Charge Form Factor and Cluster Structure of ^{6}Li Nucleus

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Abstract. The charge form factor of ^{6}Li nucleus is considered on the basis of its cluster structure. The charge density of ^{6}Li is presented as a superposition of two terms. One of them is a folded density and the second one is a sum of ^{4}He and the deuteron densities. Using the available experimental data for ^{4}He and deuteron charge form factors, a good agreement of the calculations with the experimental data for the charge form factor of ^{6}Li is obtained, including those in the region of large transferred momenta.

1 Introduction

It is known that the structure of the ^{6}Li nucleus has some peculiarities compared to the other 1p-shell nuclei (see e.g. [1–7]). The electron elastic scattering data [8] on the charge form factor and the rms radius of ^{6}Li cannot be explained in the framework of the shell-model by means of an oscillator parameter $\hbar\omega = 15 \div 16 \ MeV$, the latter providing a good description of these data for the other 1p-shell nuclei. The usage of another value of $\hbar\omega$, the same for the s- and p- nucleons, as well as of two different oscillator parameters for the s- and p-shells is also not successful. The situation is similar in the case of the inelastic form factors. The wave functions of the low-lying states of ^{6}Li are significantly different from the commonly accepted and used shell model wave functions. This fact is important for the analysis of the $(p, 2p)$, (p, pd), $(p, p\alpha)$ reactions on ^{6}Li, the photonuclear reactions, the (^{6}Li, d), (^{6}Li, α) reactions and others.

It has been estimated that ^{6}Li has a well pronounced cluster structure and is considered generally as a system consisting of $\alpha-$ and deuteron clusters in a mutual motion exchanging nucleons. The small value of the decay threshold ^{6}Li$\rightarrow \alpha + d$, the large nuclear radius, etc. give evidence that the $\alpha-$ and $d-$ clusters in ^{6}Li are quite isolated. In one of the cluster models, the Model of Nucleon Associations (MNA) (e.g. [6]), the problem of the role of the exchange has been studied by analyzing the elastic and inelastic form factors of the Coulomb electron scattering. The antisymmetrization effect turns out to be substantial only at large values of the isolation parameter $x \approx 1$, where the parameter $x = b/a$ is the ratio between the relative motion function parameter b and the α-particle function parameter a. At the real value $x = 0.3 \div 0.4$, the exchange effects are already of no importance.

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In MNA the value $x = 1$ corresponds to the shell-model structure of ^{6}Li, while $x = 0$ corresponds to the cluster model ($\alpha - d$ structure). It has been found that the isolation parameter x has different values for nuclei with cluster structure. For instance, $x = 0.5 \div 0.6$ for ^{9}Be, $x = 0.7$ for ^{12}C and $x = 0.8$ for ^{16}O [9, 10]. The elastic scattering charge form factor, although being sensitive to the value of x, can be described by different models due to the fact that it is obtained on the base of the charge density distribution that is averaged over the angular variables. The form factor of the inelastic quadrupole scattering, however, strongly depends on x and cannot be described within the shell model. The MNA provides a good rms radius of ^{6}Li [11] and with the above values of the isolation parameter ($x = 0.3 \div 0.4$) allows a proper simultaneous description of the electron elastic and inelastic scattering but only up to transferred momentum values $q \sim 2 \ fm^{-1}$.

The aim of the present work is to suggest an approach in which the $\alpha - d$ cluster structure of ^{6}Li to be checked by calculations of the charge density and the corresponding charge form factor. We construct a scheme in which the charge densities of ^{4}He and the deuteron are included and the available experimental data for them can be used to calculate the ^{6}Li charge density, the charge form factor and the latter to be compared with the experiment. In this sense, our work has a meaning of a 'theoretical experiment' in the cluster structure analysis of ^{6}Li.

In Section 2 the theoretical scheme, the results of the calculations and a discussion are presented. The conclusions are given in Section 3.

2 Charge Density and Form Factor of ^{6}Li in Relation to Those of ^{4}He and Deuteron

Considering the problems in the description of the ^{6}Li charge density and form factor briefly mentioned above, we made an attempt to study these quantities on the base of the corresponding ones for ^{4}He and the deuteron within the framework of the $\alpha - d$ cluster structure of ^{6}Li nucleus.

Our first attempt was to describe the charge density of ^{6}Li by means of folding of the charge densities of ^{4}He and the deuteron:

$$\rho_{^6Li}^{ch} (\vec{r}) = \frac{3}{2} \int d\vec{r}' \rho_{^4He}^{ch} (\vec{r} - \vec{r}') \rho_d^{ch} (\vec{r}') \ . \tag{1}$$

The charge densities in Eq. (1) are normalized to the number of protons Z ($Z = 3$, 2 and 1 for ^{6}Li, ^{4}He and the deuteron, correspondingly). Substituting ^{6}Li charge density (1) in the definition of the charge form factor

$$F^{ch} (\vec{q}) = \frac{1}{Z} \int d\vec{r} \, e^{i\vec{q} \cdot \vec{r}} \rho^{ch} (\vec{r}) \tag{2}$$

we obtain

$$F_{^6Li}^{ch}(q) = F_{^4He}^{ch}(q) \, F_d^{ch}(q) \, e^{q^2/(4A^{2/3})} \ , \tag{3}$$

Table 1. The values of the parameters α, β, γ and δ in the parametrizations I and II, obtained from the global best fit in [16] (the values of α are derived from Eq. (8)).

Set	α	β	γ	δ
I	5.75 ± 0.07	-5.11 ± 0.09	12.1 ± 0.5	1.04 ± 0.03
II	5.50 ± 0.06	-4.78 ± 0.08	12.1 ± 0.5	1.05 ± 0.03

in which the exponential factor approximately accounts for the centre-of-mass (c.m.) corrections according to [12].

In our calculations of the charge form factor of ^{6}Li (Eq. (3)) we use the available experimental data for the charge form factor of ^{4}He (see e.g. [13] and references therein), as well as the experimental data for the charge form factor of the deuteron. The latter are those from the Thomas Jefferson Laboratory experiments in which the deuteron charge form factor was measured for a first time to a transferred momentum value up to $q = 6.64 \, fm^{-1}$ and the node of the form factor was observed (Abbott et al. [14, 15]). In our calculations for the deuteron charge form factor we use a best fit parametrization obtained in [16]. It is represented by Eqs (4)-(8) [16]:

$$F_d^{ch}(q^2) = g(q^2)\overline{F}_d^{ch}(q^2) \,, \tag{4}$$

$$\overline{F}_d^{ch}(q^2) = 1 - \alpha - \beta + \alpha \frac{m_\omega^2}{m_\omega^2 + q^2} + \beta \frac{m_\Phi^2}{m_\Phi^2 + q^2} \,, \tag{5}$$

where m_ω and m_Φ are the meson masses ($m_\omega = 0.784 \, GeV$ and $m_\Phi = 1.019 \, GeV$). For any values of the two real parameters α and β

$$F_d^{ch}(0) = 1 \,. \tag{6}$$

The factor g in Eq. (4) has the form

$$g(q^2) = \frac{1}{(1 + \gamma q^2)^\delta} \tag{7}$$

and γ and δ are also real parameters.

The requirement of a node for $q_0^2 \approx 0.7 \, GeV^2$ gives the following relation between the parameters α and β:

$$\alpha = \frac{m_\omega^2 + q_0^2}{q_0^2} - \beta \frac{m_\omega^2 + q_0^2}{m_\Phi^2 + q_0^2} \,. \tag{8}$$

The values of two sets of the parameters α, β, γ and δ obtained in [16] by a best fit to the experimental data, which are used in the calculations of the present work, are given in Table 1.

Recent large-scale shell-model (LSSM) calculations of [17] and the analysis of the electron and proton elastic and inelastic scattering data from ^{6,7}Li have proved the 'clustering' behavior of these systems. In our work [18] proton, neutron, charge,
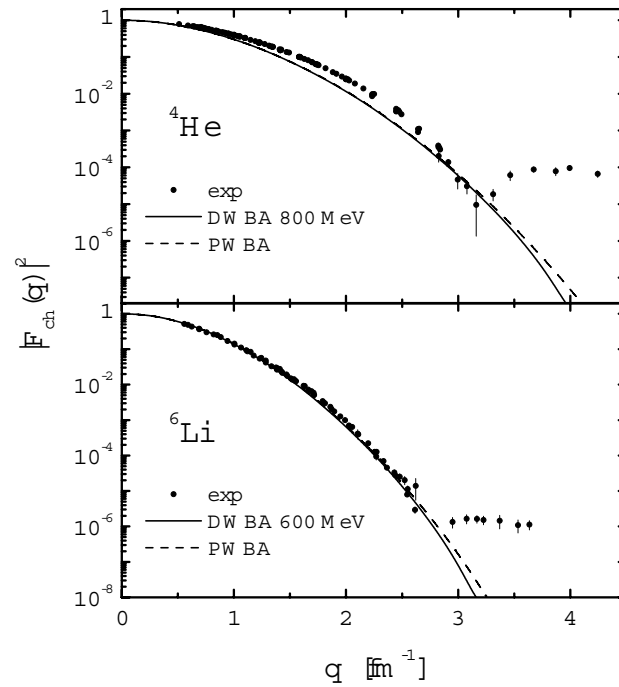
Figure 1. Charge form factors of the stable isotopes ^{4}He and ^{6}Li obtained in [18] using LSSM densities in PWBA and in DWBA calculations in comparison with the experimental data [13, 19–25].

and matter densities of a wide range of exotic nuclei obtained in the Hartree-Fock-Bogolyubov method and in the LSSM (for He and Li isotopes) have been used for Plane-Wave Born Approximation (PWBA) and Distorted-Wave Born Approximation (DWBA) calculations of the related form factors. This makes it possible to analyze the influence of the increasing number of neutrons on the proton and charge distributions in a given isotopic chain. The obtained in [18] theoretical predictions for the charge form factors of exotic nuclei are a challenge to their measurements in the future experiments on the electron-radioactive beam colliders in GSI and RIKEN in order to get detailed information on the charge distributions of such nuclei.

Our results for ^{4}He and ^{6}Li charge form factors obtained in [18], compared to the available experimental data [13, 19–25], are given in Figure 1. One can see a good agreement with the data up to $q \sim 3\,fm^{-1}$.

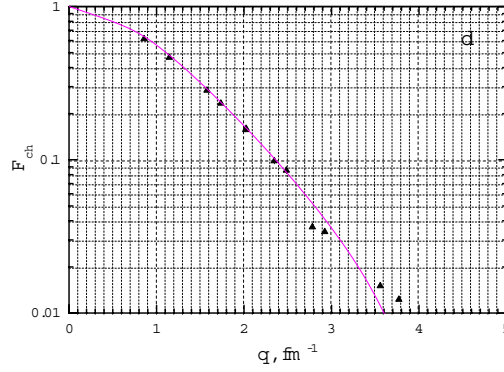Figure 2. The charge form factor of the deuteron calculated using two sets of parameters α, β, γ and δ (with values given in Table 1) and compared with the experimental data [14, 15] up to $q \approx 3.8\,fm^{-1}$.

In Figure 2 are presented the experimental data for the deuteron charge form factor [14, 15] and the result of the parametrization from [16] up to $q \approx 3.8\,fm^{-1}$ (with parameter sets I and II from Table 1).

In Figure 3 are given our results for the squared charge form factor of ^{6}Li calculated by using of Eq. (3) (taking account of the c.m. correction) and the experimental data for the charge form factors of ^{4}He and the deuteron. For the latter we used the same parametrization from [16] (Eqs (4)-(8)) with two sets of parameters I and II from Table 1. A good agreement with the experimental data in the interval of transferred momentum $0 < q \leq 2.7\,fm^{-1}$ can be seen and a disagreement with the values of the form factor for larger q's that are related to small values of r's, i.e. to the central part of the nuclear density. In other words, the central density can be different from the assumption for the folding density (Eq. (1)). We note the similarity of the results (compared with the data) of the calculated charge form factor of ^{6}Li for $q \lesssim 2.7\,fm^{-1}$ from the present work (shown in Figure 3) with those from [18] (shown in the down panel of Figure 1).

The results shown in Figure 3 were the reason to look for an extension of the approach. Our second suggestion is to consider the charge density of ^{6}Li as a superposition of a folding term and a sum of the charge densities of ^{4}He and the deuteron with weight coefficients c_1 and c_2:

$$\rho_{6_{Li}}^{ch}(\vec{r}) = \frac{3}{2}c_1 \int d\vec{r}' \rho_{4_{He}}^{ch}(\vec{r} - \vec{r}')\,\rho_d^{ch}(\vec{r}') + c_2 \left[\rho_{4_{He}}^{ch}(\vec{r}) + \rho_d^{ch}(\vec{r}) \right] . \qquad (9)$$

The normalization of the densities in Eq. (9) to Z leads to the condition for the coefficients
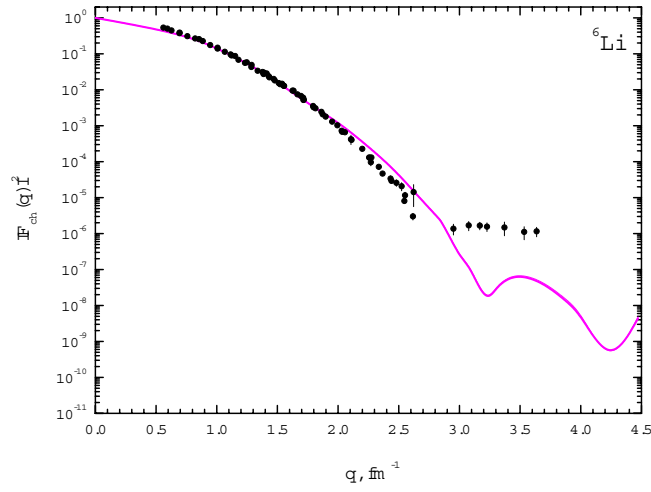
Figure 3. The charge form factor of ^{6}Li calculated by using Eq. (3) and the experimental data for the charge form factors of ^{4}He and the deuteron in comparison with the experimental data ([13, 19–21, 24, 25]).

$$c_1 + c_2 = 1 \,. \tag{10}$$

The charge density (Eq. (9)) leads to the following expression of the charge form factor of ^{6}Li (with the account for the c.m. correction):

$$F_{^6Li}^{ch}(q) = \left\{ c_1 F_{^4He}^{ch}(q) \, F_d^{ch}(q) + \frac{c_2}{3} \left[2F_{^4He}^{ch}(q) + F_d^{ch}(q) \right] \right\} e^{q^2/(4A^{2/3})} \,. \tag{11}$$

For $q = 0$

$$F_{^6Li}^{ch}(0) = 1 \,. \tag{12}$$

The squared charge form factor can be written as:

$$\mid F_{^6Li}^{ch}(q) \mid^2 = A + B + C \,, \tag{13}$$

where A, B and C represent the contributions to the charge density of ^{6}Li of the folding term (A), of the sum of the charge densities of ^{4}He and the deuteron (B) and the interference term (C). Their explicit expressions are:

$$A = c_1{}^2 \mid F_{^4He}^{ch}(q) \mid^2 \mid F_d^{ch}(q) \mid^2 e^{q^2/(2A^{2/3})} \,, \tag{14}$$

$$B = \frac{c_2{}^2}{9} \left[4 \mid F_{^4He}^{ch}(q) \mid^2 + \mid F_d^{ch}(q) \mid^2 + 4 \mid F_{^4He}^{ch}(q) \mid\mid F_d^{ch}(q) \mid \right] e^{q^2/(2A^{2/3})} \,, \tag{15}$$

$$C = \frac{2}{3}c_1 c_2 \mid F_{^4He}^{ch}(q) \mid\mid F_d^{ch}(q) \mid [\, 2 \mid F_{^4He}^{ch}(q) \mid + \mid F_d^{ch}(q) \mid] \, e^{q^2/(2A^{2/3})}. \quad (16)$$

In the following three Figures 4-6 are presented the results for the squared charge form factor of ^{6}Li calculated using Eqs (13)-(16) with the experimental data for the charge form factor of ^{4}He and the deuteron and for different sets of the values of the weight coefficients c_1 and c_2. The fit of Eq. (13) to the experimental data reveals an interval of values of $c_1 = 0.975 \div 0.985$ and, correspondingly, of $c_2 = 0.025 \div 0.015$, for which the results reasonably agree with the experimental data [13, 19–21, 24, 25] within the limits of the experimental errors. We also show the contributions of the three terms A, B and C.
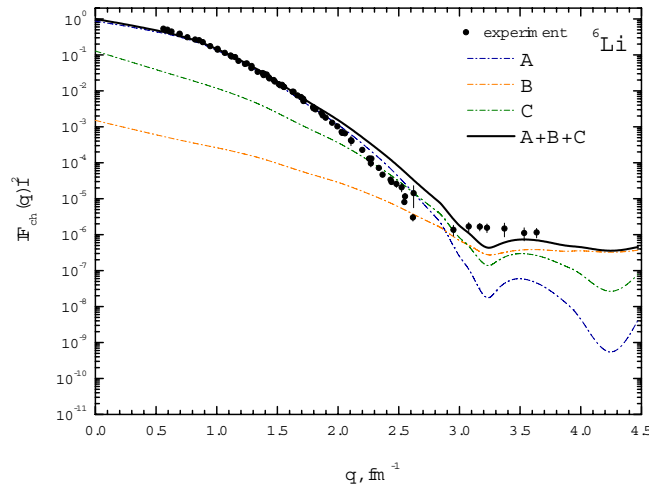

Figure 4. The charge form factor of ^{6}Li (Eqs (11)-(16)) calculated for $c_1 = 0.975$ and $c_2 = 0.025$, in comparison with the experimental data from [13, 19–21, 24, 25].

The results presented in Figures 4-6 prove that the contribution of the folding density to the charge density of ^{6}Li is about $97.5 \div 98.5\%$. This corresponds to the weight of the contribution of the sum of ^{4}He and the deuteron densities of about $2.5 \div 1.5\%$. It is seen that the term A (Eq. (14)) describes well the squared charge form factor of ^{6}Li in the interval $0 < q \lesssim 2.7\,fm^{-1}$, while the shell-model cluster density (related to the term B, Eq. (15)) is important for the description of the charge form factor of ^{6}Li for the large values of q ($q \gtrsim 3\,fm^{-1}$), related to the central nuclear density. The interference term C (Eq. (16)) has a contribution to the charge form factor of ^{6}Li for $q \gtrsim 3\,fm^{-1}$. The increase of c_1 within the above interval leads to a better description of the data for $q = 1.8 \div 2.9\,fm^{-1}$, but at
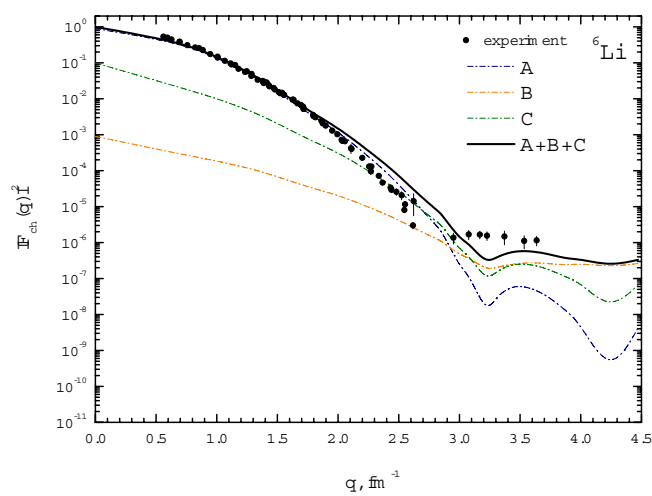
Figure 5. The same as in Figure 4 but with $c_1 = 0.979$ and $c_2 = 0.021$.

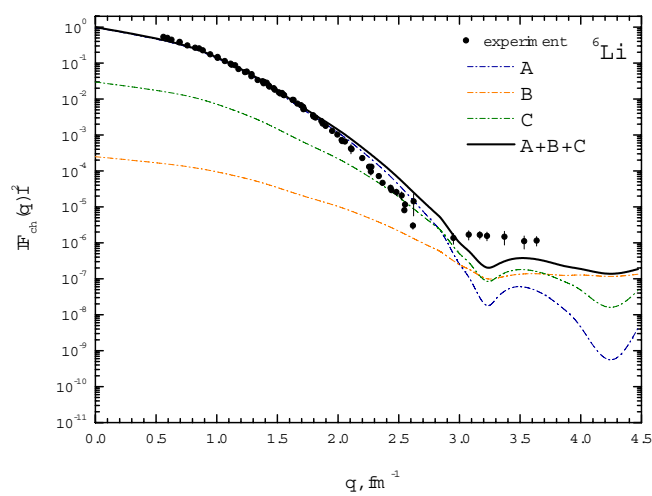Figure 6. The same as in Figure 4 but with $c_1 = 0.985$ and $c_2 = 0.015$.

the same time to a decrease of the values of the squared ^{6}Li charge form factor for $q \gtrsim 3\,fm^{-1}$, underestimating the data. Our calculations show that most reasonable results are obtained for 97.9% contribution of the folding term that corresponds to 2.1% weight of the contribution of the sum of ^{4}He and the deuteron charge densities to the ^{6}Li charge density.

As known, the value of the obtained rms radius is a test for the consistency of any approach to the description of the nuclear system structure. The charge rms radius of ^{6}Li is given by the expression:

$$\langle r_{6_{Li}}^2 \rangle = \frac{1}{3} \int \mathrm{d}\vec{r}\, r^2 \rho_{6_{Li}}^{ch}(\vec{r}) \;. \tag{17}$$

Substituting the expression for the charge density of ^{6}Li (Eq. (9)) in Eq. (17), we obtain:

$$\langle r_{6_{Li}}^2 \rangle = c_1 \left[\langle r_{4_{He}}^2 \rangle + \langle r_d^2 \rangle \right] + \frac{c_2}{3} \left[2 \langle r_{4_{He}}^2 \rangle + \langle r_d^2 \rangle \right] \;. \tag{18}$$

The usage of the experimental data for the rms radii of ^{4}He and the deuteron [20,26]:

$$\langle r_{4_{He}}^2 \rangle^{1/2} = 1.676(8)\,fm\,,$$

$$\langle r_d^2 \rangle^{1/2} = 2.116(6)\,fm$$

in Eq. (18) (with $c_1 = 0.979$ and $c_2 = 0.021$) leads to the following value for the ^{6}Li charge rms radius:

$$\langle r_{6_{Li}}^2 \rangle^{1/2} = 2.684\,fm\,,$$

which is in accordance with the experimental estimations for the charge rms radius of ^{6}Li [20, 26]:

$$\langle r_{6_{Li}}^2 \rangle^{1/2} = 2.57(10)\,fm.$$

This could be expected due to the use of the experimental charge densities of the deuteron and ^{4}He, being combined in a realistic theoretical scheme that gives a good agreement with the experimental data for the charge form factor of ^{6}Li.

3 Conclusions

In the present work we suggest a theoretical scheme for calculations of the charge density distribution and form factor of ^{6}Li in the framework of the $\alpha - d$ cluster model of this nucleus. The obtained results can be summarized as follows:

1. Our calculations show a reasonable description of the charge form factor of ^{6}Li on the basis of a superposition of two density distributions: **i)** a folding density obtained from ^{4}He and the deuteron charge densities, and **ii)** a sum of the ^{4}He and deuteron charge densities.

   Provided corresponding experimental data for both densities are used, the calculations show that a reasonable agreement with the data can be obtained when the weight of the folding density contribution is about $97.5 \div 98.5\%$

and the weight of the contribution from the sum of both densities is about $2.5 \div 1.5\%$.

2. The scheme has only one free parameter (c_1 or c_2) with a clear physical meaning, namely, it is the weight of the one of the contributions to the density of ^{6}Li.

3. The behavior of the charge form factor of ^{6}Li for $0 < q \lesssim 2.7\,fm^{-1}$ is determined mainly by the folding contribution (of ^{4}He and the deuteron densities) to the charge density of ^{6}Li (the weight is about $97.5 \div 98.5\%$).

4. The shell-model $\alpha - d$ cluster density of ^{6}Li (i.e. the sum of ^{4}He and the deuteron charge densities) is important (though with a small weight of about $2.5 \div 1.5\%$) in the central nuclear region and, correspondingly, it is responsible for the values of the charge form factor of ^{6}Li at large values of q ($q \gtrsim 3\,fm^{-1}$).

5. The calculated within the suggested scheme charge rms radius of ^{6}Li agrees with the experimental estimations of this quantity.

6. We would like to pay attention to the following facts: **i)** the minimum of the experimental charge form factor of the deuteron is at $q \approx 4.2\,fm^{-1}$ [14, 15], **ii)** the minimum of the experimental charge form factor of ^{4}He is at $q \approx 3.2\,fm^{-1}$ [13], and **iii)** the minimum of the experimental charge form factor of ^{6}Li is at $q \approx 2.9\,fm^{-1}$ [13, 19–21, 24, 25]. Based on points **i)** and **ii)**, our estimations show that the latter is determined mainly by the contribution of the charge density and the corresponding form factor of ^{4}He.

Acknowledgments

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