# Gamow-Teller Transitions in Hot Nuclei

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**Abstract.** A formalism based on the thermo field dynamics and allowing to treat thermal effects on the Gamow-Teller strength distributions in hot spherical nuclei is presented. The GT$_{\pm}$ strength distributions in $^{54,56}$Fe at temperature $T \leq 1$ MeV are calculated with the model Hamiltonian which contains a pairing BCS interaction and separable isovector $\sigma\tau$ particle-hole forces. Then $\beta^{-}$-decay and electron capture rates are calculated at temperatures and stellar media densities corresponding to an advanced stage of stellar evolution.

## 1 Motivation

Now it is well established that the Gamow-Teller (GT) resonance in atomic nuclei plays a very important role in many astrophysical processes related to weak interaction mediated reactions. The examples are nuclear beta-decays and electron captures (EC) which occur during the presupernova phase in the core collapse of a massive star and play a decisive role at the advanced stages of stellar evolution (see e.g. Ref. [1] and references therein). Under supernova conditions (high temperature and density) both the processes are mainly driven by the GT transitions and, thus, their reliable determination requires the accurate description of the GT strength distributions in nuclear spectra.

At the advanced stages of stellar evolution the capture and decay processes in stellar environment occur not only from the ground states of nuclei but also from thermally populated excited states that can increase the rates significantly [2]. Since the GT transitions from excited states are not accessible by terrestrial laboratory experiments, a reliable theoretical prediction is requested as a nuclear physics input in simulations of the stellar evolution.

A variety of approaches have been used to take into account thermal effects on the GT transition distributions in nuclei from Fe region (see e.g. [3–7] and references therein).

In the present paper we study the GT strength distributions in hot nuclei applying the quasiparticle-phonon model (QPM) [8] extended to finite temperatures exploring the thermo field dynamics (TFD) [9, 10]. Moreover, a temperature dependence of $\beta$-decay and electron capture rates for $^{54,56}$Fe nuclides is calculated. Actually the problem of extending the QPM to finite temperatures was already studied earlier in [11, 12] (see also [13]). However, there is some difference between previous papers and the present consideration.

## 2 TFD formalism

The TFD is based on the fact that the grand canonical ensemble average of any operator $\hat{A}$ can be replaced by a quantum expectation value of the same operator with respect to a temperature dependent function $|0(T)\rangle$ called a thermal vacuum state [9]. That is,

$$\ll \hat{A} \gg = Z^{-1}(T)\mathrm{Tr}(\hat{A}\, \mathrm{e}^{-H/T}) = \langle 0(T)|\hat{A}|0(T)\rangle \ . \tag{1}$$

Here $H$ is the Hamiltonian of the system (the chemical potential is included in $H$); $T$ is a temperature given in units of energy; and $Z$ is a partition function.

A thermal vacuum $|0(T)\rangle$ cannot be constructed in the Hilbert space of the original system [9, 10]. Instead, this can be done after a formal doubling of a number of system degrees of freedom which is achieved by introducing a fictitious tilde-space spanned by the tilde-states $|\tilde{n}\rangle$. Then the whole Hilbert space of the heated system is spanned by a direct product of eigenstates $|n\rangle$ of the Hamiltonian ( $H|n\rangle = E_n|n\rangle$ ) belonging to the original Hilbert space and those of the tilde Hamiltonian $\widetilde{H}$ which correspond to the same eigenvalues, i.e. $\widetilde{H}|\widetilde{n}\rangle = E_n|\widetilde{n}\rangle$ .

The time translation operator for a heated system is the thermal Hamiltonian $\mathcal{H} = H - \widetilde{H}$. In order to find the excitation spectrum of a heated system one should diagonalize $\mathcal{H}$. The thermal vacuum state is the eigenstate of $\mathcal{H}$ corresponding to zero eigenvalue. As was shown in [14] the diagonalization of $\mathcal{H}$ under certain assumptions produces the thermal Hartree-Fock-Bogoliubov equations. A linear extension of the theory produces the thermal random phase approximation. In case if the corresponding equations have several solutions one should take the solution which provides a minimum of the corresponding thermodynamic potential

$$\Omega = \langle 0(T)|H - T\hat{K}|0(T)\rangle \ , \tag{2}$$

Here, $|0(T)\rangle$ is a thermal vacuum within the corresponding approximation and $\hat{K}$ is the entropy operator of the system.

## 3 Proton-neutron RPA at finite temperatures

Here we apply the TFD formalism to evaluate the equations of the thermal proton-neutron RPA. To simplify the corresponding equations we adopt a schematic nuclear Hamiltonian with separable spin-isospin interaction in the particle-hole channel. The Hamiltonian contains three different terms

$$H = H_{\mathrm{sp}} + H_{\mathrm{pair}} + H_{\mathrm{ph}}, \tag{3}$$

where $H_{\mathrm{sp}}$ is a sum of neutron and proton mean fields and $H_{\mathrm{pair}}$ is a sum of the BCS pairing Hamiltonians for protons and neutrons. The term $H_{\mathrm{ph}}$ has the following form:

$$H_{\mathrm{ph}} = -2\chi \sum_{\mu} S^{(-)}_{\mu} (-1)^{\mu} S^{(+)}_{-\mu} \ . \tag{4}$$

where the one-body operator $S^{(-)}_\mu$ reads

$$S^{(-)}_\mu = \sum_{\substack{j_p m_p \\ j_n m_n}} \langle j_p m_p|\sigma_\mu t^-|j_n m_n\rangle a^+_{j_p m_p} a_{j_n m_n}\ , \quad S^{(+)}_\mu = (-1)^\mu (S^{(-)}_{-\mu})^+\ , \quad (5)$$

The following notations are used above: $a^+_{jm}$ and $a_{jm}$ are the creation and annihilation operators of particles with subshell quantum numbers $jm \equiv n,l,j,m$; $\chi$ is the spin-isovector coupling constant; $t^-$ is isospin lowering operator.

The Hamiltonian (3) was widely used in theoretical studies of the GT resonances and in framework of the QPM in particular [15, 16].

First, following the TFD prescriptions we construct the thermal Hamiltonian $\mathcal{H}$ and introduce thermal quasiparticle operators which diagonalize single-particle and pairing part of it. Original particle operators and the thermal quasiparticle operators are connected by two unitary Bogoliubov transformations. The first one is the standard Bogoliubov transformation which mixes the creation and annihilation operators of particle:

$$\begin{aligned} a_{jm} &= u_j \alpha_{jm} + v_j \alpha^+_{\overline{jm}}\ , \\ a^+_{jm} &= u_j \alpha^+_{jm} + v_j \alpha_{\overline{jm}}\ , \quad (u_j^2 + v_j^2 = 1)\ . \end{aligned} \quad (6)$$

Notation $\overline{jm}$ stands for the time reversed state. The same transformation (with the same $u_j$ and $v_j$) should be made for tilde-operators $\widetilde{a}^+_{jm},\ \widetilde{a}_{jm}$ thus producing tilde-quasiparticles $\widetilde{\alpha}^+_{jm}$ and $\widetilde{\alpha}_{jm}$. To take into account thermal effects the second (or thermal) Bogoliubov transformation should be made. It is the following transformation from ordinary and tilde quasiparticle operators to thermal quasiparticle operators:

$$\begin{aligned} \alpha^+_{jm} &= x_j \beta^+_{jm} + i y_j \widetilde{\beta}_{jm}\ , \\ \widetilde{\alpha}^+_{jm} &= x_j \widetilde{\beta}^+_{jm} - i y_j \beta_{jm}\ , \quad (x_j^2 + y_j^2 = 1)\ . \end{aligned} \quad (7)$$

We should stress that in contrast with previous works [9, 11–13] we prefer here to explore a complex thermal transformation. In this respect we follow to I. Ojima [17] who has shown that this type of thermal rotation for fermion operators allow to get right asymptotic behavior for RPA operators of thermal phonons (details will be given elsewhere [18]).

The coefficients $u_j,\ v_j$ and $x_j,\ y_j$ are determined demanding a minima of the free energy of a nucleus governed by the Hamiltonian

$$\mathcal{H}_{\rm sqp} = H_{\rm sp} + H_{\rm pair} - \widetilde{H}_{\rm sp} - \widetilde{H}_{\rm pair}$$

on the thermal vacuum state $|0(T)\rangle_{\rm sqp}$ defined as follows: $\beta_{jm}|0(T)\rangle_{\rm sqp} = 0$. As a result one obtains the well known BCS equations at finite temperature for the neutron and proton pairing gaps $\Delta_{\rm n,p}$ and chemical potentials $\lambda_{\rm n,p}$ (see for details [11–13]). The coefficients $x_j,\ y_j$ appear to be directly connected with the Fermi-Dirac thermal occupation numbers for the Bogoliubov quasiparticles, namely

$$y_j = \left[1 + \exp\left(\frac{\varepsilon_j}{T}\right)\right]^{-1/2} .$$

where $\varepsilon_j$ is the energy of thermal quasiparticle state $\beta^+_{jm}|0(T)\rangle_{\rm sqp}$. With the coefficients $u_j,\ v_j,\ x_j,\ y_j$ the Hamiltonian $\mathcal{H}_{\rm sqp}$ becomes diagonal in terms of thermal quasiparticles

$$\mathcal{H}_{\rm sqp} = \mathcal{H}_{\rm sp} + \mathcal{H}_{\rm pair} \simeq \sum_{jm} \varepsilon_j (\beta^+_{jm}\beta_{jm} - \widetilde{\beta}^+_{jm}\widetilde{\beta}_{jm}), \tag{8}$$

i.e. $\mathcal{H}_{\rm sqp}$ describes a system of independent thermal quasiparticles.

At the next step we take into account a long range particle-hole forces $H_{\rm ph}$ (4) within the thermal version of the proton-neutron RPA approximation. To proceed, we introduce the following thermal GT phonon operator:

$$\begin{aligned} Q^+_{\mu i} = \frac{1}{\sqrt{2}} \sum_{j_p j_n} &\Big(\psi^i_{j_p j_n}[\beta^+_{j_p}\beta^+_{j_n}]_\mu + \widetilde{\psi}^i_{j_p j_n}[\widetilde{\beta}^+_{\overline{j_p}}\widetilde{\beta}^+_{\overline{j_n}}]_\mu - i\eta^i_{j_p j_n}[\beta^+_{j_p}\widetilde{\beta}^+_{\overline{j_n}}]_\mu - i\widetilde{\eta}^i_{j_p j_n}[\widetilde{\beta}^+_{\overline{j_p}}\beta^+_{j_n}]_\mu\Big) \\ &+ \Big(\phi^i_{j_p j_n}[\beta_{\overline{j_p}}\beta_{\overline{j_n}}]_\mu + \widetilde{\phi}^i_{j_p j_n}[\widetilde{\beta}_{j_p}\widetilde{\beta}_{j_n}]_\mu - i\xi^i_{j_p j_n}[\beta_{\overline{j_p}}\widetilde{\beta}_{j_n}]_\mu - i\widetilde{\xi}^i_{j_p j_n}[\widetilde{\beta}_{j_p}\beta_{\overline{j_n}}]_\mu\Big) . \end{aligned} \tag{9}$$

Above, the square brackets $[\ ]_\mu$ stand for coupling of single-particle angular momenta of proton and neutron thermal quasiparticles $j_p,\ j_n$ to the total angular momentum 1 and its projection $\mu$. Further, it is assumed that thermal phonons can be treated as bosons. This produces some constrains on the phonon amplitudes [12].

Then the thermal RPA equations are obtained by applying a variational principle

$$\delta\Big[\langle\Psi_0(T)|Q_{\mu i}\mathcal{H}Q^+_{\mu i}|\Psi_0(T)\rangle - \frac{\omega_i}{2}\Big(\langle\Psi_0(T)|Q_{\mu i}Q^+_{\mu i}|\Psi_0(T)\rangle - 1\Big)\Big] = 0 . \tag{10}$$

Here $\mathcal{H}$ is the full Hamiltonian expressed in terms of thermal quasiparticle operators; the function $|\Psi_0(T)\rangle$ is a new thermal vacuum state which is treated as the phonon vacuum. The Lagrangian factor $\omega_i$ plays a role of the one-phonon state energy. After variation over phonon amplitudes one gets a homogeneous system of linear equations which has a solution if $\omega_i$ is a root of the following secular equation:

$$\big[\chi X^{(+)}(\omega_i) - 1\big]\big[\chi X^{(-)}(\omega_i) - 1\big] - \big[\chi X^{(0)}(\omega_i)\big]^2 = 0 , \tag{11}$$

Functions $X^{(\pm)}(\omega_i)$ , $X^{(0)}(\omega_i)$ are defined as

$$X^{(\pm)}(\omega_i) = \frac{2}{3}\sum_{j_p j_n} f^2_{j_p j_n}\left\{\frac{\varepsilon^{(+)}_{j_p j_n}(u^{(\pm)}_{j_p j_n})^2}{(\varepsilon^{(+)}_{j_p j_n})^2 - \omega_i^2}(1 - y^2_{j_p} - y^2_{j_n}) - \frac{\varepsilon^{(-)}_{j_p j_n}(v^{(\mp)}_{j_p j_n})^2}{(\varepsilon^{(-)}_{j_p j_n})^2 - \omega_i^2}(y^2_{j_p} - y^2_{j_n})\right\} ,$$

$$X^{(0)}(\omega_i) = \frac{2}{3}\,\omega_i\sum_{j_p j_n} f^2_{j_p j_n}\left\{\frac{u^{(+)}_{j_p j_n}u^{(-)}_{j_p j_n}}{(\varepsilon^{(+)}_{j_p j_n})^2 - \omega_i^2}(1 - y^2_{j_p} - y^2_{j_n}) - \frac{v^{(-)}_{j_p j_n}v^{(+)}_{j_p j_n}}{(\varepsilon^{(-)}_{j_p j_n})^2 - \omega_i^2}(y^2_{j_p} - y^2_{j_n})\right\} .$$

The following notations are introduced: $\varepsilon^{(\pm)}_{j_p j_p} = \varepsilon_{j_p} \pm \varepsilon_{j_n}$, $u^{(\pm)}_{j_p j_p} = u_{j_p}v_{j_n} \pm v_{j_p}u_{j_n}$, $v^{(\pm)}_{j_p j_p} = u_{j_p}u_{j_n} \pm v_{j_p}n_{j_n}$ and $f_{j_p j_n} = \langle j_p\|\sigma t^-\|j_n\rangle$.

The poles $\varepsilon^{(-)}_{j_p j_n}$ in Eq. (11) which do not exist in the pnRPA equations at $T = 0$ arise due to crossover terms $\beta^+\widetilde{\beta}^+$ in the thermal phonon structure (9). It should be stressed also that solutions of TRPA equations corresponding to negative eigenvalues should be interpreted as tilde-phonon states $\widetilde{Q}^+_{\mu i}|\Psi_0(T)\rangle$.

The secular equation (11) was evaluated applying TFD technique in [12] as well. However, applying the variation principle in the form (10) one cannot determine unambiguously phonon amplitudes $\psi, \phi, \eta, \xi$ and their tilde-counterparts. This point has been missed in [11, 12].

The uncertainty arises due to invariance of the thermal Hamiltonian

$$\mathcal{H} = \sum_{\mu\, i} \omega_i (Q^+_{\mu i} Q_{\mu i} - \widetilde{Q}^+_{\mu i}\widetilde{Q}_{\mu i}) \tag{12}$$

under the following thermal rotation:

$$Q^+_{\mu i} \to X_i Q^+_{\mu i} - Y_i \widetilde{Q}_{\mu i}\ , \quad \widetilde{Q}^+_{\mu i} \to X_i \widetilde{Q}^+_{\mu i} - Y_i Q_{\mu i}\ , \qquad (X_i^2 - Y_i^2 = 1)\ .$$

In full analogy with the case of thermal pairing Hamiltonian when the coefficients $x_j, y_j$ have been determined unambiguously finding the minima of the free energy (see [6, 12] for details), the coefficients $X_i$ and $Y_i$ (and consequently the thermal phonon structure) can be determined by minimizing the corresponding thermodynamic potential as well [9, 10]. The variation procedure gives the following result:

$$Y_i = \left[\exp\left(\frac{\omega_i}{T}\right) - 1\right]^{-1/2}\ , \qquad X_i = \left[1 - \exp\left(-\frac{\omega_i}{T}\right)\right]^{-1/2}\ . \tag{13}$$

Thus the factor $Y_i^2$ is nothing else but the thermal Bose-Einstein occupation factor for the thermal phonon with the energy $\omega_i$.

Rates of GT excitation and deexcitation processes involving hot nuclei are calculated with operators $S^{(-)}_{\lambda\mu}$ or $S^{(+)}_{\lambda\mu}$ (5). Nuclear structure factors for transitions from the thermal vacuum $|\Psi_0(T)\rangle$ to usual and tilde thermal one-phonon states read

$$\begin{aligned} \Phi^{(\mp)}_i &= \left|\langle \Psi_0(T) Q_{\mu i} \| S^{(\mp)}_\mu \| \Psi_0(T)\rangle\right|^2 = \frac{9}{4}\frac{(1 \pm \mathcal{Y}_i)^2}{\chi^2 \mathcal{N}_i} X_i^2, \\ E^{(\mp)}_i &= \omega_i \mp (\Delta\lambda_{np} + \Delta m_{np}), \end{aligned} \tag{14}$$

$$\begin{aligned} \widetilde{\Phi}^{(\mp)}_i &= \left|\langle \Psi_0(T) \widetilde{Q}_{\mu i} \| S^{(\mp)}_\mu \| \Psi_0(T)\rangle\right|^2 = \frac{9}{4}\frac{(1 \mp \mathcal{Y}_i)^2}{\chi^2 \mathcal{N}_i} Y_i^2, \\ \widetilde{E}^{(\mp)}_i &= -\omega_i \pm (\Delta\lambda_{np} + \Delta m_{np}), \end{aligned} \tag{15}$$

The values $E^{(\mp)}_i$ and $\widetilde{E}^{(\mp)}_i$ are the energies of corresponding transitions. They include thermal phonon energies $\omega_i$ and the differences between neutron and proton chemical potentials $\Delta\lambda_{np}$ in the parent nuclei as well as the mass difference of neutron and proton $\Delta m_{np}$. The values $\mathcal{Y}_i$, $\mathcal{N}_i$ are normalization factors of thermal one-phonon wave function.

Factor $\Phi_i^{(\mp)}$ (14) is valid for endoenergetic processes, i.e. for transitions to the states lying higher than the thermal vacuum. In stellar environment these processes are electron and positron captures. Factor $\widetilde{\Phi}_i^{(\mp)}$ (15) should be used for exoenergetic processes, i.e. for transitions to the states lying lower than the thermal vacuum. These processes are $\beta^{\pm}$ decays.

## 4 Numerical results

Numerical calculations are made within a rather schematic model for the nuclides $^{54,56}$Fe which are stable against a beta-decay in their ground states. We apply the Hamiltonian from Refs. [15, 16] which consists of the phenomenological neutron and proton mean-fields of the Woods-Saxon form, neutron-neutron and proton-proton pairing interactions with a constant matrix elements $G_n, G_p$ (i.e. of the BCS type) and a spin-isospin separable interaction 4 in the particle-hole channel. Since only the isovector part of the latter is taken into account, the effective spin-isospin interaction contains the only coupling constant $\chi$. The constant is used to fit the experimental values of the Gamow-Teller resonance energy. The empirical value of $\chi$ appears to be in agreement with the estimations of [15]. The theoretical strengths collected in the GT resonances in $^{54,56}$Fe exceed the corresponding experimental values from [19, 20]. To achieve an agreement with experimental data we introduce a quenching factor $q$ in the corresponding transition operator (5). The $q$ value is close to that from other theoretical studies of GT resonance, $q = 0.74$.

In Figs. 1, 2 the strength distributions of the Gamow-Teller transitions of the GT$_-$ and GT$_+$ type built on the $^{54,56}$Fe nuclides at different temperatures $T$ are displayed.

At $T = 0$ all the GT-strengths are at the energies higher then the ground state of the parent nuclides $^{54,56}$Fe. The strengths are concentrated mainly in one or a few one-phonon states, i.e. in the corresponding GT resonances[1]. With temperature increasing a small amount of both the GT$_-$ and GT$_+$ strengths appear at negative energies $E$. These GT-strength fractions correspond to tilde-phonon excitations and appears to be responsible for $\beta^+$ or $\beta^-$ decays of $^{54,56}$Fe from thermal ground states.

The appearance of GT-strength at negative $E$ values is one of the reason of a decrease of the GT resonance energy centroid at finite temperatures. The other ones are diminishing of pairing correlations and a thermal smearing of the Fermi surface which makes possible GT transitions with low energies. The decrease of the GT$_+$ resonance energy is important for EC rates making them higher at low densities of degenerate electron gas in stellar media. The decrease of the GT$_-$ resonance energy noticeably affects r-process of nucleosynthesis.

The GT-strength distributions presented above are used to calculate EC and $\beta^-$ decay rates for the same nuclei in stellar media at presupernova conditions. In this approach it is assumed that the parent nuclei $^{54,56}$Fe are in a thermal equilibrium

[1] Please, note the logarithmic scale of the abscissae axis in Figs. 1 and 2.

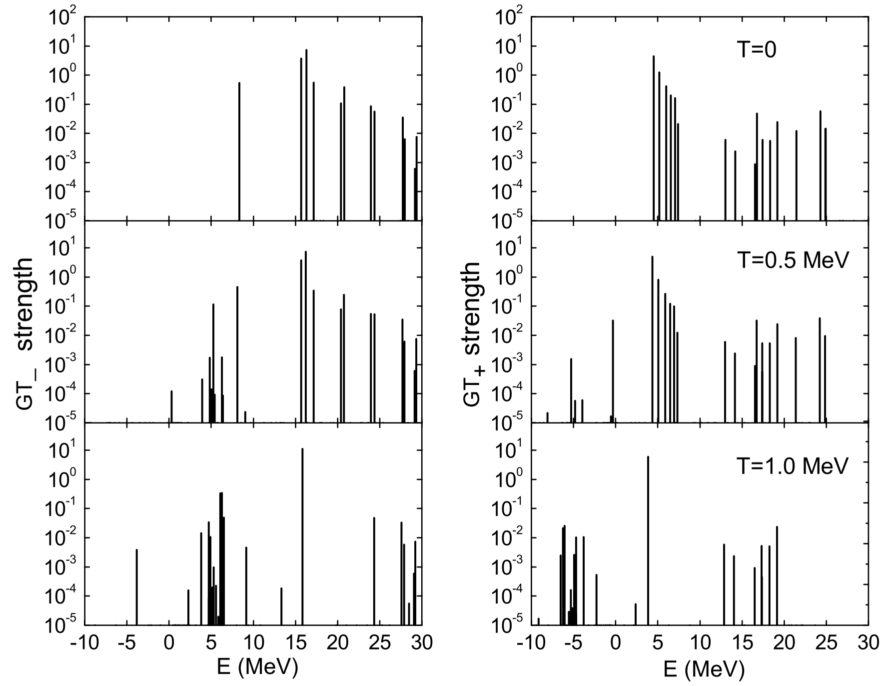


Figure 1. $\mathrm{GT}_-$ (left panel) and $\mathrm{GT}_+$ (right panel) strength distributions built on the top of $^{54}$Fe at different temperatures. $E$ – energy transferred to the target nucleus.

state which is treated as a thermal vacuum state (thermal phonon vacuum $|\Psi_0(T)\rangle\rangle$. Electron capture process corresponds to transition $|\Psi_0(T)\rangle \to Q^+_{\mu i}|\Psi_0(T)\rangle$ and $\beta^-$ decay to $|\Psi_0(T)\rangle \to \widetilde{Q}^+_{\mu i}|\Psi_0(T)\rangle$. Certainly, only allowed GT transitions are taken into account in this consideration.

Moreover, at temperatures typical for the considered stellar evolution stage $(1 \div 10) \times 10^9$ K$^o$ atoms are fully ionized and nuclei are embedded in degenerate gas of electrons. Under these circumstances the total rate of EC or $\beta^-$ decay reads

$$\lambda^\alpha = \frac{\ln 2}{D} \sum_{j=i,\ \widetilde{i}} B_j F^\alpha_j, \quad \alpha = \mathrm{EC}, \beta^-. \tag{16}$$

The sum in (16) is running over all thermal one-phonon states, both corresponding to the positive roots as well as negative ones (i.e. tilde states); $B_j$ is a reduced transition probability of the corresponding transition, and $F^\alpha_j$ is a phase space integral.

The expression for $B_j$ is the following:

$$B_j = \left(\frac{g_\mathrm{A}}{g_\mathrm{V}}\right)^2_{\mathrm{eff}} \Phi^{(\mp)}_j(\mathrm{GT}) \,, \quad (`+\text{' refers to EC}, \quad `-\text{'to } \beta^- \text{ decay}).$$

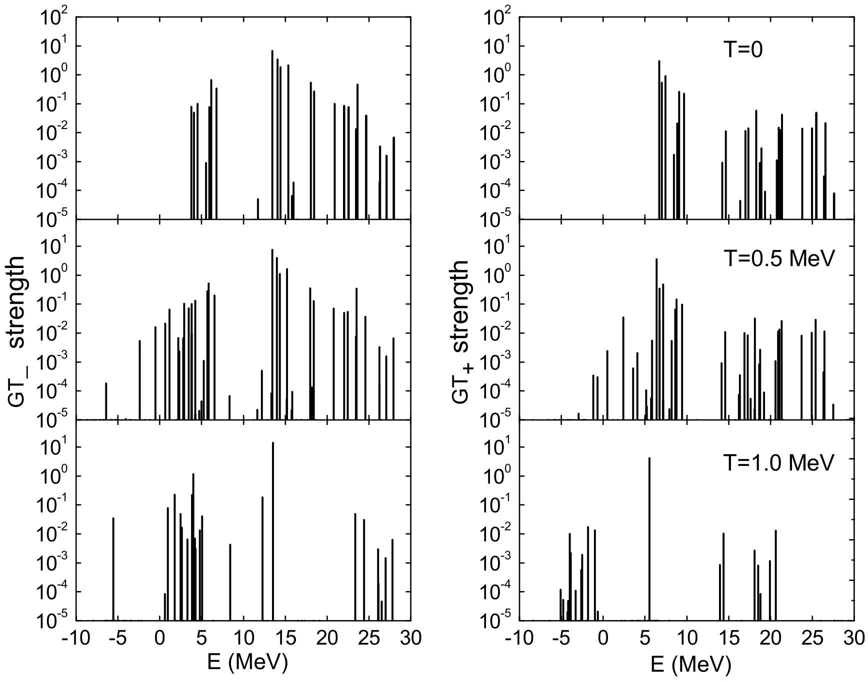


Figure 2. $\mathrm{GT}_-$ (left panel) and $\mathrm{GT}_+$ (right panel) strength distributions built on the top of $^{56}$Fe at different temperatures. $E$ – energy transferred to the target nucleus.

The phase space integral $F_j^\beta$ for $\beta^-$ decay reads

$$F_j^\beta = \int_1^{q_j} w^2(q_j - w)^2 G(Z+1, w)(1 - S_\mp)dw$$

whereas for EC it is

$$F_j^{\mathrm{EC}} = \int_{w_l}^{\infty} w^2(q_j + w)^2 G(Z, w) S_\mp dw.$$

The new notations above are the following: $q_j = -E_j^{(\pm)}/m_e c^2$ is the decay (capture) total energy (or the energy of j-th thermal phonon state, negative or positive); $w_l = 1$ if $q_j \geq -1$, or $w_l = -q_j$ if $q_j < -1$ is the capture threshold; $G(Z, w)$ - the Fermi function taking into account a distortion of electron wave function in the Coulomb field of a nucleus; $S_-$ is a thermal distribution function of electrons over energy in a stellar media. It has the following shape:

$$S_- = \left[\exp\left(\frac{\omega - 1 - U_F}{kT}\right) + 1\right]^{-1},$$

Here $k$ is the Boltzman constant, $U_F$ - the Fermi energy of electrons which should be found at given values of $T$ and $\rho$ from integral equation.

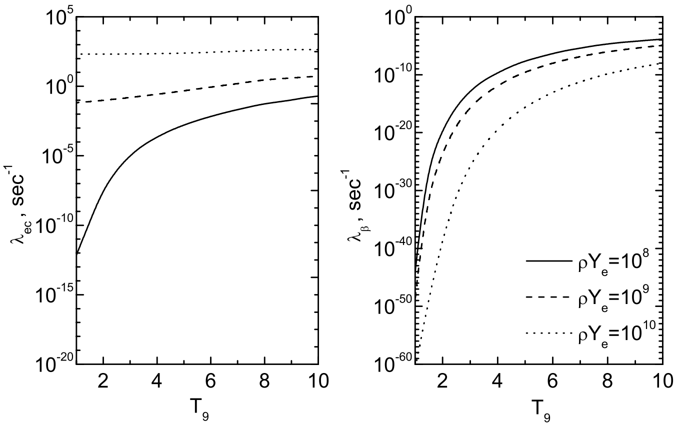


Figure 3. Electron capture (left panel) and $\beta^-$-decay (right panel) rates for $^{54}$Fe as a function of temperature $T$ at three values of the electron density $\rho Y_e$. $T$ is in units of $T_9 = 10^9$ K; $\rho Y_e$ is in units of moles/cm$^3$.

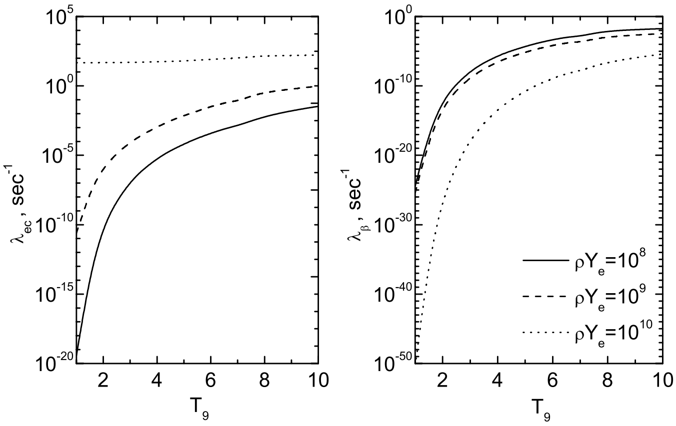


Figure 4. Electron-capture and $\beta^-$-decay rates for $^{56}$Fe as a function of temperature $T$ at three values of the density $\rho Y_e$. $T$ is in units of $T_9 = 10^9$ K; $\rho Y_e$ is in units of moles/cm$^3$.

The calculated $\lambda_{\rm EC}$ and $\lambda_\beta$ for $^{54,56}$Fe are displayed in Figs. 3 and 4, respectively. The EC rates increase when temperature as well as the density increase. The reason is an increase of a number of electrons with energies near or higher the $\rm GT_+$ resonance energy. A decrease in the $\rm GT_+$ resonance energy with $T$ increasing also contributes to increase in $\lambda_{\rm EC}$. The EC rates displayed in Figs. 3 and 4 agree well with the results of shell-model calculations in Refs. [1].

At the same time $\lambda_\beta$ is affected by increase of $T$ and $\rho Y_e$ in opposite ways. A rise of density $\rho Y_e$ suppresses the $\beta^-$ decay rate due to diminishing a phase space available for escaping electrons. However, increase in temperature weakens the Pauli blocking and enhances a contribution of the $\rm GT_-$ transitions from excited states of the parent nuclei. In contrast with the EC rate our $\lambda_\beta$ values appear to be much smaller at low temperatures than those from [1]. The reason is much larger $\rm GT_-$ strength in the $\beta^-$ decay energy window predicted in our calculations. We surmise that more adequate description of the GT-strength distribution, e.g. incorporation in our scheme a coupling of GT-phonons with more complex configurations combining approaches of Ref. [16] and [11, 18] will improve the agreement.

## 5 Conclusions

An approach based on the thermo field dynamics and adapted to describe thermal effects on GT strength distributions in nuclei was presented. The thermal neutron-proton RPA equations were evaluated and examples of GT-resonance behavior at finite temperatures were given. Our calculations of electron capture and $\beta^-$ decay rates for hot nuclei $^{54,56}$Fe in stellar media gave results in reasonable agreement with the previous ones [1]. However, further development of the approach is necessary. A desirable improvement would be an inclusion of a coupling of the RPA GT-states with more complex ones.

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