Systematics of the Behavior of the Ground and Octupole Bands of the Deformed Even-Even Lanthanides

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Abstract. We investigate systematically the behavior of the energies of collective states from the ground and octupole bands of the deformed even-even lanthanides, which are described very accurately by the $U(6) \supset U(3) \otimes U(2) \supset SO(3)$ dynamical symmetry of the symplectic extension to $Sp(12, R)$ of the Interacting Vector Boson Model. Our approach is based on the assumption that the phenomenological parameters of the Hamiltonian, obtained for each individual nucleus, should reflect the dependence of the energies on the nuclear characteristics including, especially, the dependence on number of neutrons N and protons Z that build it. Hence we employ a two-step fitting procedure. First we fit the parameters of the Hamiltonian as first or second order polynomials in the changing values of N for the isotopic chains (fixed Z) of the deformed Nd, Sm, Gd, Dy, Er, Yb and Hf nuclei. The next step is to approximate each of the polynomial coefficients obtained with their uncertainties by a linear regression with respect to the number of valence protons Z. As a result we obtain a unified expression for the energies of the ground and octupole bands for the deformed nuclei in the rare earth region. This allows us to relate the phenomenological parameters of the Hamiltonian to the fundamental nuclear structure and to interpret the development of collectivity across the broad region of nuclei that we explore.

1 Introduction

Algebraic models like the IBM [1], which are based on the symmetry principles, have proven themselves appropriate for the description of a large amount of data [2] on nuclear structure. Even more, they have established benchmarks for understanding the evolution of the nuclear structure [3] and have developed the necessary technique for the description of the sometimes sharp (phase/shape) transitions from one to another collective mode [4]. Nevertheless, along the way some of the beauty and simplicity of the group theoretical approach to the nuclear structure problems is somewhat lost and mainly geometrical considerations in terms of the collective model's [4, 5] variables are advanced.

In the early eighties, a boson number preserving version of the phenomenological algebraic Interacting Vector Boson Model /IVBM/ [6] was successfully ap-

plied [7] for a description of the low-lying collective rotational spectra of the even-even medium and heavy mass nuclei. With an aim to extend these application in order to incorporate the new experimental data on the states with higher spins and on the various new excited bands, we explore the symplectic extension of the IVBM, for which the group of dynamical symmetry is the $Sp(12,R)$. The extension is realized and has its physical interpretation over the basis of its maximal compact subgroup $U\left(6\right) \subset Sp(12,R)$. This naturally leads to an additional description [8] of not only the energies, but of the staggering effects between the states of the ground and octupole bands up to very high spins.

In this particular limiting case, as well as in the other two dynamical symmetry limits [9, 10] of the symplectic extension of the IVBM, we obtain exact analytic solutions for the eigenvalues of the Hamiltonian, expressed in terms of the first and second order Casimir invariants of the subgroups in the respective chain, in the basis labelled by the quantum numbers of their irreducible representations. To advantage this further, we also use in the applications the possibility of changing the number of phonons building the collective states and as a consequence gain larger basis spaces that incorporate more and more complex nuclear spectra. In practice the applications for a description of nuclear collective spectra is achieved by a fit of the parameters of the model Hamiltonian to the experimental energies of each nucleus considered. An obvious drawback of this phenomenological approach, is that the results obtained do not contain any specific dependence on the individual nuclear characteristics. In other words, we have a set of model parameters for each nucleus. Hence the aim of the preliminary results that we present in this talk is to investigate the behavior of the phenomenological parameters for a sequence of nuclei and to establish if they reflect some dependence on a specific characteristic of the nuclear structure.

2 Description of the Ground and Octupole Bands in the Unitary Limit of the IVBM

First we describe the algebraic structure of the IVBM, which is realized in terms of creation (annihilation) operators, $u_m^+(\alpha)(u_m(\alpha))$, in a 3- dimensional oscillator potential ($m = 0, \pm 1$) of two types of bosons differing by the value of the "T-spin" projection $\alpha = 1/2(p)$, $\alpha = -1/2(n)$. The bilinear products of the creation and annihilation operators of the two vector bosons generate the boson representations of the non-compact symplectic group $Sp(12,R)$ [6]. The set of operators:

$$A_M^L(\alpha,\beta) = \sum\nolimits_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m(\beta), \tag{1}$$

where C_{1k1m}^{LM} are the usual Clebsch-Gordon coefficients for $L = 0, 1, 2$ and $M = -L, -L+1, ...L$, define their transformation properties under rotations and close under commutation the algebra of the maximal compact subgroup of $Sp(12,R) \supset U\left(6\right)$ [11]. The linear invariant of $U(6)$ is the number operator,

$$N_b = \sqrt{3}(A^0(p,p) + A^0(n,n)) = N_+ + N_-, \tag{2}$$

that counts the total number of bosons. Hence this operator (2) splits the boson representations of $Sp(12,R)$ into a countless number of symmetric unitary irreducible representations (UIR) of the type $[N_b,0,0,0,0,0] = [N_b]_6$, where $N_b = 0,2,4,...$ for the even UIR and $N_b = 1,3,5,...$ for the odd ones.

In this work we investigate the behavior of the free parameters a, b, α_3, α_1 and β_3 of the Hamiltonian:

$$H = aN_b + bN_b^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2, \tag{3}$$

which is constructed by the first and second order invariants of the subgroups of the "unitary" limit of the symplectic $Sp(12,R)$,

$$U(6) \supset SU(3) \otimes U(2) \supset SO(3) \otimes U(1) \tag{4}$$

$$[N_b] \qquad (\lambda,\mu) \quad (N,T) \;\; K \;\; L \qquad T_0, \tag{5}$$

where the quantum numbers (5) of the $U(6)$-basis that is used are given below each of the subgroups of the chain (4). The first two terms in (3) are related to the $U(6)$ invariants, and the L^2 term is the second-order invariant of the $SO(3)$ group of the angular momentum L. Since $SU(3)$ and $U(2)$ in (4) are mutually complementary groups, their Casimir invariants are related and we don't include $C_2(SU(3))$ in the Hamiltonian, but rather only $C_2(SU(2)) \sim T^2$ of the important group of the $T-$ spin.

The quantum numbers of $SU(3)$ and $U(2)$ are related in the following way:

$$T = \frac{\lambda}{2}, N = 2\mu + \lambda, \tag{6}$$

so we can write the basis as $|\,[N_b]_6;(\lambda,\mu);K,L,M;T_0\rangle = |\,(N_b,T);K,L,M;T_0\rangle$. The ground state of the system is the vacuum state $|0\rangle$ with $N_b = 0, T = 0, K = 0, L = 0, M = 0, T_0 = 0$. The basis states associated with the even irreducible representation of the $Sp(12,R)$ can be constructed by the application on it of even powers, given by N_b of the creation operators $u_m^+(\alpha)$ [12]. Therefore the complete spectrum of the system is calculated through the diagonalization of the Hamiltonian in the subspaces of all the UIR of $U(6)$, belonging to a given UIR of $Sp(12,R)$, which solves the problem of dealing with noncompact representations. Then the possible values for the $T-$spin for each fixed value of N_b-even are $T = \frac{N_b}{2}, \frac{N_b}{2} - 1, \ldots$ 0. Thus when N_b and T are fixed, $2T+1$ equivalent representations of the group $SU(3)$ arise, for which λ and μ are obtained from (6). Each of them is labelled by the eigenvalues of the operator $T_0 : -T, -T+1, \ldots, T$. The usefulness of obtaining the $SU(3)$ representations (λ,μ) lies in the availability of the standard reduction rules for the $SU(3) \supset SO(3)$ chain. The multiplicity index K appearing in this reduction (5) is related to the projection of L in the body fixed frame [13] and is used with the parity (π) to label the different bands (K^π) in the energy spectra of the nuclei. We define the parity of the states as $\pi = (-1)^T$, which allows us to describe both positive and negative bands. The Hamiltonian (3) is obviously diagonal in the basis labelled by the quantum numbers of the subgroups of the chain (4).

3 Application to Real Nuclear Spectra

A successful application of the theory requires a correct identification of the experimentally observed bands with the sequences of basis states for the even representation of $Sp(12, R)$. In [8] for the description of the ground band, which starts with the vacuum state $|0\rangle$, we used the basis states with $L = 0, 2, 4, 6, \ldots$ created with $N_b = 0, 4, 8, \ldots$ bosons with $T = 0, (\lambda = 0)$ and $T_0 = 0$, which correspond according to (6) to the sequence of $SU(3)$ multiplets $(0, \mu = N_b/2)$. While for the octupole band we have the following identification for its states with $L = 3, 5, 7, \ldots$ and negative parity $T = 1, (\lambda = 2)$; $T_0 = 0$ and $N_b = 8, 12, ...$, which means that the $SU(3)$ multiplets $(2, \mu-1)$ are used. Further we define the energies of each state with given L as yrast energy with respect to N_b in the two considered bands. Hence, taking into account the reduction rules [8], their minimum values are obtained at $N_b = 2L$ for the ground band, and $N_b = 2L+2$ for the octupole band, respectively.

According to the correspondence identified above the employed basis states have $T_0 = 0$ and the last term in the Hamiltonian (3) vanishes. Its eigenvalues give the energies of the collective states of ground state band /gsb/ as:

$$E_g(L) = (2a - 4b)L + (4b + \beta_3)L(L + 1) \, , \tag{7}$$

and for the octupole band as:

$$E_o(L) = (2a + 4b)L + (4b + \beta_3)L(L + 1) + (2a + 4b + 2\alpha_3) \, . \tag{8}$$

From (7) and (8), it can be seen that the eigenvalues of states of the first positive and negative bands mix the rotational $L(L + 1)$ and vibrational L collective modes. The rotational interaction is with equal strength $(4b + \beta_3)$ in both of the bands. For the octupole band there is an additive constant that is the only dependent on the parameter α_3.

Further the application to real nuclear systems is realized by means of the evaluation of the phenomenological model parameters a, b, α_3, and β_3 by a fit to the experimental data [14]. Hence we obtain for each of the considered nuclei a set of these four parameters of the Hamiltonian, which according to (7) and (8) reproduce very accurately the energies in their ground and octupole bands up to rather high spins [8].

4 Dependence of the Parameters on the Nuclear Characteristics

As mentioned above, a drawback of our model approach is that although the description of the collective bands we obtain is very good, the model parameters are fit for each nucleus and there is no dependence in them on the specific nuclear characteristics. In the present case the quantum numbers that define the states depend upon the chosen dynamical symmetry and as a consequence the result are directly linked to that symmetry limit, and therefore should not be subject to a broader interpretation as is typically done, for example, within an IBM framework. In order to overcome

this apparent model dependency and to establish that the parameters of the theory are linked to the nuclear structure, we must systematically investigate the behavior of the parameters for a sequence of nuclei. For this purpose we start with the collection of the experimental data on the energies of the ground and octupole bands in the heavy nuclear systems, and specifically for rare-earth and actinide nuclei [14]. For present purposes, we selected nuclei for which there are at least 3 states that are known experimentally in the ground band as well as in the first negative parity band, so as to have good statistics in evaluating the four parameters of the Hamiltonian. While the actinides are better investigated and the observed bands longer [15, 16], the data on the lanthanides is richer with respect to the number of isotopic chains for which data is available and correspondingly there is more relevant information on their deformation and geometric shapes. Hence in this work, we presents and discuss a systematic study of the behavior of the Hamiltonian parameters in the following isotopic chains of the lanthanides:$_{60}$Nd with $N = 84-92$; $_{62}$Sm with $N = 84-94$; $_{64}$Gd with $N = 88 - 96$; $_{66}$Dy with $N = 90 - 96$; $_{68}$Er with $N = 90 - 100$; $_{70}$Yb with $N = 86 - 104$ and $_{72}$Hf with $N = 86 - 100$.

We start by plotting the values of the four hamiltonian parameters a, b, β_3 and α_3 for each of the isotopic chains (fixed Z) as functions of the number of neutrons N, denoted by shapes on the Figures: 1, 2, 3 and 4. Although in the isotopic chains considered we have different change ranges of the number of neutrons N, there are rough features of the behavior of the parameters that can be clearly observed and analyzed. First of all the changes in the parameters a, b and β_3 are very similar in all isotopic chains, being relatively small and smooth which stands in contrast to the rather sharp increase with N in the value of α_3.

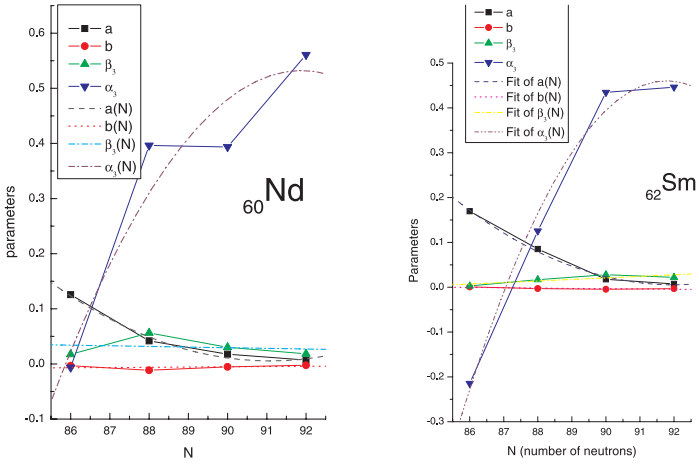

Figure 1. Behavior of the parameters a, b, α_3, and β_3 of the energies of the ground and octupole bands and their approximation with linear and polynomial functions of the changing values of N for the isotopic chains of the Nd and Sm.

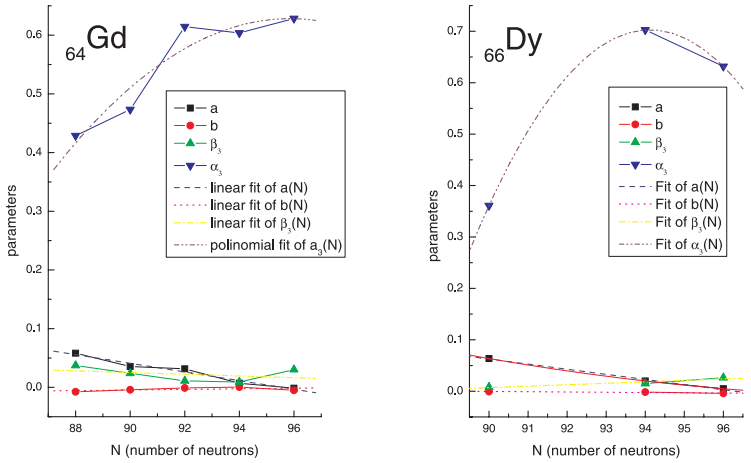

Figure 2. Same as on Figure 1 for the isotopic chains of the Gd and Dy.

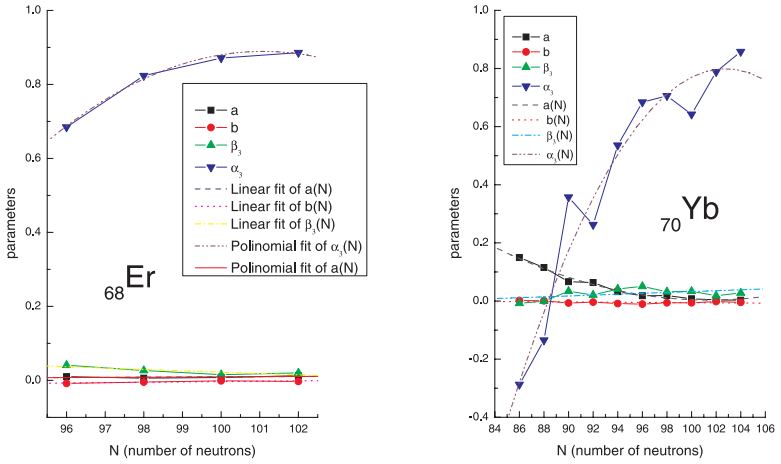

Figure 3. Same as on Figure 1 for the isotopic chains of the Er and Yb.

The observed changes in the parameters within the isotopic chains as a function of the number of the neutrons in the nucleus gives us an opportunity to approximate them with functions of N in the following way:

$$a(N) = A_0 + A_1 N + A_2 N^2; \qquad b(N) = B_0 + B_1 N; \tag{9}$$

$$\beta_3(N) = C_0 + C_1 N; \qquad \alpha_3(N) = D_0 + D_1 N + D_2 N^2 \tag{10}$$

Hence b and β_3 are linear functions of N and a and α_3 are second order polynomials. We realize that the approximations with smooth functions will not reflect the sometimes sharp changes in the parameters. A particularly illustrative example are these changes for α_3 between $N = 88$ and $N = 90$, which are well-known [17] as

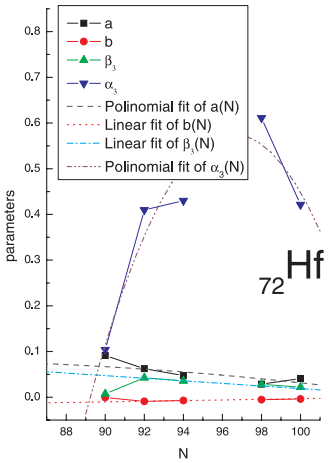

Figure 4. Same as on Figure 1 for the isotopic chain of the Hf.

regions of nuclei at critical points. This critical behavior is reflected in the octupole band, as can be seen for the Nd, Sm and Yb isotopes on Figures 1 and 3.

In the chosen approximations (9) and (10) for the parameters we obtain in the first fitting procedure the respective coefficients $A_i, i = 0, 1, 2$, $B_i, i = 0, 1$, $C_i, i = 0, 1$ and $D_i, i = 0, 1, 2$ and their uncertainties. Then on the panels (i changing) of Figures 5, 6, 7 and 8 we plot each of the values obtained for the isotopic chains considered, that is, Z changing from 60 to 72. While a quick look would suggest there is no systematic behavior among the coefficients, with at most simply a change of the scale with the degree of N noted, on closer scrutiny one can see from a straight-line interpolation through the points that there is a staggering about such linearity that depends upon whether $Z/2$ is even or odd.This can be related to the staggering of the energies of ground band states in even-even nuclei [18], but there needs to be more careful investigation of the phenomena. In this situation, changes in these parameters as a function of Z is not simple, but taking into account relatively short range changes in the parameters it makes sense to do the simplest linear approximation in Z (doted lines on Figures 5, 6, 7 and 8) as follows:

$$A_i(Z) = +A_{i0} + A_{i1}Z \qquad B_i(Z) = +B_{i0} + B_{i1}Z \tag{11}$$

$$C_i(Z) = +C_{i0} + C_{i1}Z \qquad B_i(Z) = +B_{i0} + B_{i1}Z \tag{12}$$

So, in the second fitting procedure we evaluated the values of the parameters $A_{ik}, i = 0, 1, 2$, $B_{ik}, i = 0, 1$, $C_{ik}, i = 0, 1$ and $D_{ik}, i = 0, 1, 2$ for $k = 0, 1$ (20 in total) in expressions (11) and (12) and their uncertainties. The values we obtained are given in Tables 1, 2, 3 and 4.

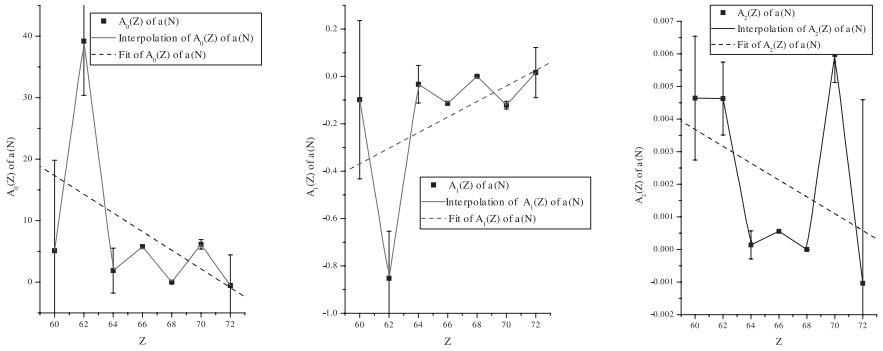

Figure 5. Behavior of the coefficients $A_i, i = 0, 1, 2$ in the expression for the parameter $a(N)$ (left of (9)) and their linear approximations as functions of Z.

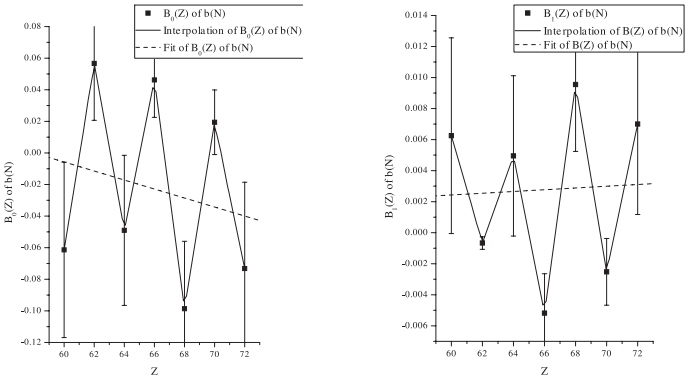

Figure 6. Same as on Figure 5 for the coefficients $B_i, i = 0, 1$ for the parameter $a(N)$ (right of (9)).

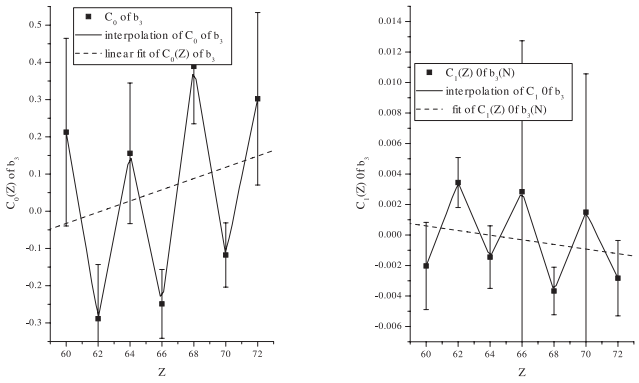

Figure 7. Same as on Figure 5 for the coefficients $C_i, i = 0, 1$ for the parameter $\beta_3(N)$ (left of (10)).

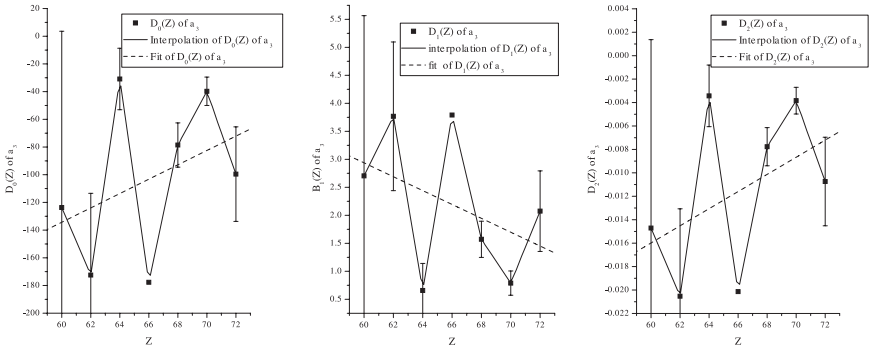

Figure 8. Same as on Figure 5 for the coefficients $D_i, i = 0, 1$ for the parameter $\alpha_3(N)$ (right of (10)).

5 Results and Discussion

As a result of our empirical study of the behavior of the Hamiltonian parameters in the isotopic chains of deformed even-even rare earth nuclei, we obtain the following general expressions for them as a function of the number of protons Z and neutrons N in each nuclear species:

$$\begin{aligned} a(N,Z) &= A_{00} + A_{01}Z + (A_{10} + A_{11}Z)N + (A_{20} + A_{21}Z)N^2, \\ b(N,Z) &= B_{00} + B_{01}Z + (B_{10} + B_{11}Z)N, \\ \beta_3(N,Z) &= C_{00} + C_{01}Z + (C_{10} + C_{11}Z)N, \\ \alpha_3(N,Z) &= D_{00} + D_{01}Z + (D_{10} + D_{11}Z)N + (D_{20} + D_{21}Z)N^2. \end{aligned} \tag{13}$$

Introducing these values into the expressions for the energy functions of the ground and octupole bands (7) and (8) we describe the energies of 423 states of 43 nuclei

Table 1. $A_{ik}, i = 0, 1, 2; k = 0, 1$

A_{00}	ΔA_{00}	A_{01}	ΔA_{01}
101.81988	93.85576	-1.40442	1.42632
A_{10}	ΔA_{10}	A_{11}	ΔA_{11}
-2.20426	2.0675	0.03052	0.03142
A_{20}	ΔA_{20}	A_{21}	ΔA_{21}
0.01715	0.0195	-2.23382E-4	2.96385E-4

Table 2. $B_{ik}, i = 0, 1; k = 0, 1$

B_{00}	ΔB_{00}	B_{01}	ΔB_{01}
0.16519	0.41831	-0.00285	0.00633
B_{10}	ΔB_{10}	B_{11}	ΔB_{11}
-9.39097E-4	0.03784	5.61948E-5	5.72233E-4

Table 3. $C_{ik}, i = 0, 1; k = 0, 1$

C_{00}	ΔC_{00}	C_{01}	ΔC_{01}
-0.93744	1.81635	0.01508	0.02747
C_{10}	ΔC_{10}	C_{11}	ΔC_{11}
0.0097	0.01907	-1.51786E-4	2.88403E-4

Table 4. $D_{ik}, i = 0, 1, 2; k = 0, 1$

D_{00}	ΔD_{00}	D_{01}	ΔD_{01}
-445.17911	371.07499	5.18106	5.61205
D_{10}	ΔD_{10}	D_{11}	ΔD_{11}
10.36298	8.05326	-0.12378	0.1218
D_{20}	ΔD_{20}	D_{21}	$\Delta D21$
-0.05986	0.04372	7.3125E-4	6.61214E-4

which exhibit a rather complex transitional spectra. Analyzing the values of the parameters that were so determined, given in the Tables 1, 2, 3 and 4, we were able to conclude that their biggest weight comes from the free coefficients. The smallest parameters multiply the third-order terms ZN^2, which have rather large values for the nuclei considered. In general there is in all the parameters of the Hamiltonian a dependence on both N and Z and their product NZ, that reflects on the importance of the proton neutron interactions which were initially included in the interpretation of the models assumptions.

Of course in order to test our procedure, we will have to further establish the accuracy of our results by comparing the energies obtained with the new 20 parameters for the states of the two bands of each nucleus (the corresponding values of (N,Z)) with the experimental data. Another advantage of this approach is the possibility to predict by interpolation the values of some parameters and to calculate with their help the not yet measured energies of the states from the bands under consideration.

This work was supported by the Bulgarian National Foundation for scientific research under Grant Number $\Phi - 1501$, a grant for scientific collaboration with the Bogolyubov Laboratory for Theoretical Physics of JINR, Dubna and the U.S. National Science Foundation, Grant Number 0500291.

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