Signatures of deformation on single and double beta decay

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Abstract. We study Gamow-Teller strength distributions within a QRPA approach performed on top of a deformed Skyrme-Hartree-Fock+BCS single particle basis. We include residual spin-isospin interactions in both particle-hole and particle-particle channels. We address different problems of interest in Nuclear Structure and Nuclear Astrophysics, such as the $\beta-$decay properties of proton-rich medium-mass nuclei of astrophysical interest and the deformation dependence of the Gamow-Teller strength distributions in neutron deficient Pb isotopes. We also discuss the role of deformation in the two-neutrino double beta decay.

1 Introduction

Exploring the properties of nuclei with unusual proton to neutron ratios is nowadays one of the most active topics in Nuclear Structure Physics and Nuclear Astrophysics, both theoretical and experimentally. The new experimental facilities involving radioactive ion beams provide new information and correspondingly new theoretical efforts to its description. In particular, the decay properties and nuclear reactions of exotic nuclei are essential ingredients to understand the late phases of the stellar life. Since this input cannot be determined experimentally for the extreme conditions of temperature and density holding in the stellar medium, reliable theoretical calculations for these processes are absolutely necessary. As an example, the decay properties of proton rich nuclei are fundamental to understand X-ray bursts and their associated rp-processes.

The theoretical framework used to describe these processes has been explained in detail elsewhere [1]. We construct first the quasiparticle basis selfconsistently from a deformed Hartree-Fock (HF) calculation with density-dependent Skyrme forces and pairing correlations. Then we solve the QRPA equations with separable residual interactions in both particle-hole (ph) and particle-particle (pp) channels.

In this work, we stress the role played by the nuclear deformation in various selected cases of special interest. We discuss the results obtained for the β-decay of a series of proton rich isotope chains approaching $N = Z$ in the mass region $A = 70$, in the iron mass region and in the neutron-deficient even isotopes in the lead and neighboring nuclei. We compare our results to available data for Gamow-Teller (GT) strength distributions and half-lives, as well as with other theoretical approaches. We also discuss the sensitivity of our calculations to deformation, pairing, and residual interactions.

Finally, we apply the formalism to the simultaneous description of single beta Gamow-Teller strength distributions and to the two-neutrino double beta decay matrix elements in double beta emitters. We discuss the sensitivity of the double beta decay matrix elements to the deformed mean field and to the residual interactions. Nuclear deformation is found to be a mechanism of suppression of the two-neutrino double beta decay.

2 Skyrme HF+BCS+QRPA formalism in deformed nuclei

It is well known that the Skyrme HF approximation gives a very good description of ground-state properties for both spherical and deformed nuclei [2] and it is at present the most reliable mean field description. Most of the results in this work are obtained with the force SLy4 [3].

The single-particle HF solutions for axially symmetric deformed nuclei are expanded into the eigenfunctions of an axially deformed harmonic-oscillator potential using twelve harmonic oscillator major shells N. Truncation effects are minimized by a proper choice of the oscillator parameters of the basis. We include pairing between like nucleons in the BCS approximation with fixed gap parameters for protons Δ_π, and neutrons Δ_ν, which are determined phenomenologically from the odd-even mass differences.

To describe Gamow-Teller transitions we add to the mean field a spin-isospin residual interaction, which is expected to be the most relevant force for that purpose. This interaction contains two parts ph and pp. The ph part is responsible for the position and structure of the GT resonance [1,4]. It may be derived consistently from the same energy density functional (and Skyrme interaction) as the HF equation, in terms of the second derivatives of the energy density functional with respect to the one-body densities. The ph residual interaction is written in a separable form by averaging the Landau-Migdal resulting force over the nuclear volume, as explained in Refs. [1, 5].

$$V_{GT}^{ph} = 2\chi_{GT}^{ph} \sum_{K=0,\pm1} (-1)^K \beta_K^+ \beta_{-K}^- , \qquad \beta_K^+ = \sum_{\pi\nu} \langle \nu | \sigma_K | \pi \rangle \, a_\nu^+ a_\pi \, . \quad (1)$$

The coupling strength χ_{GT}^{ph} is determined by the Skyrme parameters, the nuclear radius, and the Fermi momentum [1]

$$\chi_{GT}^{ph} = -\frac{3}{8\pi R^3} \left\{ t_0 + \frac{1}{2} k_F^2 \, (t_1 - t_2) + \frac{1}{6} t_3 \rho^\alpha \right\} = \frac{3G_0'}{2\pi R^3 N_0} \, . \quad (2)$$

The particle-particle part is a neutron-proton pairing force in the $J^\pi = 1^+$ coupling channel. We introduce this interaction in the usual way [4], that is, in terms of a separable force with a coupling constant κ_{GT}^{pp}, which is fitted to the phenomenology.

$$V_{GT}^{pp} = -2\kappa_{GT}^{pp} \sum_K (-1)^K P_K^+ P_{-K} \,, \qquad P_K^+ = \sum_{\pi\nu} \left\langle \pi \left| (\sigma_K)^+ \right| \nu \right\rangle a_\nu^+ a_{\bar{\pi}}^+ \,. \quad (3)$$

The proton-neutron QRPA phonon operator for GT excitations in even-even nuclei is written as

$$\Gamma_{\omega_K}^+ = \sum_{\pi\nu} \left[X_{\pi\nu}^{\omega_K} \alpha_\nu^+ \alpha_{\bar{\pi}}^+ + Y_{\pi\nu}^{\omega_K} \alpha_{\bar{\nu}} \alpha_\pi \right] \,, \tag{4}$$

where ω_K are the excitation energies, and $X_{\pi\nu}^{\omega_K}, Y_{\pi\nu}^{\omega_K}$ the forward and backward amplitudes, respectively. They are obtained by solving the QRPA equations. For even-even nuclei the GT transition amplitudes in the intrinsic frame connecting the QRPA ground state $|0\rangle$ $[\Gamma_{\omega_K}|0\rangle = 0]$ to one phonon states $|\omega_K\rangle$ $\left[\Gamma_{\omega_K}^+|0\rangle = |\omega_K\rangle\right]$, are given by

$$\left\langle \omega_K | \beta_K^\pm | 0 \right\rangle = \mp M_\pm^{\omega_K} \,, \tag{5}$$

with

$$M_-^{\omega_K} = \sum_{\pi\nu} \left(q_{\pi\nu} X_{\pi\nu}^{\omega_K} + \tilde{q}_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right) \,, \quad M_+^{\omega_K} = \sum_{\pi\nu} \left(\tilde{q}_{\pi\nu} X_{\pi\nu}^{\omega_K} + q_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right) \,, \tag{6}$$

$$\tilde{q}_{\pi\nu} = u_\nu v_\pi \Sigma_K^{\nu\pi} \,, \quad q_{\pi\nu} = v_\nu u_\pi \Sigma_K^{\nu\pi} \,, \tag{7}$$

v's are occupation amplitudes ($u^2 = 1 - v^2$) and $\Sigma_K^{\nu\pi}$ spin matrix elements connecting neutron and proton single-particle states with spin operators

$$\Sigma_K^{\nu\pi} = \langle \nu | \sigma_K | \pi \rangle \,. \tag{8}$$

Once the intrinsic amplitudes $\left\langle f \left| \beta_K^\pm \right| i \right\rangle$ are calculated, the Gamow-Teller strength $B(GT)$ in the laboratory system for a transition $I_i \to I_f$ can be obtained as

$$B\left(GT^\pm\right) = \sum_{M_i, M_f, \mu} \left| \left\langle I_f M_f \left| \beta_\mu^\pm \right| I_i M_i \right\rangle \right|^2 \,, \tag{9}$$

in units of $g_A^2/4\pi$. The initial and final states in the laboratory frame are expressed in terms of the intrinsic states $|\phi_K\rangle$ using the Bohr-Mottelson factorization. In the case of even-even parent nuclei, $I_i = K_i = 0$, $I_f = 1$, and $K_f = 0, 1$, we obtain

$$B\left(GT^\pm\right) = \frac{g_A^2}{4\pi} \left\{ \delta_{K_f,0} \left\langle \phi_{K_f} \left| \beta_0^\pm \right| \phi_0 \right\rangle^2 + 2\delta_{K_f,1} \left\langle \phi_{K_f} \left| \beta_1^\pm \right| \phi_0 \right\rangle^2 \right\} \,. \tag{10}$$

The total half-life $T_{1/2}$ for allowed β decay from the ground state of the parent nucleus is obtained by summing the strengths over all the final states involved in the process with excitation energy below the $Q-$value,

$$T_{1/2}^{-1} = \frac{\kappa^2}{6200} \sum_{\omega} f\left(Z,\omega\right) \left| \left\langle 1_\omega^+ \left\| \beta^+ \right\| 0^+ \right\rangle \right|^2 , \tag{11}$$

where $f\left(Z,\omega\right)$ is the Fermi integral. We include effective factors

$$\kappa^2 = \left[\left(g_A/g_V\right)_{\rm eff} \right]^2 = \left[0.77 \left(g_A/g_V\right)_{\rm free} \right]^2 . \tag{12}$$

3 Gamow-Teller strength distributions and half-lives

We have investigated the GT properties of nuclei in the region of medium masses around A=56, which are of special importance because they are the main constituents of the stellar core in presupernovae formations. In the last decades, GT_+ strength distributions in this mass region have been studied experimentally via (n,p) charge exchange reactions at forward angles. We find that the present pnQRPA calculations are able to reproduce the main features of the GT properties in this mass region [6], reinforcing the confidence in the method and in its predictive power. Our pnQRPA results are comparable to the results obtained from full shell model calculations [7]. One advantage of the QRPA approach is that it can be extended to much heavier nuclei beyond the present capability of the full shell model calculations, without increasing the complexity of the calculations.

We have also investigated the proton-rich medium-mass region around mass A=70, which is characterized by a large variety of competing nuclear shapes. The HF theory gives a single solution, which is the Slater determinant of lowest energy. To study shape coexistence one has to extend the theory to a constrained HF theory. In this way, minimization of the HF energy under the constraint of holding the nuclear deformation fixed is carried out over a range of deformations. When more than one local minimum occurs for the total energy as a function of deformation, shape coexistence results. This type of constrained calculations has been done in Ref. [1] with the result that most neutron-deficient Ge, Se, Kr, and Sr isotopes are candidates for shape coexistence.

The sensitivity of the GT strength distributions to the deformation of the decaying nucleus has been exploited as an alternative tool to extract information on the shape of exotic nuclei. Comparison of theoretical GT strength distributions obtained from various shapes with recent experimental data from CERN-ISOLDE [8] shows that while the data are incompatible with an oblate shape and favors strongly a prolate ground state in the case of ^{76}Sr, in the case of ^{74}Kr the data indicate an oblate/prolate shape mixing.

We have calculated the half-lives of waiting point nuclei involved in rp-processes in the mass region A=60-90 within the QRPA formalism [9]. We have analyzed the half-lives as a function of deformation. The results obtained indicate that this formalism is a useful method for reliable calculations of half-lives, which is especially interesting for applications to cases where no experimental information is available and to nuclei that are beyond the capability of full shell model calculations.

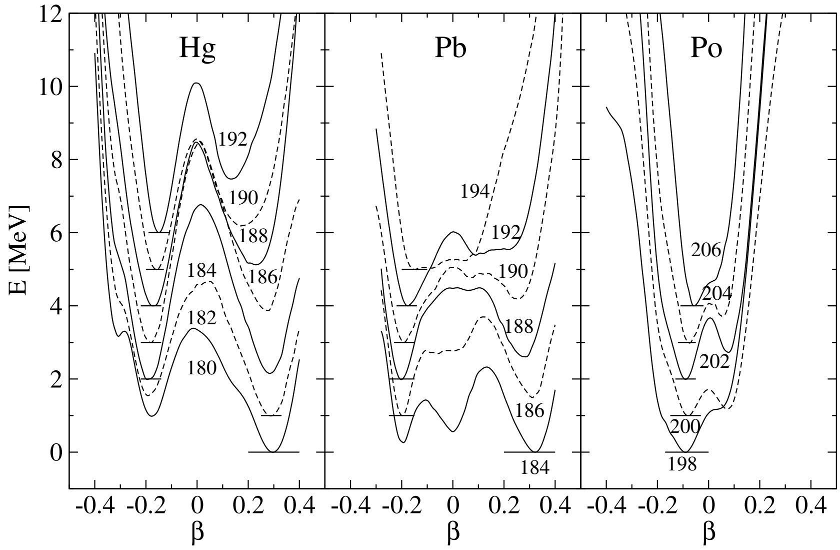

Figure 1. HF energy scaled to the ground-state energy as a function of the quadrupole deformation.

This type of analysis has been recently extended to the study of the GT strength distributions in neutron-deficient even Hg, Pb, and Po isotopes, where the phenomenon of shape coexistence has been experimentally observed [10]. By analyzing the sensitivity of the GT strength distributions to the various ingredients in the formalism, we conclude that the β-decay of these isotopes could be a useful tool to look for fingerprints of nuclear deformation [11].

Fig. 1 shows the results obtained from a constrained HF calculation in this mass region. By investigating the sensitivity of our results to the forces used, we arrive to the conclusion that the energy deformation curves are sensitive to the Skyrme and pairing forces used. The ground state shape predicted may be modified with different choices for these forces. Nevertheless, the deformations at which the minima occur are stable and the shape coexistence between the various shapes is a constant feature in these isotopes. We also observe that the GT strength distributions show specific characteristics for each deformation that remain against changes of the Skyrme and pairing forces. The effect of the deformation on the GT strength distributions is much stronger than the effects coming from the Skyrme or pairing forces used. As a consequence, we have identified clear signatures of deformation on the GT strength distributions in Hg, Pb and Po isotopes [11]. These signatures are related to the profiles of the GT strength distributions, which are peaked at different energies depending on the shape of the decaying nucleus. An example of these results can be seen in Fig. 2 (for Hg isotopes), Fig. 3 (for Pb isotopes) and Fig. 4 (for Po isotopes).

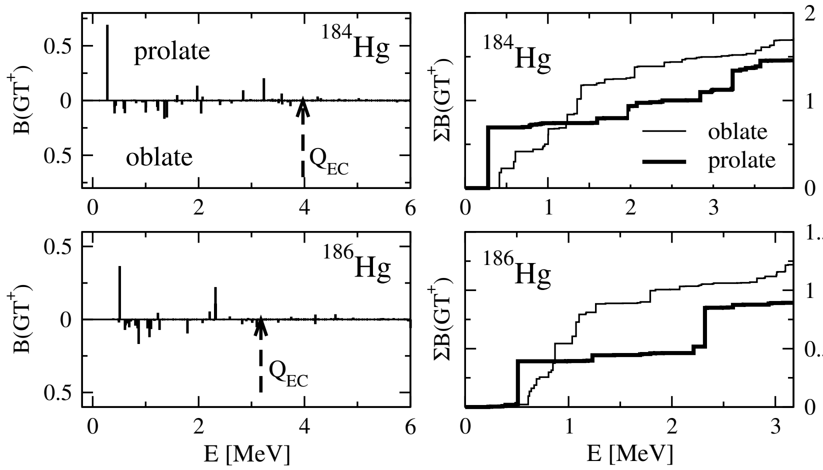

Figure 2. (Left) GT strength distributions in Hg isotopes for prolate (upward) and oblate (downward) shapes. (Right) Accumulated GT strength for prolate and oblate shapes plotted up to Q_{EC} energies.

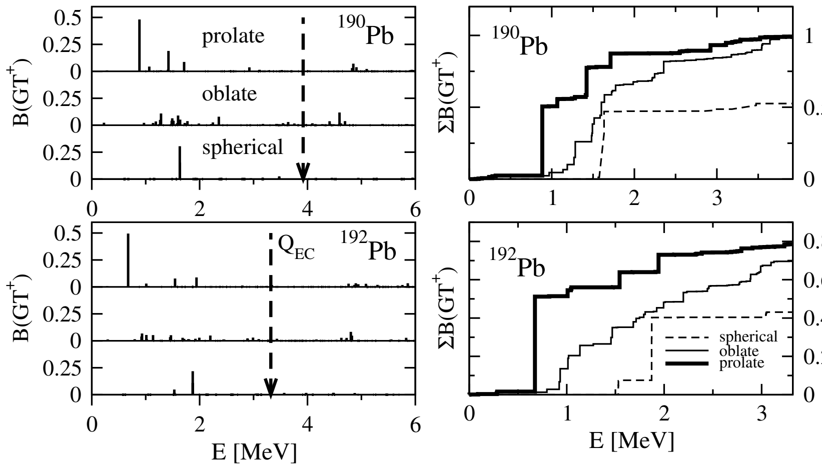

Figure 3. Same as in Fig. 2 for Pb isotopes. In this case we also include results for spherical shapes.

We can also see in Table 1 the comparison of the calculated half-lives with the experimental ones. First we show the experimental percentage of the β^+/EC involved in the total decay. In the next column we show the total experimental half-lives [s] and within brackets, the β^+/EC experimental half-lives extracted from the percentage. The table also contains the theoretical half-lives obtained with the force SLy4 as well as the $B(GT^+)$ strength $[g_A^2/(4\pi)]$ summed up to Q_{EC} energies.

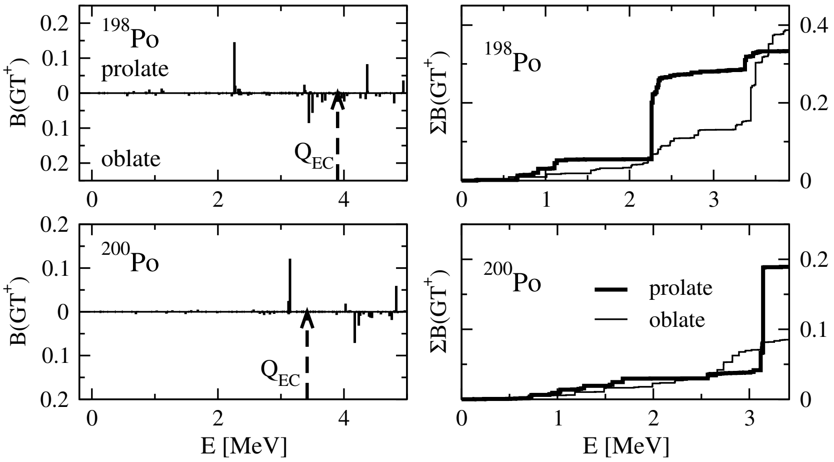

Figure 4. Same as in Fig. 2 for Po isotopes.

Table 1. Comparison of experimental and theoretical half-lives for some selected isotopes.

Isotope		% (β^+/EC)	$T^{\rm total}_{1/2,exp}$ (β^+)	$T^{\beta^+/EC}_{1/2,SLy4}$	$\Sigma_{Q_{EC}} B(GT^+)$
^{184}Hg	obl	98.89	30.6 (30.9)	17.5	1.71
	prol			17.0	1.46
^{186}Hg	obl	100	82.8 (82.8)	47.2	1.22
	prol			68.4	0.89
^{190}Pb	sph	99.6	71 (71)	80.0	0.54
	obl			41.7	0.99
	prol			26.4	0.99
^{192}Pb	sph	100	210 (210)	251.7	0.41
	obl			93.4	0.70
	prol			45.6	0.79
^{198}Po	obl	43	106 (247)	342	0.387
	prol			160	0.333
^{200}Po	obl	88.9	690 (776)	1390	0.085
	prol			1066	0.189

4 Two-neutrino double beta decay

Nuclear double beta decay is a rare second order weak interaction process that takes place when the transition to the intermediate nucleus is energetically forbidden or highly retarded. Two main decay modes are expected in this process. The two neutrino mode, involving the emission of two electrons and two neutrinos, and the neutrinoless mode with no neutrino leaving the nucleus. While the first type of process is perfectly compatible with the Standard Model, the second one violates lepton number conservation and its observation is linked to the existence of a massive Majorana neutrino. For this reason, considerable experimental and theoretical effort is being devoted to the study of this process [12].

From the theoretical point of view, one particular source of uncertainty is the evaluation of the nuclear matrix elements involved in the process. They have to be calculated as accurately as possible to obtain reliable estimates for the limits of the double beta decay half-lives. Since the nuclear wave functions and the underlying theory for treating the neutrinoless and the two-neutrino modes are similar, the usual procedure is to test first the nuclear structure component of the two-neutrino mode against the available experimental information on half-lives.

We use a deformed QRPA formalism [13] to describe simultaneously the energy distributions of the single beta Gamow-Teller strength that build up the double beta process and the two-neutrino double beta decay ($2\nu\beta\beta$) matrix elements. Calculations are performed using deformed Woods-Saxon potentials and deformed Skyrme Hartree-Fock mean fields in those cases where the ($2\nu\beta\beta$) half-lives have been measured, namely, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{136}Xe and ^{150}Nd, and their corresponding partners.

The $2\nu\beta\beta$ decay is described in second order perturbation of the weak interaction as two successive Gamow-Teller transitions via virtual intermediate 1^+ states. The half-life of the $2\nu\beta\beta$ decay

$$T_{1/2}^{2\nu}\left(0^+_{\rm gs}\rightarrow 0^+_{\rm gs}\right)=\left[G^{2\nu}\left|M^{2\nu}_{\rm GT}\right|^2\right]^{-1} \tag{13}$$

is given as a product of a phase space integral $G^{2\nu}$ and the Gamow-Teller transition matrix element $M^{2\nu}_{\rm GT}$, which contains the nuclear structure effects. For a transition connecting initial and final ground states, it is given by

$$M^{2\nu}_{\rm GT}=\sum_K\sum_{m_i,m_f}\frac{\left\langle 0^+_f\,||\,\sigma_K t^-\,||\,\omega_K^{m_f}\right\rangle\left\langle\omega_K^{m_f}\,|\,\omega_K^{m_i}\right\rangle\left\langle\omega_K^{m_i}\,||\,\sigma_K t^-\,||\,0^+_i\right\rangle}{\left(\omega_K^{m_f}+\omega_K^{m_i}\right)/2}\,, \tag{14}$$

where $K=0,\pm1$ and m_i, m_f label the number of intermediate 1^+ RPA states $\omega_K^{m_i}$, $\omega_K^{m_f}$ reached from the initial $|0^+_i>$ and final $|0^+_f>$ nuclear ground states, respectively. The overlap $<\omega_K^{m_f}|\omega_K^{m_i}>$ is needed to take into account the non-orthogonality of the intermediate states reached from the initial ground state to those reached from the final ground state.

We discuss the sensitivity of the parent and daughter Gamow-Teller strength distributions in single beta decay, as well as the sensitivity of the double beta decay matrix elements to the deformed mean field and to the residual interactions. In the case of HF the deformation is determined selfconsistently and we are able to reproduce the experimental charge radii and quadrupole moments. In the case of WS the input deformation is taken from experiment.

Nuclear deformation is found to be a mechanism of suppression of the two-neutrino double beta decay [13]. The double beta decay matrix elements are found to have maximum values for about equal deformations of parent and daughter nuclei. They decrease rapidly when differences in deformations increase.

We find that we need different strengths of the ph force to reproduce the position of the GT resonance, depending on the HF or WS basis. It should also be mentioned that the GT^- strength of the parent nucleus and the GT^+ strength of the daughter are located at different energies, a feature that is relevant for double beta decay because it introduces a reduction of the double beta decay probabilities. It would be very useful to improve and complete the experimental information on GT strength distributions by (p,n) and (n,p) charge exchange reactions on nuclei participating in double beta decay.

5 Conclusions

We have applied a deformed HF+BCS+QRPA formalism with density-dependent two-body effective Skyrme interactions to the description of β-decay properties in various isotopic chains of medium-mass proton-rich nuclei, as well as to the study of Gamow-Teller strength distributions in the iron and lead mass regions.

Very reasonable agreement with ground state, $\beta-$decay properties, as well as with GT and spin $M1$ strength distributions is obtained. In the $A \sim 70$ mass region we have found shape isomerism in most of the isotopes studied and a nice agreement with available experimental data on Q_{EC} values, half-lives, and GT strength distributions. From our study of the dependence on the shape of the GT strength distributions we conclude that information on the shape of the parent nucleus can be gained by comparison with experiment in the whole Q_{EC} window.

In the iron mass region, the comparison of pnQRPA results to data on GT strength distributions provides a fairly sound basis to safely apply this method to the estimates of GT strengths and particularly of $\beta-$decay properties of highly unstable nuclei in other mass regions. Though the GT strength distributions obtained from full shell model calculations may agree better with experiment in some details, the present HF+BCS+pnQRPA results are, on the overall, of comparable quality.

The half-lives of N=Z waiting point nuclei of relevance in the rp-process have been calculated within the QRPA formalism and analyzed as a function of deformation. The agreement of the results obtained with the available experimental information indicates that the formalism is useful to extrapolate to regions unexplored and to different conditions of densities and temperatures.

The effect of deformation on the $2\nu\beta\beta-$decay matrix elements has been studied first by considering the deformations of both parent and daughter as free parameters. It is found that the matrix elements are suppressed with respect to the spherical case. More precisely, it is found a sizable reduction effect that scales with the deformation difference between parent and daughter. This suppression mechanism, which is ignored in spherical treatments, may play an important role in approaching the theoretical estimates to experiment.

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