Energy Distributions from 3-Body Decaying Resonances

R. Álvarez-Rodríguez[1]*, A. S. Jensen[1], E. Garrido[2], D. V. Fedorov[1], and H. O. U. Fynbo[1]

[1] Department of Physics and Astronomy, Aarhus University, Ny Munkegade Bygning 1520 DK-8000 Aarhus C, Denmark
[2] Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas, Serrano 123
E-28006 Madrid, Spain

Abstract. We show the energy distributions of the fragments after the 3-body decay of many body resonances. We use them to study the decay mechanisms. The wave functions are computed by the complex-scaled hyperspherical adiabatic expansion method. The large-distance part of the wave functions is crucial and must be accurately computed. We illustrate by showing the results for ^{12}C and ^{9}Be resonances.

1 Introduction

The 3-body decay of many-body resonances can be measured very accurately in complete kinematics. This allows us to study different decay mechanisms, often divided into direct versus sequential. Direct decay takes place when the three particles leave simultaneously their interaction regions, while sequential decay proceeds via an intermediate 2-body configuration. The two cases we shall use here as examples (^{12}C and ^{9}Be) each present their own intriguing problems but they are moreover requested for astrophysical appplications.

The ^{12}C resonances below the proton separation threshold have been studied over many years because of its astrophysical interest, since triple α process is one of the key processes in stellar nucleosynthesis [1]. Unfortunately many unanswered questions still remain, *i.e.* what are the energies, angular momenta and decay properties of these low-lying resonances. Morinaga conjectured in the 1950s [2] the existence of a 2^+ resonance as a member of a rotational band of the Hoyle state (0^+ at 7.65 MeV of excitation energy), but no agreement has yet been reached theoretically or experimentally for the position of the first excited 2^+ state of ^{12}C.

Experimentalists discuss with no agreement the decay of the $5/2^-$ resonance of ^{9}Be at 2.4 MeV of excitation energy as sequential via ^{8}Be or ^{5}He. The only known procedure used by experimentalists in order to analyze 3-body decay experiments is R-matrix analysis, and it assumes sequential decay via intermediate configurations.

* Present address: Istituto Nazionale di Fisica Nucleare, Largo B. Pontecorvo 3, I-56127 Pisa, Italy

Moreover the intermediate paths are not observables in quantum mechanics. The interpretation of the data are then used to derive the reaction rates of the inverse process, and the classification into decay modes are the results of interpretation or model computations. A correct interpretation and understanding of the data is crucial to get a reliable application.

The energy distribution of the fragments after the decay is the only experimental information that allows us to study the decay path. This information is contained in the large-distance part of the wave function, which has then to be accurately computed. We will show results for individual particle energy distributions, energy correlations of Dalitz plots and angular distributions.

2 Theoretical Framework

Since the resonances under study decay into three particles, we are dealing with a 3-body problem. The Faddeev equations describe a 3-body system. We solve them by using the complex-scaled adiabatic hyperspherical expansion method [3]. The advantage of using complex scaling is that the resonances can be treated as if they were bound states. The appropriate coordinates are the so-called hyperspherical coordinates and consist of the hyperradius $\rho^2 = 4\sum_{i=1}^{3}(\vec{r}_i - \vec{R})^2$, and five hyperangles. In the adiabatic hyperspherical expansion method the angular part of the Faddeev equations,

$$(T_\Omega - \lambda_n)\Phi^{(i)}_{nJM} + \frac{2m}{\hbar^2}\rho^2 V_i \Phi_{nJM} = 0 \qquad i = 1, 2, 3 \; , \tag{1}$$

is solved first and the angular eigenfunctions are then used as a basis to expand the total wave function

$$\Psi^{JM} = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho)\Phi_{nJM}(\rho, \Omega) \; . \tag{2}$$

The ρ-dependent expansion coefficients, $f_n(\rho)$, are the hyperradial wave functions obtained from the coupled set of hyperradial equations

$$\left(-\frac{\partial^2}{\partial\rho^2} + \frac{15/4}{\rho^2} + \frac{2m}{\hbar^2}[W_n(\rho) + V_{3b}(\rho) - E]\right) f_n(\rho) = \sum_{n'=1}^{\infty} \hat{P}_{nn'} f_{n'}(\rho) \; . \tag{3}$$

$W_n(\rho) = \hbar^2\lambda_n/2m$ are the angular eigenvalues of the 3-body system Hamiltonian with fixed ρ, V_{3b} is the 3-body potential, E is the 3-body energy and $P_{nn'}$ are the non adiabatic terms. The eigenvalues $W_n(\rho)$ of the angular equations Eq. (1) serve as effective potentials.

We have chosen a fenomenological Ali-Bodmer $\alpha{-}\alpha$ potential slightly modified in order to reproduce the s-wave resonance of ^{8}Be [4]. For the $\alpha - n$ interaction we use a potential constructed to reproduce the low-energy $\alpha - n$ phase-shifts. The energy of the resonance is corrected by including a diagonal 3-body interaction

$V_{3b} = S\exp(-\rho^2/b^2)$. This effective 3-body potential provides a continuation of the three-body formulation to small distances where N-body degrees of freedom possibly are more appropriate. The structure of the resonance is maintained since V_{3b} only depends on ρ.

The momentum distribution of the decay fragments is determined by the Fourier transform of the coordinate-space wave function. The hyperspherical harmonics transform into themselves after Fourier transformation. It has been shown [5] that the angular amplitude of the momentum-space wave function of the resonance is directly proportional to the coordinate-space one for a large value of ρ. Numerically converged results in the appropriate region of ρ-values are then needed in order to have a reliable computation. The probability distribution is obtained after integration over the four hyperangles describing the directions of the momenta,

$$P(k_y^2) \propto P(\cos^2\alpha) \propto (\sin 2\alpha) \int d\Omega_x \, d\Omega_y \, |\Psi(\rho, \alpha, \Omega_x, \Omega_y)|^2 \, . \qquad (4)$$

The asymptotic behavior is reached for hyperradii larger than about 60 fm. There is a small variation of the distribution from 70 to 100 fm, that shows the stability and convergence of the computation. We have chosen 80 fm as the value of ρ where the energy distributions should be computed. We have performed a Monte Carlo integration over the phase space to get the probability distributions.

3 Decay of ^{12}C Resonances

We find fourteen resonances for ^{12}C below the proton separation threshold for most angular momenta and both parities. The short-distance part of the wave function provides us with information of the energies and widths of the resonances and of their partial wave decomposition [6].

The individual α energy distributions show the probability of finding one α particle with a given energy. The Dalitz plots contain more information than the individual distributions, by showing how the three particles share the energy after the decay. And finally the angular distribution provides us with information about the preferred direction of the decaying particles and the angular momentum.

Natural-parity states. These states can breakup sequentially via ^{8}Be(0^+). When this happens one of the adiabatic components of the wave function must approach the 2-body behavior. This means that this component can be identified and therefore the amount of sequential decay can be estimated. Unfortunately the momentum distributions related to this component are not accurate, because the 2-body asymptotics is not reached for $\rho \sim 100$ fm. The contribution from this component can be then replaced by the known 2-body asymptotic behavior. This is illustrated in Figure 1 for the 4^+ resonance. We estimate a 20% of sequential decay and the rest (80%) direct decay into the 3-body continuum. In the figure corresponding to the sequential decay we can clearly see how one of the α particles tries to carry all the available

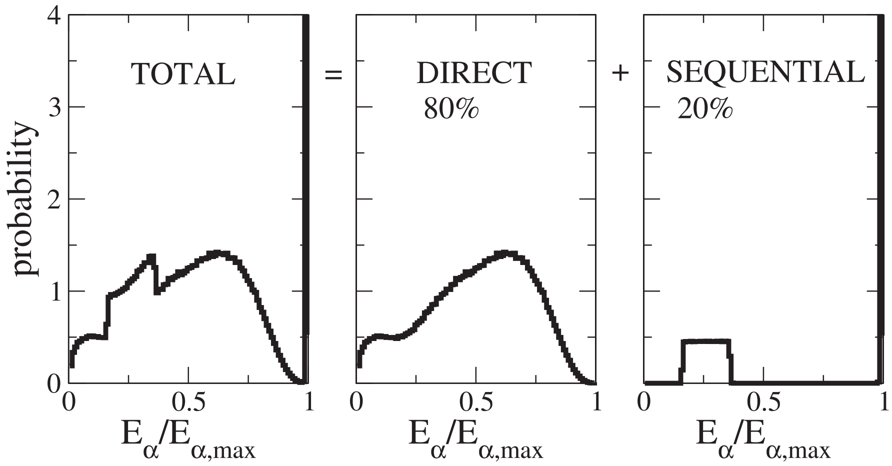

Figure 1. The α particle energy distribution for the 4^+ resonance of 12C at an excitation energy of 14.10 MeV (or 6.83 MeV above the 3α threshold). On the left side the total distribution. In the middle the contribution to the energy distribution after removal of the sequential part. On the right side the sequential energy distribution.

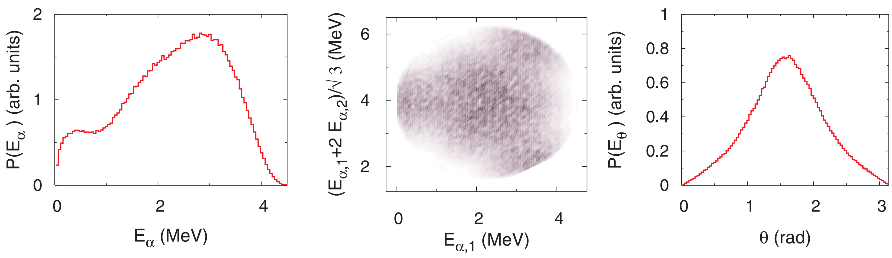

Figure 2. The α particle energy distribution (left), the Dalitz plot (middle) and the angular distribution (right) for the 4^+ resonance of ^{12}C at an excitation energy of 14.10 MeV (or 6.83 MeV above the 3α threshold). θ is the angle between the direction formed by two of the particles and the direction formed by their center of mass and the third particle.

energy ((the narrow peak at maximum energy in sequential decay), while the other two share the remaining energy (the square distribution).

In order to make a clearer comparison with the experiment, we remove the sequential decay via the ^{8}Be ground state. In Figure 2 we show the energy distribution for one α particle, the Dalitz plot and the angular distribution [7]. The distribution of the kinetic energy of the particles is rather diffuse (middle of figure), in contrast with the sequential distribution showed in Figure 1. The angular distribution exhibits a smooth peak around $\pi/2$.

Unnatural-parity states. Angular momentum and parity conservation forbid the decay of these states via the ^{8}Be ground state. Figure 3 shows the energy distribution [8], Dalitz plot and angular distribution for the 1^+ resonance at 11.7 MeV of

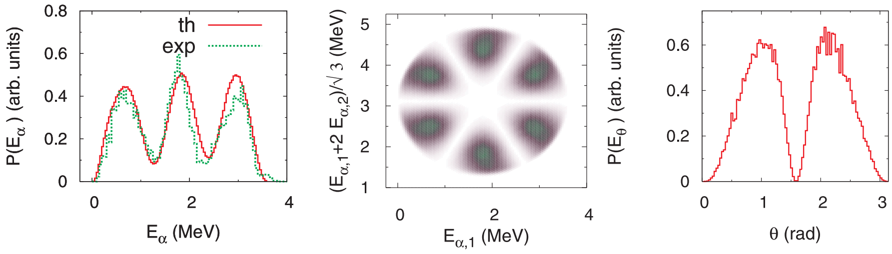

Figure 3. The α particle energy distribution (left), the Dalitz plot (middle) and the angular distribution (right) for the 1^+ resonance of 12C at an excitation energy of 11.7 MeV (or 5.4 MeV above the 3α threshold). θ is the angle between the direction formed by two of the particles and the direction formed by their center of mass and the third particle. The experimental data is from [9].

excitation energy. We also show experimental data for the energy distribution [9]. The Dalitz plot can also be compared to the experimental one showed in [10]. In both cases the agreement is impressive. Moreover, this state is known to be a "shell-model" state with no significant cluster structure, but it is clear that the 3-body cluster model provides a good description of its decay into three α particles.

We have also compared our computations with the experimental distributions obtained from the reaction ^{11}B(^{3}He,$d\alpha\alpha\alpha$) that has been studied at CMAM (Madrid, Spain) in March 2008 [11]. A preliminary analysis of the data shows a very nice agreement with our predictions, both for energy distributions, Dalitz plots and angular distributions.

4 Decay of ^{9}Be Resonances

The low-lying ^{9}Be resonances decay into one neutron and two α particles. The same formalism used for ^{12}C can then be applied here. The $5/2^-$ state of ^{9}Be can decay sequentially via the narrow ^{8}Be(0^+). The other sequential decay channels for this state lies above the energy of the resonance, but due to their broad width, a virtual (energy forbidden) decay through their tails could also contribute. These 2-body states are ^{8}Be(2^+), ^{5}He($p_{3/2}$) and ^{5}He($p_{1/2}$). The decay of this resonance has been controversial, since experimental groups do not agree over its path. The authors of [12] claim that the decay occurs mainly (86%) via ^{8}Be(2^+), in agreement with the theoretical prediction from [13] but in contrast with [14].

We estimate that at large distances only 3% of the wave function corresponds to the adiabatic component related to the ground state structure of ^{8}Be [15]. The dominating adiabatic component represents 96% of the wave function at large distances. Figure 4 shows the partial wave decomposition for this dominating component in the two different Jacobi sets that we can consider in this case. On the left side we can see that at large distances the structure is almost entirely $2\hbar$ between the two α particles coupled to $1\hbar$ between the neutron and the center of mass of the α parti-

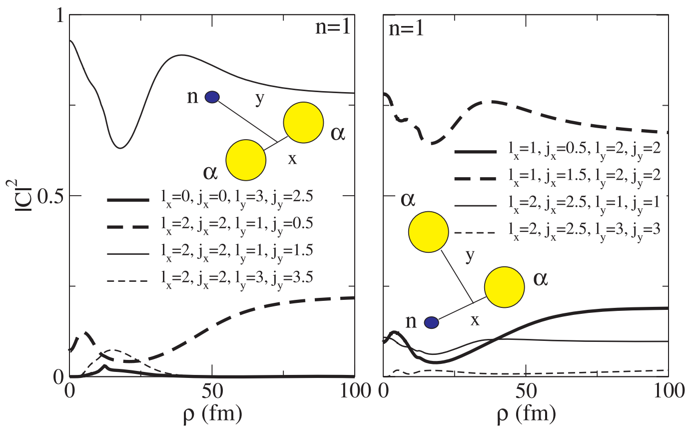

Figure 4. The probabilities for different angular momentum combinations in the two Jacobi sets for the dominating adiabatic component (n=1).

cles. On the right side the same wave function written in the other Jacobi set reveals ^{5}He structure of $1\hbar$ between the neutron and one of the α particles in turn coupled to $2\hbar$ between these two and the other α.

We plot in Figure 5 the neutron energy distribution and the angular distribution compared to the experimental ones from [12]. Even though our distribution lies a bit above the experimental one and at high energies a bit below at low energies, we are able to reproduce nicely the experimental data.

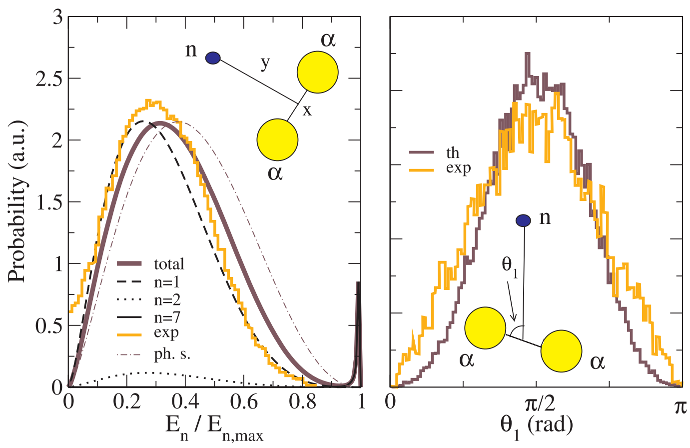

Figure 5. The theoretical and experimental [12] distribution for the neutron energy (left) and the angular distribution (right) after the decay of the ^{9}Be$(5/2^-)$ resonance at 2.4 MeV of excitation energy.

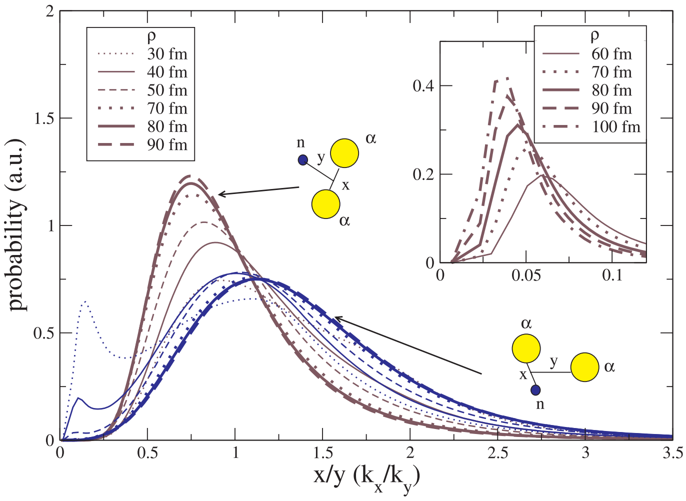

Figure 6. The evolution with the hyperradius for the two different sets of Jacobi coordinates of the probability distributions for the ratios between distances or momenta of two particles and their center of mass and the third particle. The inset shows the contribution from the sequential decay through ^{8}Be(0^+).

We usually compute the energy distributions from the large-distance part of the wave function, that is where the asymptotical behavior has been reached and the distribution is stable as function of hyperradius. But we would also like to find out about the evolution of the distribution from small to large distances. We show in Figure 6 the probability of finding a given value of the ratio of the distance between two particles and the distance between their center of mass and the third particle. We plot this probability for several values of the hyperradius, from small distance (30 fm) to large distance (90 fm). In the first set of Jacobi coordinates there is a peak at small x/y (see inset of Figure 6). The distance between the two α's is constant with increasing hyperradius. This is the behavior that one expects when there is a ^{8}Be structure, and it is related to the adiabatic component that gives the 3% of sequential decay through ^{8}Be(0^+). In the other Jacobi set we find a peak at small x/y related to ^{5}He structure that disappears as we increase the hyperradius. We can then clearly distinguish one of the possible decay modes while the other two possible decay modes can not be distinguished from each other or from the direct decay into the 3-body continuum. Reaction rates extracted from derived branching ratios can then be misleading.

Figure 7 shows the energy correlations exhibited by Dalitz plots. We plot both $\alpha - \alpha$ and $\alpha - n$ combinations. The figure on the left is symmetric, since the α's are identical particles. The small region at low energy is related to the sequential decay via ^{8}Be(0^+), in which the neutron takes most of the available energy and the two α's share the small amount of remaining energy. This fact is also reflected on

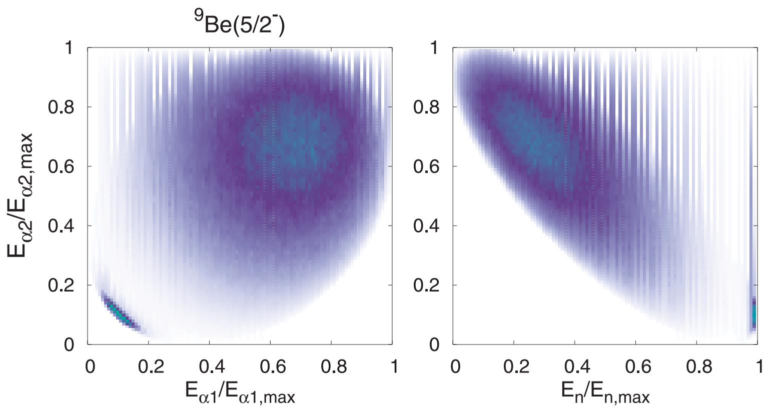

Figure 7. The Dalitz plots showing α-α (left) and n-α (right) energy correlations for the ^{9}Be$(5/2^-)$ resonance.

the right side of the figure as a small shadow around the maximum energy of the neutron.

We have seen that sequential descriptions via ^{8}Be(2^+) and ^{5}He(p) both are equally consistent with angular momentum couplings. However, at large distances we do not see intermediate configurations where two particles are spatially close and the third one is far away. A direct decay picture is more consistent.

5 Summary

We have computed the energy distributions of 3-body decaying resonances using the hyperspherical adiabatic expansion method combined with complex scaling. These distributions reflect the structure related to the decay mechanism. We are able to distinguish allowed sequential decays via 2-body configurations.

In general, decays via high-lying or broad resonances are very unlikely because the corresponding components would be depopulated already at small hyperradii where the coupling is strong. Sequential decays via energy allowed low-lying and narrow resonances are possible but not unavoidable. Virtual sequential decays are possible when the two-body resonance energy and width both are small and the effective barrier therefore very thick. Several descriptions in terms of different basis sets may be equally efficient.

We illustrate by application to the decay of ^{12}C and ^{9}Be resonances. We show individual particle energy distributions, Dalitz plots and angular distributions. When experimental information is available, we find a good agreement with the data.

Acknowledgments

R.A.R. acknowledges support by a post-doctoral grant from Ministerio de Ciencia e Innovación (Spain). The authors would like to thank M. Alcorta and O. Tengblad for providing their preliminary experimental results.

References

1. H.O.U. Fynbo *et al.*, Nature **433**, 136 (2005).
2. H. Morinaga, Phys. Rev. **101**, 254 (1956).
3. E. Nielsen, D.V. Fedorov, A.S. Jensen and E. Garrido, Phys. Rep. **347**, 373 (2001).
4. S. Ali and A.R. Bodmer, Nucl. Phys. **80**, 99 (1966).
5. D.V. Fedorov, H.O.U. Fynbo, E. Garrido and A.S. Jensen, Few-body systems **34**, 33 (2004).
6. R. Álvarez-Rodríguez, E. Garrido, A.S. Jensen, D.V. Fedorov and H.O.U. Fynbo, Eur. Phys. J A **31**, 303 (2007).
7. R. Álvarez-Rodríguez, A.S. Jensen, E. Garrido, D.V. Fedorov and H.O.U. Fynbo, Phys. Rev. C **77**, 064305 (2008).
8. R. Álvarez-Rodríguez, A.S. Jensen, D.V. Fedorov, H.O.U. Fynbo and E. Garrido, Phys. Rev. Lett. **99**, 072503 (2007).
9. C. Aa. Diget. *Ph.D. Thesis* Univeristy of Aarhus (2006).
10. H.O.U. Fynbo *et al.*, Phys. Rev. Lett. **91**, 082502 (2003).
11. M. Alcorta *et al.*, to be published.
12. P. Papka *et al.*, Phys. Rev. C **75**, 045803 (2007).
13. P. Descouvemont, Eur. Phys. J. A, **12**, 413 (2001).
14. L.V. Grigorenko and M.V. Zhukov, Phys. Rev. C **72** 015803 (2005).
15. R. Álvarez-Rodríguez, H.O.U. Fynbo, A.S. Jensen and E. Garrido, Phys. Rev. Lett. **100**, 192501 (2008).