On E0 Transitions in even-even nuclei

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Abstract. The reanimation of the investigations dedicated to 0^+ states energies and $E0$ transitions between them is provoked by new and more precise experimental technics that not only made revision of the previous data but also gave a possibility to obtain a great amount of new 0^+ states and conversion electrons data. We suggest a phenomenological model for estimation of the $E0$ transition nuclear matrix elements. Recently theoretical calculations [1] predicted the existence of a 0^+ state with energy 0.68 MeV in ^{160}Dy nucleus. Arguments in favor of existence of 681.4 keV state in ^{160}Dy nucleus are presented.

1 Introduction

The nature of the low lying 0^+ states bands in deformed nuclei still remains a mystery under debate. The improvements in technology enable new spectroscopic and life-time measurements of a large number of K^π=0^+ bands in nuclei which were previously inaccessible [2]. Some authors point out importance of studying anharmonic effects in microscopic way in deformed nuclei [3], quadrupole and pairing vibrational modes in conversional electrons and internal pair decay [4], or exact diagonalization in the restricted space of collective phonons of different types [6].
The energies and electromagnetic decay properties of the excited 0^+ states are important for determining the applicability and are a test of theoretical models - the Shell model, the Cluster-vibrational model, the Quasi-particle - phonon model, the Deformed configuration mixing shell model, the Interacting boson approximation, the Pairing quadrupole correlations, O(6)limit of the IBA. There are some calculations [4, 7, 11, 12] devoted to the estimation of $E0$ nuclear matrix elements ρ^2 between different 0^+ states in the same nucleus. For instance in [12] It is found that $\rho^2(0_2^+ \to 0_1^+)$ is very small in comparison with $\rho^2(0_3^+ \to 0_1^+)$ which indicates that the 0_3^+ state is more collective than 0_2^+. It should be very important to determine experimentally the half-lives of the 0^+ states, because, it would allow more definitive conclusions of the structure of the excited 0^+ states [11]. Often the first excited 0^+ state in nuclei is considered as less collective than the next states with higher excitation energy. For instance the 0^+ state observed in ^{158}Gd with excitation energy of 0,2548 MeV ($n = 20$) is much more collective than the 0^+ state with energy of 0,5811 MeV ($n = 1$) [1].

In this talk we illustrate the predictable power of our approach by comparing the theoretical results of E0 transition probabilities and distribution of excitation energies of the 0^+ states in ^{160}Dy with new experimental data.

2 E0 transition matrix elements

In this paper we will estimate the E0 transition probability by factorizing the electronic and nuclear wave functions, but we will take into account the influence of the nuclear charge distribution on the electronic factor Ω. Let us consider the simple description of the K-electrons conversion process starting with the atomic Hamiltonian of the form [17]:

$$H = H_{nucl} + H_{elect} - \sum_{p,e} \frac{\alpha}{|r_p - r_e|} \tag{1}$$

Here H_{nucl} is the nuclear Hamiltonian, H_{elect} - the Hamiltonian of the electron system. The last term is the electrons-nuclear interaction:

$$H' = -\sum_{p,e} \frac{\alpha}{|r_p - r_e|} \tag{2}$$

The matrix element between the initial state $|i\rangle$ and the final state $|f\rangle$ of the system is equal to:

$$\langle i\,|H'(L=0)|\,f\rangle =$$
$$-\sum_{p,e}\alpha\left[\int d\tau_{nuc}\int_0^{r_p} d\tau_e \phi_f^* \psi_f^* \frac{1}{r_p}\phi_{\mathbf{i}}\psi_{\mathbf{i}} + \int d\tau_{nuc}\int_{r_p}^{\infty} d\tau_e \phi_f^* \psi_f^* \frac{1}{r_e}\phi_{\mathbf{i}}\psi_{\mathbf{i}}\right] \tag{3}$$

In the expression above we replace the electron initial and final wave functions respectively by $|\phi_i\rangle\sim e^{-ar}$ and $|\phi_f\rangle\sim e^{i\mathbf{kr}}$ in infinity. In case of cut-off nuclear charge density distribution $d_0\Theta(R-r)$, for K-electrons we find the result of the above integration (3) as

$$\mathbf{F_{nuc,el}}(\mathbf{k},\mathbf{R}) = \frac{\mathbf{16}\pi^{\mathbf{2}}\alpha\left(\mathbf{kR}\left(\mathbf{k^2R^2}+\mathbf{3}\right)\cos(\mathbf{kR}) - \mathbf{3}\sin(\mathbf{kR})\right)}{\mathbf{3k^5}} \tag{4}$$

It is very important for the further consideration that this nucleus-electron factor depends on the nuclear size R and on the electron impulse k defined by nuclear transition energy. Before introducing the collective degrees of freedom in our calculations of the $E0$ nuclear transition matrix elements (4) we present two illustrative examples:

– Let consider a nuclear characteristic $d(r,R)$ which depends on the nuclear size R. Then for this function we can write the identity:

$$d(r, R) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d(r, R_0 + x) e^{ip(x - \Delta R} dp dx \qquad (5)$$

Following the receipt of paper [18] the expectation values of this characteristic between different collective states is determined by the matrix element

$$< n_1 \mid d(r, R) \mid n_2 >$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d(r, R_0 + x) e^{ipx} < n_1 \mid e^{-ip\Delta R(b^+, b)} \mid n_2 > dp dx \qquad (6)$$

If we assume that $d(r, R) = d_0 \theta(R - r)$ is the nuclear density distribution, then the matrix element (6) for the ground state has the form:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d_0 \theta(R - r - x)}{2\pi} e^{ipx - \frac{p^2 s^2}{2}} dp dx = \frac{1}{2} d_0 Erfc(\frac{r - R}{\sqrt{2} s}) \qquad (7)$$

As a result of integration by $dpdx$ in (6) we obtain instead of the starting cut-off distribution a function which depends on the nuclear surface diffuseness S (see Figure-1.):

$$\theta(R - r) \quad \Rightarrow \quad \frac{1}{2} d_0 \, Erfc[\frac{r - R_0}{\sqrt{2} S}] \qquad (8)$$

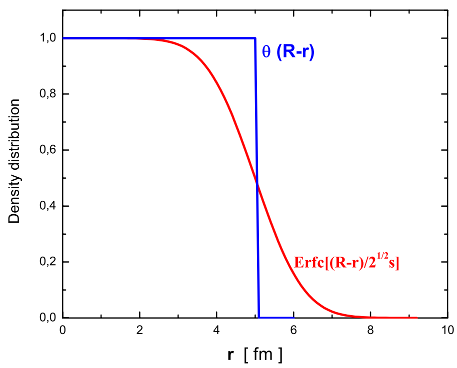

Figure 1. Density distributions before and after introducing the diffuseness S as a collective degree of freedom.

The mean square radius corresponding to the density distribution (8) is equal to:

$$\langle r_{ms}^2 \rangle_{00}^{1/2} = \sqrt{\frac{0.6 R_0^5 + 6 R_0^3 S^2 + r R_0 S^4}{R_0^3 + 3 R_0 S^2}} \qquad (9)$$

Small vibrations of nuclear shapes around the equilibrium can give rise to states at low to moderate excitation energies. In the case of $S = 0$ we have the cut-off density mean square radius:

$$\langle r_{ms}^2 \rangle_{00}^{1/2} = \sqrt{\frac{3}{5}}\, R_0 \tag{10}$$

– A classification of large amount of experimental data in terms of integer classification parameter recently has been done based on phenomenological monopole part of collective Hamiltonian for the single level approach [1, 5]. The analysis have shown that the experimental energies of low lying excited 0^+ states in nuclei can be presented by a parabolic distribution function of number of collective excitations:

$$E_n = An - Bn^2 + C \tag{11}$$

We can label every 0^+ state by an additional characteristic n - the number of monopole bosons determining it's collective structure. Now, going back to the transition matrix elements (4) we have the defined collective structure of the 0^+ states. In Figure-2 the experimental data for the excitation energies of the 0^+ states in ^{160}Dy are arranged on a parabolic function of the number of monopole bosons.

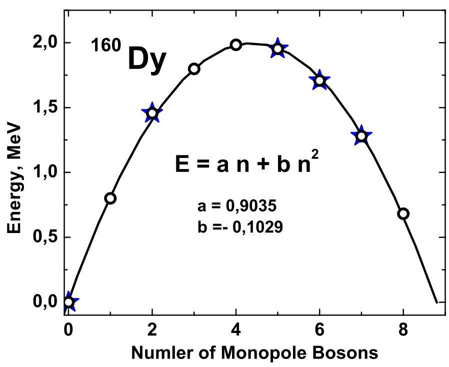

Figure 2. 0^+ states of ^{160}Dy fitted to a parabola (solid line). Circles are the regions of predicted states, blue stars present experimental data.

The transition energy $E(n) - E(m)$(i.e. electron impulse k) between different excited 0^+ states $\frac{1}{\sqrt{m!}}(b^\dagger)^m|0\rangle$ and $\frac{1}{\sqrt{n!}}(b^\dagger)^n|0\rangle$ in the same nucleus is determined automatically from (11) .

The transitional matrix elements of (4) are calculated in the following way:

$$f(m,n,p,w) =< n \mid e^{-ip\Delta R(b^\dagger,b)} \mid m > \frac{1}{\sqrt{n!m!}} \langle 0|b^n e^{-ip\Delta R(b^+,b))}(b^\dagger)^m|0\rangle \tag{12}$$

Using the derived in [19] expressions

$$\sum_{l=0}^{n-1} \frac{m!}{(m-n+l)!}\binom{n}{l}(b^\dagger)^{m-n+l}b^l \qquad n \leqslant m \tag{13}$$

$$\sum_{l=0}^{m-1} \frac{n!}{(n-m+l)!}\binom{m}{l}(b^\dagger)^{l}b^{n-m+l} \qquad n \geqslant m \tag{14}$$

and obtain

$$f(m,n,p,w) = \frac{e^{\frac{p^2\,{R_0}^2\,w^2}{2}}}{\sqrt{n!m!}} \sum_{k=0}^{\infty}(R_0 w)^{2k+m-n}(ip)^{2k+m-n}\frac{(m+k)!}{k!(m+k-n)!} \tag{15}$$

The summation by k in (15) gives:

$$f(m,n,p,w) = \frac{e^{\frac{p^2\,{R_0}^2\,w^2}{2}}\,(ip)^{m-n}\,(R_0\,w)^{m-n}}{2\,\pi\,\sqrt{m!\,n!}\,\Gamma(1+m-n)} \cdot \tag{16}$$

$$\Gamma(1+m)\,{}_1F_1(1+m, 1+m-n, -p^2\,{R_0}^2\,w^2)$$

Finally the nuclear $E0$ transition matrix elements are nothing but:

$$\rho_{mn} = \frac{A_{norm}}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{F_{nuc,el}}(\mathbf{k},\mathbf{R_0}+\mathbf{x})e^{ipx}f(m,n,p,w)dpdx \tag{17}$$

Here we present analytical expressions of some matrix elements $f(m,n,p,w)$:

$$f(m,m,p,w) = \frac{e^{\frac{p^2w^2}{2}}\Gamma(m+1)\,{}_1F_1\left(m+1;1;-p^2w^2\right)}{2\pi}$$

$$f(m,0,p,w) = \frac{e^{-\frac{1}{2}p^2w^2}(ip)^m w^m}{2\pi}$$

$$f(m,m-1,p,w) = \frac{ie^{\frac{p^2w^2}{2}}pw\Gamma(m+1)\,{}_1F_1\left(m+1;2;-p^2w^2\right)}{2\pi}$$

$$f(m,m-2,p,w) = -\frac{e^{\frac{p^2w^2}{2}}p^2w^2\Gamma(m+1)\,{}_1F_1\left(m+1;3;-p^2w^2\right)}{4\pi}$$

Thus, for chosen m and n we can perform integration by $dpdx$ in (17) and calculate corresponding $\rho_{m\to n}$.

For instance:

$$\rho_{1\to0} = -\frac{5}{3}k^4\pi w^9 - \frac{20}{3}k^4\pi R_0^2 w^7 + \frac{16}{3}k^2\pi w^7 - \frac{10}{3}k^4\pi R_0^4 w^5 + 16k^2\pi R_0^2 w^5 - 8\pi w^5 - \frac{4}{9}k^4\pi R_0^6 w^3 + \frac{16}{3}k^2\pi R_0^4 w^3 - 16\pi R_0^2 w^3 - \frac{1}{63}k^4\pi R_0^8 w + \frac{16}{45}k^2\pi R_0^6 w - \frac{8}{3}\pi R_0^4 w$$

$$\rho_{2\to1} = -\frac{50}{3}k^4\pi w^9 - \frac{160}{3}k^4\pi R_0^2 w^7 + \frac{128}{3}k^2\pi w^7 - 20k^4\pi R_0^4 w^5 + 96k^2\pi R_0^2 w^5 - 48\pi w^5 - \frac{16}{9}k^4\pi R_0^6 w^3 + \frac{64}{3}k^2\pi R_0^4 w^3 - 64\pi R_0^2 w^3 - \frac{2}{63}k^4\pi R_0^8 w + \frac{32}{45}k^2\pi R_0^6 w - \frac{16}{3}\pi R_0^4 w$$

$$\rho_{4\to1} = -\frac{400}{3}k^4\pi w^9 - 320k^4\pi R_0^2 w^7 + 256k^2\pi w^7 - 80k^4\pi R_0^4 w^5 + 384k^2\pi R_0^2 w^5 - 192\pi w^5 - \frac{32}{9}k^4\pi R_0^6 w^3 + \frac{128}{3}k^2\pi R_0^4 w^3 - 128\pi R_0^2 w^3$$

The results for different transitions defined by the number of monopole bosons m and n are analytical but because of length of the expressions we wont present all of them here. The behavior of nuclear matrix elements $\rho^2_{m\to n}$ for different $E0$ transitions is shown in Figure-3.

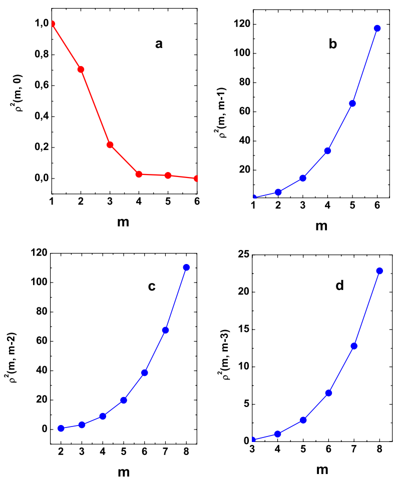

Figure 3. Behavior of calculated matrix elements. **a** - $\rho^2_{m\to0}$, **b** - $\rho^2_{m\to m-1}$, **c** - $\rho^2_{m\to m-2}$, **d** - $\rho^2_{m\to m-3}$; m is the number of monopole bosons constructed corresponding 0^+ excited state. All the values of ρ^2 divide $\rho^2_{1\to0}$

$\rho^2_{m\to 0}$ decrease rapidly with increase of number of monopole bosons m (Figure 3a), while $\rho^2_{m\to n\neq 0}$ increases with increase of m (Figures 3b, 3c, 3d).

In spite of very small values of ρ^2 for $E0$ transition from the exited states placed on the right part of the distribution shown in Figure-2 to the nuclear ground state (see Figure-3a) we decided to check experimental evidence that could vindicate our theoretical predictions. We start with the excited state with energy of 0.68 MeV in ^{160}Dy with $n = 8$ monopole bosons. For finding-out of existence of mentioned above 0^+ state in ^{160}Dy nucleus we measure β-spectrograms of [20] DLNP JINR for fractions Er (two photographic plates) and Ho (one photographic plate) using universal installation MAC-1 in ITEP [21]. At the analysis it was found out, that in all three photographic plates to the left of known line EIK with energy 682.3 keV below by energy on 1 keV, the peak comparable by intensity with the specified line is confidently observed. Our attempts to carry the mentioned peak to a conversion line or to any known from the literature [22] transitions in ^{160}Dy nucleus had no success. Then we proposed, that this peak is probably caused by new transition with energy 681.3 keV, unloading the corresponding new raised state with energy 681.3 keV to the ground state. Except for the specified state, from experiment the states with excitation energies 1280.0, 1456.7, 1708.2 and 1952.3 keV are known. Considering, that from these levels transitions to the entered by us 681.3 keV level are possible, we have undertaken searches of such transitions. As a result such transitions with energy 1822.5 (1822.4(3) and the intensity $I = 0.24$), between 2^+ state 2503.8 keV and 0^+ state 681.3 keV, and the transition from 681.3 keV state to 2^+ with energy 86.8 keV (594.5 and $I < 0.3$) have been found out. Even these facts already are powerful argument in favor of existence of the excited state with the energy of **681.3 keV** in ^{160}Dy nucleus. We proceed the searches of other transitions and will try to prove the existence of another excited 0^+ states (see Figure 2) in ^{160}Dy nucleus.

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