Superscaling analysis of neutral-current neutrino quasielastic cross sections within the Relativistic Impulse Approximation

M.C. Martínez[1], J.A. Caballero[2], T.W. Donnelly[3], and J.M. Udías[1]

[1] Grupo de Física Nuclear, Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid,
28040 Madrid, Spain
[2] Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, 41080 Sevilla, Spain
[3] Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. Superscaling properties of neutral-current (NC) neutrino-nucleus quasielastic (QE) cross sections are investigated in the Relativistic Impulse Approximation (RIA). First- and second-kind scaling are analyzed for neutrino beam energies ranging from 1 to 3 GeV for the cases of ^{12}C, ^{16}O, ^{40}Ca. Several detection angles of the outgoing nucleon are considered in order to sample different outgoing nucleon energy regimes. Superscaling is well fulfilled both in the plane-wave limit and when including final-state interaction (FSI) by means of a relativistic mean field (RMF) for the final states. The existence of superscaling within a model for NC neutrino-nucleus interactions beyond the relativistic Fermi gas opens the door for investigations of the validity of the universal character of the scaling function for electroweak processes on nuclei.

1 Introduction

Analyses of on-going and future experimental studies of neutrino reactions and oscillations at intermediate energies [1] inevitably involve nuclear targets and require accurate control of nuclear effects. One approach is to employ direct neutrino-nucleus modeling, which, while seen to be roughly correct, is typically incapable of yielding high enough quality predictions, given the demands of present experiments. A second approach that has been recently proposed takes advantage of scaling ideas.

Scaling has been extensively employed to analyze inclusive QE electron-nucleus scattering data [2, 3]. The data, when appropriately organized, scale to a function that is not only relatively independent of the momentum transfer (scaling of the first kind), but also independent of the nuclear target (scaling of the second kind). The simultaneous occurrence of both kinds of scaling is known as superscaling [3].

In order to avoid the nuclear uncertainties inherent in any neutrino-nucleus reaction model, the authors in [4, 5] have proposed the idea of a universal scaling function which is valid for both (ν, μ) and (e, e') reactions at similar kinematics. In this phenomenological SuperScaling Approach (SuSA) the scaling function is determined using electron scattering data and then carried forward to make predictions for neutrino-induced processes. These phenomenological predictions, which incorporate nuclear information provided directly by the analysis of (e, e') experimental data, are believed to be much more robust than those coming from most approaches involving direct modeling.

To date, most applications of scaling ideas to neutrino-nucleus cross sections involved charged current (CC) processes [6–13], whose kinematics parallel the electron scattering case. However, the interaction of neutrinos with matter is mediated not only by $W^{\pm}$ bosons, but also by the neutral Z^0 boson. NC processes are relevant for determining the strangeness content of the nucleon and for oscillation experiments — for instance, it is expected that they contribute as the third most important event type for the MiniBooNE experiment at Fermilab [1]. As in the case of CC processes, predictions based on scaling ideas, when possible, are clearly demanded.

The identification of CC events is relatively simple via the outgoing *charged* lepton, similar to what happens in inclusive (e, e') scattering. This means that the energy and momentum transferred at the leptonic vertex are known and thus the scaling analysis of CC neutrino-nucleus cross sections proceeds in a way identical to the electron case. However, in the case of NC events, the scattered neutrino is not detected and identification of the NC event is usually made when i) no final charged lepton is found and ii) a nucleon ejected from the nucleus is detected. Even in the case that the nucleon energy and momentum can be measured, the transferred energy and momentum at the leptonic vertex will remain unknown. The kinematics of the NC process is thus different from both electron scattering and its CC neutrino counterpart, rendering the derivation of scaling less obvious. Nevertheless, the translation of the scaling analysis to NC processes was recently outlined in [13]. There it was shown that the superscaling analysis of NC reactions in the case of the Relativistic Fermi Gas (RFG) and scattering of 1 GeV neutrinos from ^{12}C is feasible. Said study showed how to extend the scaling analysis to NC processes. Going a step further, predictions based on scaling were also provided, as well as in [14], where the Coherent Density Fluctuation Model scaling function was used to predict NC cross sections. The RFG (e, e') response exhibits perfect superscaling by definition [15], but it is not in accord with the magnitude or with the shape of the experimental scaling function. It has been shown that strong final-state interactions (FSI) are needed to describe successfully the magnitude and shape of the superscaled data, introducing also small deviations from the extracted superscaling behaviour.

In this work, we address a crucial question which arise when extending SuSA analyses to NC neutrino scattering in the QE region: i) does superscaling hold for NC neutrino-nucleus cross sections when strong FSI are present? To answer this question we use predictions from the Relativistic Impulse Approximation (RIA) [7, 8, 16–19], based on strong relativistic mean field potentials for both the bound and ejected nucleons (RIA-RMF). This model, as well as its corresponding semirelativistic version [9], reproduces the shape and magnitude of the experimental scaling curve extracted from QE (e, e') data, elusive for other theoretical models. Furthermore, RIA-RMF predicts a universal scaling function for both electron and

CC neutrino scattering [7,8,10]. Here, we verify for the first time that NC QE neutrino cross sections exhibit superscaling properties even in presence of strong FSI.

2 Results

We follow the general procedure of superscaling analyses, namely we first evaluate inclusive cross sections within a model and then obtain scaling functions by dividing them by the relevant single-nucleon cross sections [3,20]. The so-obtained scaling function is plotted against an appropriate scaling variable, and its scaling properties (first- and second-kind scaling) analyzed.

Let us start by evaluating inclusive NC neutrino-nucleus cross sections. In NC QE neutrino scattering an outgoing nucleon (mass m_N) having energy E_N, kinetic energy $T_N = E_N - m_N$ and angle θ_{kp_N} with respect to the momentum $\mathbf{k}$ of the beam is assumed to be detected. The beam energy ε is also assumed to be known. These variables determine the kinematics of the process. We assume the inclusive cross section to be basically given as the integrated semi-inclusive one-nucleon (proton or neutron) knockout cross sections. This approximation, which is implicit in scaling analyses, has been shown to work successfully in the kinematic region dominated by QE scattering. In other words, we construct the inclusive $A(\nu, N)\nu' X$ cross section by integrating the $A(\nu, \nu' N)X$ cross section over the unobserved scattered neutrino variables.

With regards to the model we employ, the NC QE neutrino-nucleus scattering is described within the relativistic impulse approximation (RIA) [21]. Our RIA model has been used to describe NC neutrino-nucleus reactions in previous work [16,18]. Here we simply summarize those aspects which are of most relevance for later discussion of the scaling properties.

The first basic assumption of the RIA is that the process occurs through the exchange of a single vector boson; this is known as the first Born Approximation (BA). In BA, a neutrino scatters off an A-body nucleus via the exchange of a Z^0. In the scattering process, a nucleon is knocked out, leaving behind an (A-1)-body daughter nucleus, generally in an excited state. The RIA also assumes the impulse approximation, *i.e.,* the incident neutrino interacts with only one nucleon, which is subsequently emitted. The nuclear current is written as a sum of single-nucleon currents. Then, the transition matrix elements from which the cross section is computed can be cast in the following form:

$$\langle J^\mu \rangle = \int d\mathbf{r}\ \overline{\phi}_F(\mathbf{r}) \hat{J}^\mu(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \phi_B(\mathbf{r})\ , \tag{1}$$

where ϕ_B and ϕ_F are relativistic bound-state and scattering wave functions, respectively. $\hat{J}^\mu$ is the relativistic one-body current operator modeling the coupling between the virtual Z^0 and a bound nucleon (see [16, 18] for details concerning the operator and nucleon form factors; in all results presented in the next section we have not allowed for strangeness content in the nucleon). We describe the bound nucleon states as self-consistent Dirac-Hartree solutions, derived within a relativistic mean field (RMF) approach using a Lagrangian containing σ, ω and ρ mesons [22].

On the one hand, if one ignores all distortions due to FSI, the scattering wave function for the outgoing nucleon is a relativistic plane wave. That is, one has the Relativistic Plane-Wave Impulse Approximation (RPWIA), which obviously entails an oversimplified description of the reaction mechanism. On the other hand, in the present work when accounting for final-state interactions between the ejected nucleon and the residual nucleus, the outgoing nucleon wave function is computed using the same relativistic mean field used to describe the initial bound states. We denote this approach as RMF.

Using these ingredients, we evaluate the six-differential cross section $d^6\sigma/d\varepsilon' d\Omega_{k'} dE_N d\Omega_N$. To get the inclusive cross section one must integrate over the three momenta of the undetected particles. In NC neutrino scattering the outgoing nucleon is assumed to be the only particle detected in the final state, and hence one integrates over the scattered neutrino variables ε' and $\Omega_{k'}$. A sum over all shells from which the nucleon may originate is also performed.

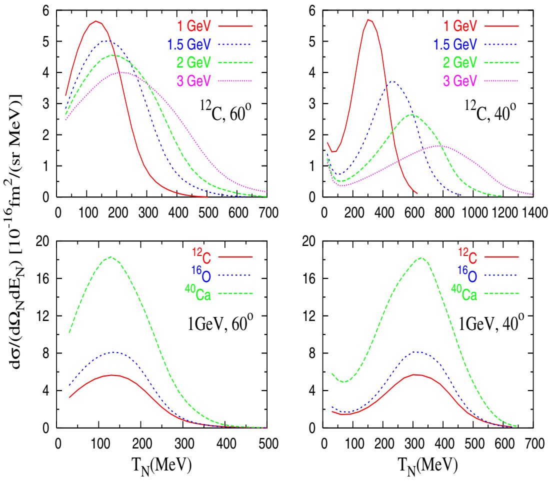

Figure 1. NC QE differential cross section $d\sigma/(dE_N d\Omega_N)$ versus the outgoing proton kinetic energy T_N for the reaction (ν, p) for different beam energies for ^{12}C (top panels) and different target nuclei at 1 GeV (bottom panels). The left-hand panels correspond to $\theta_{kp_N} = 40^o$ and the right-hand panels to 60^o.

In Fig. 1 we show the strong dependence of NC neutrino QE inclusive cross sections on the beam energy (provided that θ_{kp_N} is fixed), and on the target selected. The results are obtained with the RIA-RMF model; however a large amount of this variation is essentially due to the neutrino-nucleus coupling strength and the variation in the position of the quasielastic peak for the different beam energies. If superscaling holds, most of this dependence disappears when dividing these cross sections by the NC single-nucleon cross section given in Eq. (20) of [13] and plotting against the dimensionless scaling variable ψ^u extracted from the RFG analysis in NC kinematics (see Eq. (26) in [13] for its explicit expression). The differences in nuclear species should also been taken into account by the superscaling analysis. Results for the so-obtained scaling function $f(\psi^u)$ are presented in Fig. 2.

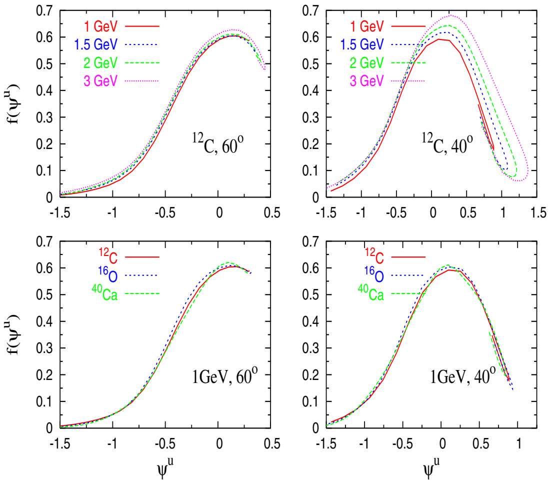

Figure 2. NC scaling functions corresponding to the differential cross sections in Fig. 1.

In the top panels of Fig. 2, one can see that first-kind scaling is well respected within RIA-RMF in the case of 60^o degrees, and also for 40^o if the region of negative ψ^u-values is considered. In other words, the large variations in the cross sections observed for different neutrino energies, are accounted for by the single-nucleon part of the cross sections, which has been factored-out in obtaining the scaling function. Furthermore, the peak of the superscaling response appears approximately at the same point for all the kinematics. However, first-kind scaling is not perfect, as for 40^o there is a sizeable increase in the height of the peaks of the curves, as well as a shift to $\psi^u > 0$ for increasing beam energy. This is similar to what is observed in RIA-RMF for the inclusive (e, e') case. Actually, the experimental (e, e') data do leave room for some breaking of first-kind scaling in the region of positive scaling variable. First-kind scaling is very well fulfilled for electron, CC and NC cases *in the absence of FSI* [5, 7, 8, 13, 23]. Therefore, the breakdown of scaling in the top-right panel of Fig. 2 must be ascribed (within IA) to FSI. In the plane-wave limit, the dependence of the cross section on the energy of the outgoing nucleon comes mainly from kinematical effects that are taken into account in the scaling analysis. However, FSI involve a redistribution of strength that depends on the energy of the final nucleon. In other words, FSI introduce an additional, non-kinematical, dependence of the cross section on T_N. If the kinematics of the process are such that the range of energies of the ejected nucleon depends strongly on the beam energy, the nucleon will be subject to different FSI for each ε, and a visible breakdown of first-kind scaling will show up. This is what happens for $\theta_{kp_N} = 40^o$, where there is a strong shift of the position of the peak of the cross section with incoming beam energy. However, for those kinematics for which the range of T_N remains approximately the same when considering different beam energies, as for $\theta_{kp_N} = 60^o$, first-kind scaling is obtained even with FSI included, as FSI effects on the knockout nucleon are similar for different beam energies.

Incidentally, in Fig. 2 we also observe that $f(\psi^u)$ can be, for pure kinematical reasons, a bivalued function of the scaling variable ψ^u, as the same value of ψ^u may be obtained, at fixed beam energy and nucleon angle, for two different values of the outgoing nucleon energy. In the absence of FSI (as in ref. [13]), superscaling is a good approximation and the two values of the superscaling function for these ψ^u are nearly equal. When FSI are present, and if the kinematics prevents superscaling (as for $\theta_{kp_N} = 40^o$), the bivalued nature of the superscaled function is revealed.

Results for scaling of the second kind are presented in the bottom panels of Fig. 2. The superscaling functions obtained for several nuclei are almost identical, in spite of the strong difference in magnitude of the corresponding cross sections (*cf.* Fig. 1). That is, the dependence on the nuclear species is well accounted for by the superscaling analysis. Scaling of second kind is seen to be very robust, thereby opening up a means of taking into account nuclear effects for different nuclei employing superscaling ideas.

3 Conclusions

In this work we have used the relativistic impulse approximation (RIA) to analyze neutral-current neutrino-nucleus scattering from nuclei. Inclusive differential cross sections have been evaluated for various choices of kinematics and nuclei. An essential ingredient in our model comes from the description of the final-state interactions (FSI) between the emitted nucleon and the residual nucleus. We describe the outgoing nucleon wave functions making use of the same relativistic mean field potential (RMF) already used for describing the initial bound nucleon states. This approach to FSI has been shown to be very successful in describing the inclusive electron scattering superscaling functions.

Our main aim in this work has been the analysis of scaling and superscaling properties for NC neutrino scattering within the RIA. Up to now, only the RFG model had been shown to scale, and results had been illustrated for scattering of 1 GeV neutrinos from ^{12}C. In our model, we evaluate the superscaling function $f(\psi^u)$ in a realistic nuclear model that includes FSI, and display it as a function of the scaling variable ψ^u for various values of beam energy and for various target nuclei. Proceeding in this way we investigate scaling of the first kind, *i.e.,* independence of the scaling function on the *transferred momentum* , and scaling of the second kind (no dependence with the target nucleus selected). From our results, we conclude that results with the RMF approach show that scaling of the second kind works extremely well. However, small violations of first kind scaling are observed for $\theta_{kp_N} = 40^o$, particularly in the region of positive ψ^u-values (low nucleon kinetic energies). This is in accordance with what is observed for inclusive electron scattering, where the RMF results show a moderate violation of first-kind scaling that, on the other hand, is not excluded from the experimental scaling function. Importantly, if the kinematics is such that the range of energies spanned by the ejected nucleon is nearly independent of the incoming neutrino energy, as for $\theta_{kp_N} = 60^o$, first-kind scaling is well respected even in the presence of strong FSI. The existence of superscaling within a model for NC neutrino-nucleus interactions including strong FSI opens the door for investigations of the validity of the universal character of the scaling function for electroweak processes on nuclei.

Acknowledgments

This work was partially supported by DGI (Spain) and FEDER funds: FIS2005-01105, FPA2006-07393, FPA2006-13807 and FPA2007-62216, by the Junta de Andalucía, and by the INFN-CICYT collaboration agreement “Study of relativistic dynamics in electron and neutrino scattering”, and by the Comunidad de Madrid and UCM (910059 ’Grupo de Física Nuclear’ and PR1/07-14895). It was also supported in part (TWD) by U.S. Department of Energy Office of Nuclear Physics under contract No. DE-FG02-94ER40818. M.C.M acknowledges a ’Juan de la Cierva’ contract from MEC. The computations were performed in the ’cluster for physics’ of UCM, funded by UCM and FEDER funds.

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