inconsistent with the chiral scenario for well deformed nuclei with stable $\gamma = 30^\circ$ deformation. However, they do not rule out the possibility for chirality in the moderately deformed $^{198}$Tl nucleus (with quadrupole deformation of $\beta_2 \sim 0.13$).

Figure 3. Excitation energy (left panel) and staggering, $S(I)=|E(I)-E(I-1)|/2$ (right panel), in the yrast and side bands in $^{198}$Tl

The description of these bands in terms of chirality depends crucially on the non-axiality of the nuclear shape. The chiral system is mostly favoured for stable triaxial nuclear shapes with maximum triaxiality ($\gamma=30^\circ$). The total Routhian surface (TRS) calculations performed for the three lowest lying negative parity bands in $^{198}$Tl with configurations (e,f)A, (e,f)B and (g,h)A, (where the proton routhians e, f, g and h
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Figure 4. Experimental quasi-particle alignments (top left panel), Routhians (top right panel),
kinematic moment of inertia (bottom left) and $\text{B(M1)/B(E2)}$ transition probabilities (bottom
right panel) for the yrast and side bands in $^{198}\text{Tl}$

originates from the $\text{h}_9/2$ orbitals with $\Omega = 9/2$ and 7/2, and the neutron routhians A
and B originate from the $\text{i}_{13/2}$ orbital with $\Omega=1/2$) show nearly axially symmetric
shapes with $\gamma \sim -65^\circ$. Axially symmetric shapes are predicted by this model also for
the neighbouring Tl and Hg isotopes. However, low-lying $\gamma$ bands were found in the
even-even Hg isotopes and large non-axialities (with $\gamma \sim 36^\circ$) were derived from the
excitation energies and independently from the B(E2) transition probabilities [16] in
the framework of the triaxial-rigid-rotor model. Furthermore the excitation energies
of the low lying states in the neighbouring odd Tl nuclei could not be reproduced
if the rotation of an axially deformed nucleus was assumed or by interpreting the
data in terms of harmonic vibrations (for $^{199}\text{Tl}$ see [17]). However, if the nuclear
shape was assumed to have the same $\gamma$ deformation as derived for the neighbouring


even-even core (i.e. for $^{199}$Tl $\beta_2=0.15$, $\gamma=36^\circ$ is assumed), the description of low lying states improved dramatically [17]. Thus, conflicting predictions are suggested by these theoretical models and the question about the non-axiality of the nuclear shape in $^{198}$Tl remains unresolved.

Several alternative descriptions of the partner bands in $^{198}$Tl are possible. The most obvious alternative is to assume different single particle configuration for the side band. Such description could be attempted using for instance TRS and cranked shell model (CSM), two-particle-plus-rotor model, tilted axis cranking (TAC) model, etc. TRS calculations suggest nearly the same deformation of $\beta_2=0.13$ and $\gamma=-65^\circ,-62^\circ$ for the (e,f)A, (e,f)B and (g,h)A bands. The excitation energy for these bands, as predicted by TRS is shown in Fig. 5. The excitation energy of the (e,f)B band grows fast at high angular momenta, thus the side band is more likely to have a configuration (g,h)A. The relative excitation energy between the yrast and the side band according to TRS is about 0.45 MeV, which matches well the experimental data. However, it remains unclear how to explain the difference in the staggering patterns of the two bands and also the similarity of the B(M1)/B(E2) transition probabilities as observed in the experimental data. Such differences do not occur naturally in the TRS and CSM calculations.

![Figure 5. Excitation energy as a function of the spin I for the three lowest lying negative parity bands in $^{198}$Tl using TRS calculations](image)

A good description of the signature inversion in the yrast band in $^{198}$Tl can be obtained using the particle-plus-rotor model. In this model signature inversion results from the proton-neutron interaction and the Coriolis and centrifugal effects. Such calculations were performed for the yrast band in $^{198}$Tl assuming axially symmetric shape [18]. Within this interpretation it remains unclear why these effects are not present in the side band in $^{198}$Tl.
The TAC model is known to work well for bands with no staggering, built on particle-hole configurations and for nuclei with small deformation. It is thus possible that tilted rotation is involved in the side band, and spin in this band is obtained by aligning the angular momenta of the odd proton and neutron towards the direction of the total angular momentum, while collective rotation generates the spin in the yrast band. It seems however, that the "blades" (i.e. the angular momenta of the odd proton and neutron) are too short to generate the spin in the side band, because the maximum angular momentum when the "blades" are completely closed is only $11\hbar$. It is also unclear why the $B(M1)/B(E2)$ transition probabilities would have similar values for the two bands in this scenario. It is expected that for tilted rotation the $B(M1)$ values decrease considerably faster compared with those for the case of collective rotation.

Another alternative description could be based on the assumption that the odd proton and neutron couple to the $\gamma$ band of the core. In well deformed nuclei the $\gamma$ band represents a coupling of $K = 2\gamma$ vibrations to the rotation of the core. The relative excitation energy difference between the side and the yrast bands in $^{198}$Tl of about 500 keV is not far from the gap between the energies of the $2^+_1$ and $2^+_2$ states in $^{198}$Hg of 676 keV. However it remains unclear how to explain the similarity of the $B(M1)/B(E2)$ ratios and of the quasi-particle alignments in the partner bands, because in this case the two bands correspond to configurations with different $K$ values. It should be noted that the nuclei in the $A = 190$ mass region are not well deformed and that the $\gamma$ band (or rather the "quasi" $\gamma$ band) is not expected to have $K$ as a good quantum number. It would be interesting to find out what properties a side band, obtained as a coupling to a quasi $\gamma$ band would have. It seems unlikely though that a coupling of to a $\gamma$ vibration may result in no changes in the quasi-particle alignments and $B(M1)/B(E2)$ ratios. It should be noted that data on quasi $\gamma$ bands in $^{198}$Hg and in the other neighbouring Hg isotopes is very sparse, and no direct comparisons of the experimental excitation energies or transition probabilities can be done at present. Analysis of possible interpretations for this pair of bands in $^{198}$Tl is still in progress.

5 Conclusions

The odd-odd $^{198}$Tl nuclei were studied using $\gamma$-ray spectroscopy carried out at iThemba LABS, South Africa, and electron-$\gamma$ spectroscopy performed at Orsay, France. The same reaction, $^{197}$Au$(\alpha,3n)^{198}$Tl at a beam energy of 40 MeV was used in both experiments. The previously known level scheme of $^{198}$Tl was extended to moderate and high spin and two new bands were found. One of them, a weak band feeding into the yrast $\pi h_{9/2} \otimes i_{13/2}$ band through many links, looks like a partner to the yrast band, and thus a chiral interpretation might be possible. The two bands have excitation energy displacement of about 500 keV, and definite difference in the staggering pattern, which might be a result of the moderate quadrupole deformation of $^{198}$Tl. Their other properties, i.e. quasi-particle alignments, moments of inertia,
and B(M1)/B(E2) transition probabilities look very similar, and are thus in agreement with a chiral interpretation of the bands. The shape of $^{198}$Tl, and in particular whether it is axially asymmetric, remains unresolved, with conflicting predictions by TRS (axial symmetry), and triaxial-rigid-rotor calculations (strong non-axiality). Thus, the possibility of chirality for this nucleus needs further examination. Alternative interpretations for the side band include single-particle excitation (in terms of CSM, particle-plus-rotor model and TAC model), or coupling of the odd particles to the quasi $\gamma$ band of the core. It seems, however, that none of these descriptions is very successful in accounting for all the properties of the partner bands in $^{198}$Tl.

Acknowledgments

References