Boson and Fermion Degrees of Freedom in the Orthosymplectic Extension of the IVBM: Odd-Odd Nuclear Spectra

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Abstract. The dynamical symmetry group $Sp(12, R)$ of the Interacting Vector Boson Model (IVBM) is extended to the orthosymplectic group $OSp(2\Omega/12, R)$ in order to incorporate fermion degrees of freedom. The structure of even-even nuclei is used as a core on which the collective excitations of the neighboring odd-mass and odd-odd nuclei are build on. Hence, the spectra of odd-mass and odd-odd nuclei arise as a result of the coupling of the fermion degrees of freedom, specified by the fermion sector $SO^F(2\Omega)$ to the boson core, whose states belong to an $Sp(12, R)$ irreducible representation.

The orthosymplectic dynamical symmetry is applied for the simultaneous description of the spectra of some neighboring nuclei from rare earth region. The theoretical predictions for different low-lying collective bands with positive and negative parity are compared with the experiment. The obtained results reveal the applicability of the model and its boson-fermion extension.

1 Introduction

There have been a large number of theoretical investigations concerning odd-odd nuclei but few based on the Interacting Boson Model (IBM) [1]. The $U(5)$, $SU(3)$ and $O(6)$ dynamical symmetry limits of the IBM and the associated Bose-Fermi symmetries of the Interacting Boson-Fermion Model (IBFM) [2] have enriched our understanding of the structure of low-lying collective states in heavy even-even and odd-mass nuclei respectively [1–3]. An extension of these models for odd-odd nuclei give rise to the Interacting Boson-Fermion-Fermion Model (IBFFM) [4,5]. With a simple IBFFM Hamiltonian several numerical studies of odd-odd nuclei have been caried [3,6]. However the progress in developing dynamical symmetry limits of this model is rather slow. This is mainly because here the enumeration, understanding the significance and establishing applicability of the various symmetry limits is not straight forward. There are limited but significant applications of symmetry limits of IBFFM associated with the $U(5)$ [7], $SU(3)$ [8] and $O(6)$ [9] limits of IBM.

In the early 1980s, a boson-number-preserving version of the phenomenological algebraic Interacting Vector Boson Model (IVBM) [10] was introduced and applied successfully [11] to a description of the low-lying collective rotational spectra of the even-even medium and heavy mass nuclei. With the aim of extending these applications to incorporate new experimental data on states with higher spins and to incorporate new excited bands, we explored the symplectic extension of the IVBM [12], for which the dynamical symmetry group is $Sp(12,R)$. This extension is realized from, and has its physical interpretation over basis states of its maximal compact subgroup $U(6) \subset Sp(12,R)$, and resulted in the description of various excited bands of both positive and negative parity of complex systems exhibiting rotation-vibrational spectra [13]. In [14] an orthosymplectic extension of the IVBM was carried out in order to encompass the treatment of the odd-mass nuclei. With the present work we exploit further the boson-fermion extension [14] of IVBM for the description of the ground and first excited positive and/or negative bands of odd-odd nuclei. Thus, it is the purpose of this paper to bring intrinsic degrees of freedom explicitly into the symplectic IVBM. We approach the problem by considering the simplest physical picture in which two particles (or quasiparticles) with intrinsic spins taking a single $j-$value are coupled to an even-even nucleus whose states belong to an $Sp(12,R)$ irrep. Nevertheless, the results for the energy spectra obtained in this simplified version of the model agree rather well with the experimental data.

2 The Even-Even Core Nuclei

The algebraic structure of the IVBM is realized in terms of creation and annihilation operators $u_m^+(\alpha)$, $u_m(\alpha)$ $(m = 0, \pm 1)$. The bilinear products of the creation and annihilation operators of the two vector bosons generate the boson representations of the non-compact symplectic group $Sp(12,R)$ [10]:

$$\begin{aligned} F_M^L(\alpha,\beta) &= \sum\nolimits_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m^+(\beta), \\ G_M^L(\alpha,\beta) &= \sum\nolimits_{k,m} C_{1k1m}^{LM} u_k(\alpha) u_m(\beta), \end{aligned} \tag{1}$$

$$A_M^L(\alpha,\beta) = \sum\nolimits_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m(\beta), \tag{2}$$

where C_{1k1m}^{LM}, which are the usual Clebsch-Gordon coefficients for $L = 0, 1, 2$ and $M = -L, -L+1, ...L$, define the transformation properties of (1) and (2) under rotations. The commutation relations between the pair creation and annihilation operators (1) and the number preserving operators (2) are given in [10].

Being a noncompact group, the unitary representations of $Sp(12,R)$ are of infinite dimension, which makes it impossible to diagonalize the most general Hamiltonian. When restricted to the group $U^B(6)$, each irrep of the group $Sp^B(12,R)$ decomposes into irreps of the subgroup characterized by the partitions [12, 15]:

$$[N, 0^5]_6 \equiv [N]_6,$$

where $N = 0, 2, 4, \ldots$ (even irrep) or $N = 1, 3, 5, \ldots$ (odd irrep). The subspaces $[N]_6$ are finite dimensional, which simplifies the problem of diagonalization. Therefore the complete spectrum of the system can be calculated through the diagonalization of the Hamiltonian in the subspaces of all the unitary irreducible representations

(UIR) of $U(6)$, belonging to a given UIR of $Sp(12,R)$, which further clarifies its role of a group of dynamical symmetry.

The Hamiltonian, corresponding to the unitary limit of IVBM [12]

$$Sp(12,R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes (U(1) \otimes U(1)), \tag{3}$$

expressed in terms of the first and second order invariant operators of the different subgroups in the chain (3) is [12]:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2. \tag{4}$$

H (4) is obviously diagonal in the basis

$$\mid [N]_6; (\lambda,\mu); KLM; T_0\rangle \equiv \mid (N,T); KLM; T_0\rangle, \tag{5}$$

labelled by the quantum numbers of the subgroups of the chain (3). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12,R)$:

$$\begin{aligned} E((N,T),L,T_0) = aN + bN^2 + \alpha_3 T(T+1) \\ + \beta_3 L(L+1) + \alpha_1 T_0^2. \end{aligned} \tag{6}$$

The construction of the symplectic basis for the even IR of $Sp(12,R)$ is given in details in [12]. The $Sp(12,R)$ classification scheme for the $SU(3)$ boson representations for even value of the number of bosons N is shown on Table I in Ref. [12] (see also Table 1).

The most important application of the $U^B(6) \subset Sp^B(12,R)$ limit of the theory is the possibility it affords for describing both even and odd parity bands up to very high angular momentum [12]. In order to do this we first have to identify the experimentally observed bands with the sequences of basis states of the even $Sp(12,R)$ irrep (Table 1). As we deal with the symplectic extension we are able to consider all even eigenvalues of the number of vector bosons N with the corresponding set of $T-$spins, which uniquely define the $SU^B(3)$ irreps (λ,μ). The multiplicity index K appearing in the final reduction to the $SO(3)$ is related to the projection of L on the body fixed frame and is used with the parity (π) to label the different bands (K^π) in the energy spectra of the nuclei. For the even-even nuclei we have defined the parity of the states as $\pi_{core} = (-1)^T$ [12]. This allowed us to describe both positive and negative bands.

Further, we use the algebraic concept of "yrast" states, introduced in [12]. According to this concept we consider as yrast states the states with given L, which minimize the energy (6) with respect to the number of vector bosons N that build them. Thus the states of the ground state band (GSB) were identified with the $SU(3)$ multiplets $(0,\mu)$ [12]. In terms of (N,T) this choice corresponds to $(N = 2\mu, T = 0)$ and the sequence of states with different numbers of bosons $N = 0,4,8,\ldots$ and $T = 0$ (and also $T_0 = 0$). Hence the minimum values of the energies (6) are obtained at $N = 2L$.

The presented mapping of the experimental states onto the $SU(3)$ basis states, using the algebraic notion of yrast states, is a particular case of the so called

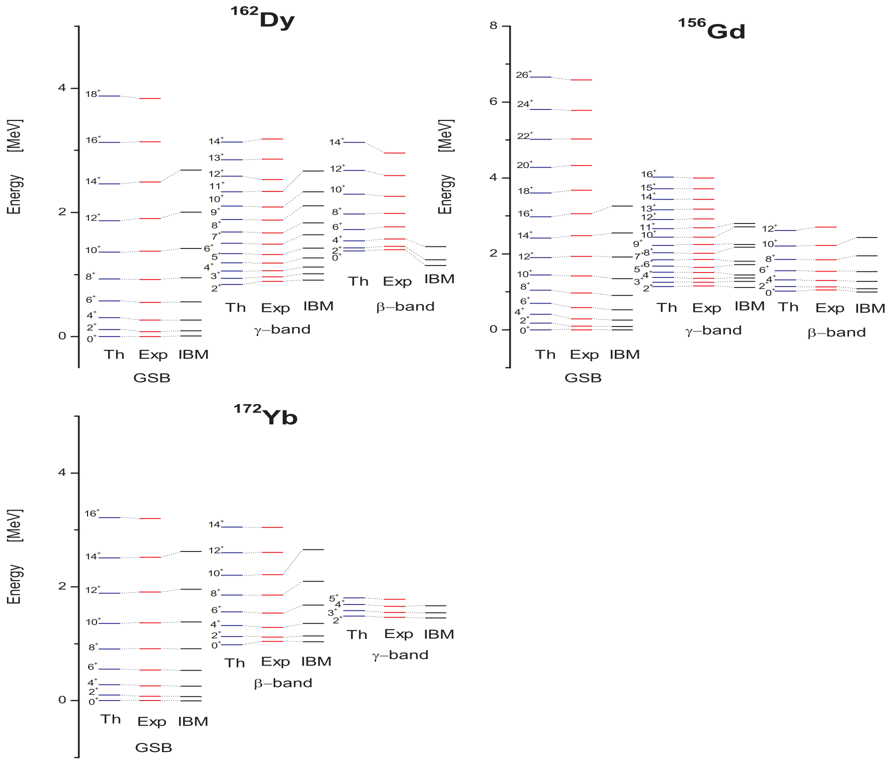

Figure 1. Comparison of the theoretical and experimental energies for the ground and first excited bands of ^{156}Gd, ^{172}Yb and ^{162}Dy core nuclei.

”stretched” states [16]. The latter are defined as the states with $(\lambda_0 + 2k, \mu_0)$ or $(\lambda_0, \mu_0 + k)$, where $N_i = \lambda_0 + 2\mu_0$ and $k = 0, 1, 2, 3, \ldots$.

It was established [17] that the correct placement of the bands in the spectrum strongly depends on their bandheads configuration, and in particular, on the minimal or initial number of bosons, $N = N_i$, from which they are built. The latter determines the starting position of each excited band.

Thus, for the description of the different excited bands, we first determine the N_i of the band head structure and develop the corresponding excited band over the stretched $SU(3)$ multiplets. This corresponds to the sequence of basis states with $N = N_i, N_i + 4, N_i + 8, \ldots$ $(\Delta N = 4)$. The values of T for the first type of stretched states ($\lambda-$changed) are changed by step $\Delta T = 2$, whereas for the second type ($\mu-$changed) $-T$ is fixed so that in both cases the parity is preserved even or odd, respectively. For all presented even-even nuclei, the states of the corresponding $\beta-$ and $\gamma-$ bands are associated with the stretched states of the first type ($\lambda-$changed).

To describe the structure of odd-mass and odd-odd nuclei, first a description of the appropriate even-even cores should be obtained. Thus, we determine the values of the five phenomenological model parameters $a, b, \alpha_3, \beta_3, \alpha_1$ by fitting the energies of the ground and few excited bands ($\gamma-$ and/or $\beta-$ bands) of the even-even nuclei to the experimental data [18], using a χ^2 procedure. The theoretical predictions for the even core nuclei are presented in the Figure 1. For comparison, the predictions of IBM (with 4 adjustable parameters) are also shown. The IBM results for ^{156}Gd and ^{162}Dy, ^{172}Yb are extracted from Refs. [19] and [20], respectively. From the figure one can see that the calculated energy levels agree rather well up to very high angular momenta with the observed data. One can see also that for high spins ($L \geq 10 - 14$), where the deviations of the IBM predictions become more significant, the structure of the energy levels of the GSB ($\beta-$ and $\gamma-$bands) is reproduced rather well.

3 Fermion Degrees of Freedom

In order to incorporate the intrinsic spin degrees of freedom into the symplectic IVBM, we extend the dynamical algebra of $Sp(12, R)$ to the orthosymplectic algebra of $OSp(2\Omega/12, R)$ [14]. For this purpose we introduce a particle (quasiparticle) with spin j and consider a simple core plus particle picture. Thus, in addition to the boson collective degrees of freedom (described by dynamical symmetry group $Sp(12, R)$) we introduce creation and annihilation operators $a^\dagger_m$ and a_m ($m = -j, \ldots, j$), which satisfy the anticommutation relations

$$\begin{aligned} \{a^\dagger_m, a^\dagger_{m'}\} &= \{a_m, a_{m'}\} = 0, \\ \{a_m, a^\dagger_{m'}\} &= \delta_{mm'}. \end{aligned} \tag{7}$$

All bilinear combinations of a^+_m and $a_{m'}$, namely

$$\begin{aligned} f_{mm'} &= a^\dagger_m a^\dagger_{m'}, \;\; m \neq m' \\ g_{mm'} &= a_m a_{m'}, \;\; m \neq m'; \end{aligned} \tag{8}$$

$$C_{mm'} = (a^\dagger_m a_{m'} - a_{m'} a^\dagger_m)/2 \tag{9}$$

generate the (Lie) fermion pair algebra of $SO^F(2\Omega)$. Their commutation relations are given in [14]. The number preserving operators (9) generate maximal compact subalgebra of $SO^F(2\Omega)$, i.e. $U^F(\Omega)$. The upper (lower) script B or F denotes the boson or fermion degrees of freedom, respectively.

3.1 Fermion dynamical symmetries

As can be seen from (9) the full number conserving symmetry of a fermion of spin j is $U^F(2j+1)$. In general, the full dynamical algebra build from all bilinear combinations (9), (8) of creation and annihilation fermion operators is the $SO(2\Omega)$ algebra (for a multilevel case $\Omega = \sum_j (2j+1)$). One can further construct a certain fermion dynamical symmetry, i.e. the group-subgroup chain:

$$SO(2\Omega) \supset G' \supset G'' \supset \dots. \tag{10}$$

In particular we are interested in the following dynamical symmetry:

$$SO^F(2\Omega) \supset Sp(\Omega) \supset SU^F(2), \tag{11}$$

where $Sp(2j+1)$ is the compact symplectic group. For one particle occupying a single level j, (11) takes the form

$$SO^F(2\Omega) \supset Sp(2j+1) \supset SU^F(2). \tag{12}$$

The dynamical symmetry (12) remains valid and for the case of two particles occupying the same level j. In this case, the allowed values of the quantum numbers I of $SU(2)$ in (12) according to reduction rules are $I = 0, 2, \dots, 2j-1$ [21]. If the two particles occupy different levels j_1 and j_2 of the same or different major shell(s), one can consider the chain

$$SO(2\Omega) \subset U(\Omega) \begin{array}{l} \nearrow U(\Omega_1) \subset Sp(2j_1+1) \subset SU_{I_1}(2) \searrow \\ \\ \searrow U(\Omega_2) \subset Sp(2j_2+1) \subset SU_{I_2}(2) \nearrow \end{array} SU^F(2) \tag{13}$$

where $\Omega = \Omega_1 + \Omega_2$. We want to point out that although the final group $SU^F(2)$ that appears in the chain (13) is the same as in (12), its content is different. Here the values of the common fermion angular momentum I are determined by the vector sum of the two individual spins I_1 and I_2, respectively. Nevertheless, for simplicity hereafter we will use just the reduction $SO(2\Omega) \subset SU^F(2)$ (i.e. dropping all intermediate subgroups between $SO(2\Omega)$ and $SU^F(2)$) and keep in mind the proper content of the set of I values for one and/or two particles cases, respectively.

3.2 Bose-Fermi symmetry

Ones the fermion dynamical symmetry is determined further the Bose-Fermi symmetries can be considered (constructed). If a fermion is coupled to a boson system having itself a dynamical symmetry (e.g., such as an IBM core), the full symmetry of the combined system is $G^B \otimes G^F$. Bose-Fermi symmetries occur if at some point the same group appears in both chains

$$G^B \otimes G^F \supset G^{BF}, \tag{14}$$

i.e. the two subgroup chains merge into one. It should be noted that (14) is true only for the diagonal subgroup $G^B \otimes G^F$, i.e. the one in which the two group elements multiplied directly are parametrized by the same parameters. In this way Bose-Fermi symmetry not only constrains parameters by the choice of particular subgroup chains in the boson and fermion sectors, but also specifies the interaction between the two.

4 Dynamical Supersymmetry

The standard approach to supersymmetry in nuclei (dynamical supersymmetry) is to embed the Bose-Fermi subgroup chain of $G^B \otimes G^F$ into a larger supergroup G, i.e. $G \supset G^B \otimes G^F$. It is our intention in this paper to do that for chains describing odd-odd nuclei.

Making use of the embedding $SU^F(2) \subset SO^F(2\Omega)$ and considerations from the proceeding section, we make orthosymplectic (supersymmetric) extension of the IVBM which is defined through the chain [14]:

$$
\begin{array}{ccccc}
OSp(2\Omega/12,R) \supset & SO^F(2\Omega) & \otimes & Sp^B(12,R) & \\
 & & & \Downarrow & \\
 & \Downarrow & \otimes & U^B(6) & \\
 & & & N & \\
 & & & \Downarrow & \\
 & SU^F(2) & \otimes & SU^B(3) \otimes U_T^B(2) & \\
 & I & & (\lambda,\mu) \Longleftrightarrow (N,T) & \\
 & & \searrow & \Downarrow & \\
 & & \otimes & SO^B(3) \otimes U(1) & \\
 & & & L \qquad\quad T_0 & \\
 & & \Downarrow & & \\
 & Spin^{BF}(3) & \supset & Spin^{BF}(2), & \\
 & J & & J_0 &
\end{array}
\qquad (15)
$$

where bellow the different subgroups the quantum numbers characterizing their irreducible representations are given. Here by $Spin^{BF}(n)$ ($n = 2, 3$) is denoted the universal covering group of $SO(n)$.

In the next section we expand the earlier application of the IVBM, developed for the description of the collective bands of even-even [12] and odd-mass [14] nuclei, in order to include in our considerations the case of odd-odd nuclei.

5 The Energy Spectra of Odd-Mass and Odd-Odd Nuclei

We can label the basis states according to the chain (15) as:

$$
\mid [N]_6; (\lambda,\mu); KL; I; JJ_0; T_0 \,\rangle \equiv \mid [N]_6; (N,T); KL; I; JJ_0; T_0 \,\rangle, \qquad (16)
$$

where $[N]_6-$ is the $U(6)$ labeling quantum number, $(\lambda,\mu)-$ are the $SU(3)$ quantum numbers characterizing the core excitations, K is the multiplicity index in the reduction $SU(3) \subset SO(3)$, L is the core angular momentum, $I-$the intrinsic spin of an odd particle (or the common intrinsic spin of two particles for the case of odd-odd nuclei), J, J_0 are the total (coupled boson-fermion) angular momentum and its third projection, and T,T_0 are the $T-$spin and its third projection, respectively. Since the $SO(2\Omega)$ label is irrelevant for our application, we drop it in the states (16).

The Hamiltonian can be written as linear combination of the Casimir operators of the different subgroups in (15):

$$H = aN + bN^2 + \alpha_3 T^2 + \beta'_3 L^2 + \alpha_1 T_0^2 + \eta I^2 + \gamma' J^2 + \zeta J_0^2 \qquad (17)$$

and it is obviously diagonal in the basis (16) labeled by the quantum numbers of their representations. Then the eigenvalues of the Hamiltonian (17), that yield the spectrum of the odd-mass and odd-odd systems are:

$$E(N;T,T_0;L,I;J,J_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta'_3 L(L+1) + \alpha_1 T_0^2 + \eta I(I+1) + \gamma' J(J+1) + \zeta J_0^2. \qquad (18)$$

We note that only the last three terms of (17) come from the orthosymplectic extension. We choose parameters $\beta'_3 = \frac{1}{2}\beta_3$ and $\gamma' = \frac{1}{2}\gamma$ instead of β_3 and γ in order to obtain the Hamiltonian form of Ref. [12] (setting $\beta_3 = \gamma$), when for the case $I = 0$ (hence $J = L$) we recover the symplectic structure of the IVBM.

The infinite set of basis states classified according to the reduction chain (15) are schematically shown in Table 1. The fourth and fifth columns show the $SO^B(3)$ content of the $SU^B(3)$ group, given by the standard Elliott's reduction rules [22], while in the next column are given the possible values of the common angular momentum J, obtained by coupling of the orbital momentum L with the spin I. The latter is vector coupling and hence all possible values of the total angular momentum J should be considered. For simplicity, only the maximally aligned ($J = L+I$) and maximally antialigned ($J = L-I$) states are illustrated in Table 1.

The basis states (16) can be considered as a result of the coupling of the orbital $\mid (N,T); KLM; T_0\rangle$ (5) and spin $\phi_{j\equiv I,m}$ wave functions. Then, if the parity of the single particle is π_{sp}, the parity of the collective states of the odd$-A$ nuclei will be $\pi = \pi_{core}\pi_{sp}$ [14]. Analogously, one can write $\pi = \pi_{core}\pi_{sp}(1)\pi_{sp}(2)$ for the case of odd-odd nuclei. Thus, the description of the positive and/or negative parity bands requires only the proper choice of the core band heads, on which the corresponding single particle(s) is (are) coupled to, generating in this way the different odd$-A$ (odd-odd) collective bands.

Further in the present considerations, the yrast conditions yield relations between the number of bosons N and the coupled angular momentum J that characterizes each collective state. For example, the collective states of the GSB $K_J^\pi = \frac{3}{2}^-$ are identified with the $SU(3)$ multiplets $(0,\mu)$ which yield the sequence $N = 2(J-I) = 0,2,4,\ldots$ for the corresponding values $J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$. The T$-$spin for the $SU(3)$ multiplets $(0,\mu)$ is $T = 0$ and hence $\pi_{core} = (-1)^T = (+)$. Here it is assumed that the single particle has $j \equiv I = 3/2$ and parity $\pi_{sp} = (-)$, so that the common parity π is also negative.

For the description of the different excited bands, we first determine the N_i of the band head structure and then we map the states of the corresponding band onto the sequence of basis states with $N = N_i, N_i+2, N_i+4, \ldots$ ($\Delta N = 2$) and $T = even = fixed$ or $T = odd = fixed$, respectively. This choice corresponds to the stretched states of the second type ($\mu-$changed).

Table 1. Classification scheme of basis states (16) according the decompositions given by the chain (15).

N	T	(λ, μ)	K	L	$J = L \pm I$
0	0	$(0, 0)$	0	0	I
2	1	$(2, 0)$	0	$0, 2$	$I;\ 2 \pm I$
	0	$(0, 1)$	0	1	$1 \pm I$
4	2	$(4, 0)$	0	$0, 2, 4$	$I;\ 2 \pm I; 4 \pm I$
	1	$(2, 1)$	1	$1, 2, 3$	$1 \pm I;\ 2 \pm I;\ 3 \pm I$
	0	$(0, 2)$	0	$0, 2$	$I;\ 2 \pm I$
6	3	$(6, 0)$	0	$0, 2, 4, 6$	$I;\ 2 \pm I;\ 4 \pm I; 6 \pm I$
	2	$(4, 1)$	1	$1, 2, 3, 4, 5$	$1 \pm I;\ 2 \pm I;\ 3 \pm I;$ $4 \pm I;\ 5 \pm I$
	1	$(2, 2)$	2	$2, 3, 4$	$2 \pm I;\ 3 \pm I;\ 4 \pm I$
			0	$0, 2$	$I;\ 2 \pm I$
	0	$(0, 3)$	0	$1, 3$	$1 \pm I;\ 3 \pm I$
8	4	$(8, 0)$	0	$0, 2, 4, 6, 8$	$I;\ 2 \pm I;\ 4 \pm I;$ $6 \pm I;\ 8 \pm I$
	3	$(6, 1)$	1	$1, 2, 3, 4, 5, 6, 7$	$1 \pm I;\ 2 \pm I;\ 3 \pm I;$ $4 \pm I;\ 5 \pm I; 6 \pm I;$ $7 \pm I; 8 \pm I$
	2	$(4, 2)$	2	$2, 3, 4, 5, 6$	$2 \pm I;\ 3 \pm I;\ 4 \pm I;$ $5 \pm I;\ 6 \pm I$
			0	$0, 2, 4$	$I;\ 2 \pm I;\ 4 \pm I$
	1	$(2, 3)$	2	$2, 3, 4, 5$	$2 \pm I;\ 3 \pm I;\ 4 \pm I; 5 \pm I$
			0	$1, 3$	$1 \pm I;\ 3 \pm I$
	0	$(0, 4)$	0	$0, 2, 4$	$I; 2 \pm I;\ 4 \pm I$
⋮	⋮	⋮	⋮	⋮	⋮

The number of adjustable parameters needed for the complete description of the collective spectra of the odd-A and odd-odd nuclei is three, namely γ, ζ and η. They are evaluated by a fit to the experimental data [18] of the GSB of the corresponding odd-A and odd-odd nucleus, respectively.

The odd-A nuclei ^{157}Gd, ^{173}Yb and ^{163}Dy can be considered as a neutron particle coupled to the even-even cores ^{156}Gd, ^{172}Yb and ^{162}Dy, respectively. The comparison between the experimental spectra for the GSB and first few excited bands and our calculations for the nuclei ^{157}Gd, ^{173}Yb and ^{163}Dy is illustrated in Figure 2. The last single particle for all of these rare earth nuclei occupies the major shell $N = 82 - 126$, where the relevant single particle levels are $2f_{\frac{7}{2}}, 2f_{\frac{5}{2}}, 3p_{\frac{3}{2}}, 3p_{\frac{1}{2}}$ having odd parity ($\pi_{sp} = -$) (excluding the intruder from the upper shell with opposite parity). In our considerations we take into account only the first available single par-

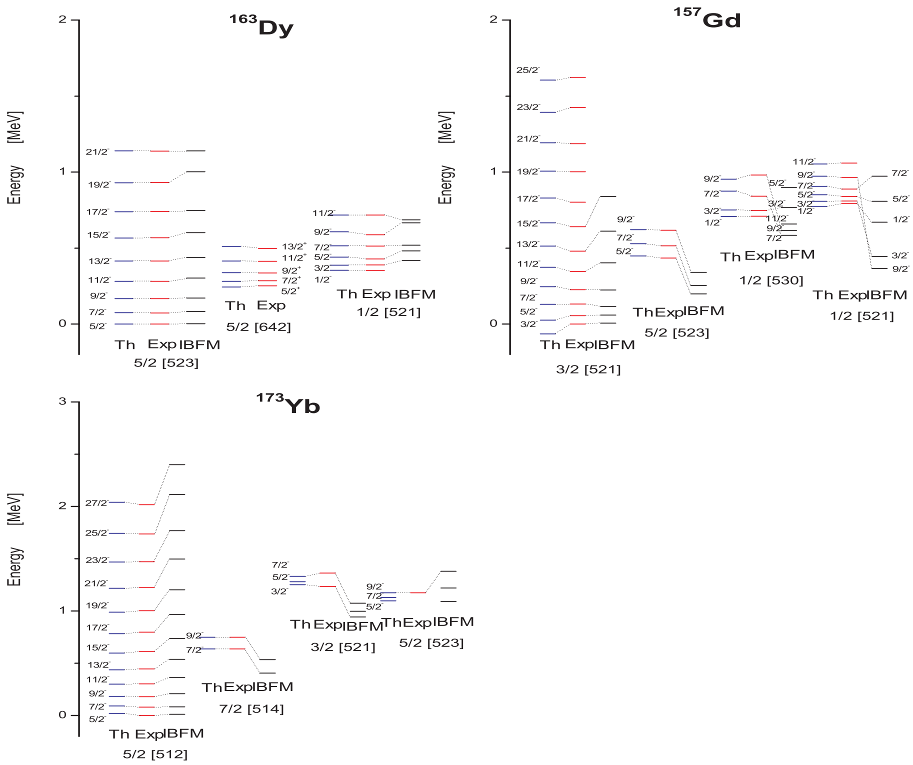

Figure 2. Comparison of the theoretical and experimental energies for the ground and first excited positive and/or negative parity bands of ^{157}Gd, ^{173}Yb and ^{163}Dy odd-mass nuclei.

ticle orbit j_1 (generating the groups $SO(2\Omega_1)$ and/or $U(\Omega_1)$ with $\Omega_1 = (2j_1+1)$), which for the first nucleus implies $j_1 = \frac{3}{2}$, while for the other two $-$ $j_1 = \frac{5}{2}$. The Nilsson asymptotic quantum numbers $\Omega[Nn_3\Lambda]$ are written bellow each band. One can see from the figure that the calculated energy levels agree rather well in general with the experimental data up to very high angular momenta. For comparison, in the Figure 2 the IBFM results (obtained by total 7 adjustable parameters) are also shown. They are extracted from Refs. [19] and [20], respectively. Note that all calculated levels, for the bands considered, are in correct order in contrast to IBFM results (for ^{157}Gd). Another difference between the IVBM and IBFM predictions is that in the former the correct placement of all the band heads is reproduced quite well.

For the calculation of the odd-odd nuclei spectra a second particle should be coupled to the core. In this paper we present results for the three odd-odd nuclei, namely ^{158}Tb, ^{164}Ho and ^{174}Lu. In our calculations a consistent procedure is employed which includes the analysis of the even-even and odd-even neighbors of the nucleus under consideration. Thus, as a first step an odd neutron was coupled to the boson core in order to obtain the spectra of the odd-mass neighbors ^{157}Gd, ^{173}Yb

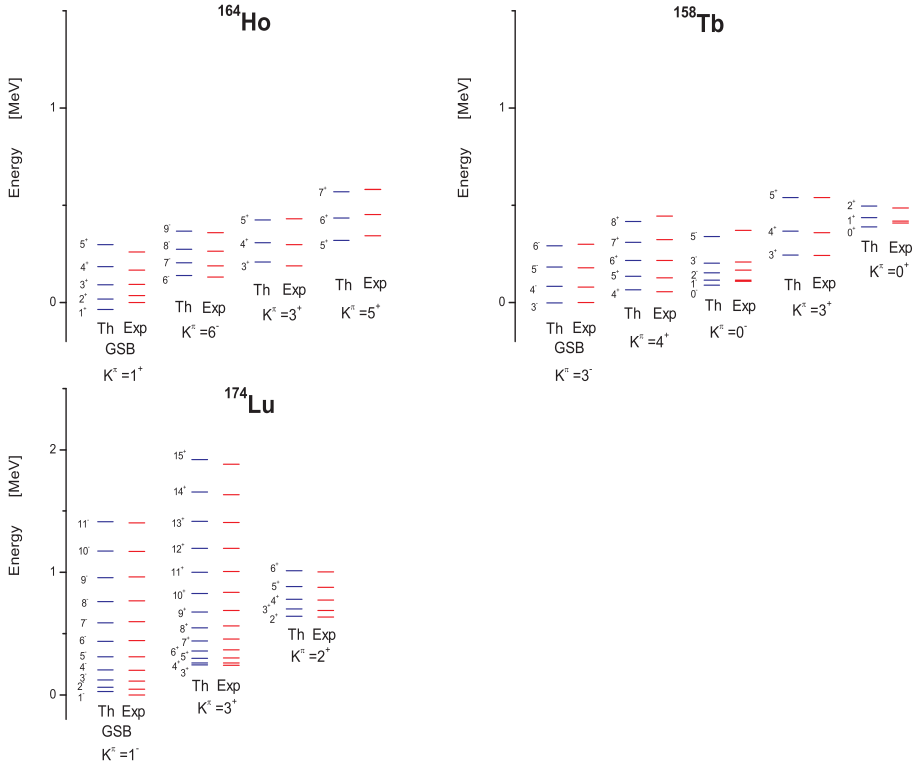

Figure 3. Comparison of the theoretical and experimental energies for the ground and first excited positive and/or negative parity bands of ^{164}Ho, ^{158}Tb and ^{174}Lu odd-odd nuclei.

and ^{163}Dy. As a second step, we consider an addition of a proton to the boson-fermion system. For the considered mass region, the relevant single particle levels for the proton are $2d_{\frac{5}{2}}, 2d_{\frac{3}{2}}, 1g_{\frac{7}{2}}, 3s_{\frac{1}{2}}$ of the major shell $N = 50-82$ with even parity ($\pi_{sp} = +$). In the calculations we take into account also only the first available level j_2 for the proton particle. Thus, we obtain the observed GSB's: $K^\pi = 3^-$, $K^\pi = 1^+$ and $K^\pi = 1^-$ for the ^{158}Tb, ^{164}Ho and ^{174}Lu, respectively. For example, the $K^\pi = 3^-$ GSB of ^{158}Tb is obtained considering $I_n = \frac{3}{2}$ and $I_p = \frac{3}{2}$ coupled to $I = 3$. Analogously, one can obtain the GSB for the other two nuclei considering $I_n = \frac{5}{2}$,$I_p = \frac{3}{2}$ and $I_n = \frac{5}{2}$,$I_p = \frac{7}{2}$ for the ^{164}Ho and ^{174}Lu, respectively. The theoretical prediction for the ground and first excited bands for the three odd-odd nuclei are presented in Figure 3. They are compared with the experimental data. From the figure one can see the good overall agreement between the theory and the experiment which reveals the applicability of the boson-fermion extension of the model.

6 Conclusions

In this paper the orthosymplectic extension of the IVBM was applied for the description of the low-lying spectra of odd-odd nuclei. For this purpose, the fermion dynamical symmetries and corresponding combined Bose-Fermi symmetries were considered in much more details.

The basis states of the odd-mass and odd-odd systems are classified by the dynamical symmetry (15) and the model Hamiltonian is written in terms of the first and second order invariants of the groups from the corresponding reduction chain. Hence the problem is exactly solvable within the framework of the IVBM which, in turn, yields a simple and straightforward application to real nuclear systems.

The structure of some odd-odd nuclear spectra has been investigated in the framework of the IVBM. The even-even nuclei are used as a core on which the collective excitations of the neighboring odd-mass and odd-odd nuclei are build on. Thus, the spectra of odd-mass and odd-odd nuclei arise as a result of the coupling of the fermion degrees of freedom to the boson core. The good agreement between the theoretical and the experimental band structures confirms the applicability of the used dynamical symmetry of the IVBM. The success is based on the (ortho)symplectic structures of the model which allow the mixing of the basic collective modes −rotational and vibrational ones arising from the yrast conditions. This allows for the proper reproduction of the high spin states of the collective bands and the correct placement of the different band heads.

The supersymmetry group $OSp(2\Omega/12, R)$ could be further used to examine the nuclear supersymmetry which might be considered in nuclear physics as proved experimentally [23]. The model can be also used for the description and systematics of other collective bands. More extensive calculations, including calculations for more extended series of isotopes should provide a more stringent test of the model. Critical phase/shape phenomena can also be analyzed within the model. They may also help us to develop a better understanding of the physical correspondences to the IVBM group-theoretical parameters.

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