Coriolis Interaction in Quadrupole-octupole Deformed Nuclei

N. Minkov[1], M. Strecker[2], and W. Scheid[2]

[1] Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Tzarigrad Road 72 , BG–1784 Sofia, Bulgaria
[2] Institut für Theoretische Physik der Justus-Liebig-Universität, Heinrich-Buff-Ring 16, D–35392 Giessen, Germany

Abstract. Based on a recent application of the collective model of coherent quadrupole-octupole oscillations and rotations in odd-mass nuclei we develop an algorithm for a microscopic calculation of the Coriolis interaction strength. It is realized by using the reflection-asymmetric deformed shell model. The single particle (s.p.) wave function is obtained in the basis of the axially deformed harmonic oscillator (ADHO). The Coriolis interaction strength is calculated after transforming the ADHO decomposition coefficients of the wave function into coefficients in the basis of the spherical harmonic oscillator (SHO). The transformation brackets relating the ADHO and SHO basis functions are numerically integrated. Calculations were implemented for several nuclei in which the parity doublet spectra are known or supposed to be built on a s.p. orbital with $\Omega = 1/2$. The results show the applicability of this approach to study the effects of Coriolis interaction in nuclei with quadrupole and octupole deformations as well as to examine their s.p. and shape characteristics.

1 Introduction

The collective spectra of nuclei with quadrupole and octupole degrees of freedom show a variety of specific structure properties [1, 2]. In even-even nuclei the collective dynamics is governed by the simultaneous vibration and rotation motion of the quadrupole-octupole shape. Due to the total rotation and space inversion invariance of the system an alternating parity structure of the spectra is observed. In odd-A nuclei the single-particle (s.p.) motion manifests together with the rotation and vibration motions of the even-even core. The conservation of the total (core+particle) symmetry in the space provides the observed parity-doublet structure of the respective spectra. The parity doublets are split due to the octupole vibration mode. This structure has been described in various model approaches [3–5].

The structure of the spectrum is determined by the conservation of the total parity of the system even when the rotation, vibration and the s.p. motions are adiabatically separated. On the other hand some basic dynamical properties of the nuclei are due to the non-adiabatic interactions between different kinds of motions. Such are the vibration-rotation interaction and the Coriolis interaction between the single particle and the even-even core. In this respect the observed split parity-doublet spectra of odd-mass nuclei provide rich information about the interaction between the collective and single-particle motion in the nuclear system. In particular, the behaviour of the energy difference between two opposite parity counterparts in dependence on the angular momentum carries specific information on the Coriolis interaction [5]. On the other hand, in some cases, the s.p. orbital on which the parity-doublet is built can not be unambiguously determined, neither from the microscopic nor from the collective point of view. Therefore, one could expect that a better understanding of the way in which the structure of the spectrum is formed can be achieved through a consistent application of both collective and microscopic approaches.

Based on the recent application of the collective model of coherent quadrupole-octupole motion [6] to odd-A nuclei [5], in the present work we consider a deformed shell model formalism as a tool to study the interaction between single-particle and collective degrees of freedom in nuclei. The aim of the work is to develop an algorithm capable of calculating the Coriolis interaction strength in the single-particle states of nuclei with pronounced quadrupole and octupole deformations. Such a consideration makes it possible to compare the capabilities of the both collective and shell model approaches for describing the effects of Coriolis interaction as well as to examine the relation between the s.p. motion and the shape characteristics of odd-A nuclei. We show that in such a way the problem for the unambiguous determination of the s.p. state, on which the collective spectrum is grounded, can be solved to a great extent. In addition we examine the possibility for a consistent application of the deformed shell model together with the collective quadrupole-octupole formalism.

2 Coherent Quadrupole–Octupole Motion with Coriolis Interaction

We consider that the even–even core of an odd-A nucleus is allowed to oscillate with respect to the quadrupole β_2 and octupole β_3 axial deformation variables mixed through a centrifugal (rotation-vibration) interaction. The unpaired nucleon contributes to the collective motion of the total system through the Coriolis interaction. The collective Hamiltonian of the nucleus can then be taken in the form [5]

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{X(I,K,\pi a)}{d_2\beta_2^2 + d_3\beta_3^2} \, , \quad (1)$$

where B_2 and B_3 are the effective quadrupole and octupole mass parameters and C_2 and C_3 are the stiffness parameters for the respective oscillation modes. The last part of (1) represents the centrifugal term in which the Coriolis interaction is taken into account

$$X(I,K,\pi a) = \frac{1}{2}\left[d_0 + I(I+1) - K^2 + \pi a \delta_{K,\frac{1}{2}}(-1)^{I+1/2}\left(I+\frac{1}{2}\right)\right] . \quad (2)$$

In [5] the decoupling factor a is taken as a fitting parameter, while d_0 characterizes the shape of the potential in the ground state of the core.

The Schrödinger equation for the Hamiltonian (1) was solved under the assumption of a coherent interplay between the quadrupole and octupole modes, which allows one to obtain the energy spectrum in the following analytic form [5]

$$E_{n,k}(I,K,\pi a) = \hbar\omega\left[2n+1+\sqrt{k^2+bX(I,K,\pi a)}\right],\ \ n=0,1,2,... \qquad (3)$$

where ω and b are related to the mass and stiffness parameters. The quantum number k switches between 1 and 2 in dependence on the parity π, providing an energy difference between states with the same angular momentum and opposite parity.

Eq. (3) has been applied to describe the split parity doublet structure of the spectra in a wide range of odd-mass nuclei [5]. The model analysis shows that the parity-doublet splitting, given by the quantity $\Delta E(I^{\pm}) = E(I^{+}) - E(I^{-})$ exhibits a staggering behaviour as a function of I for $K = 1/2$ when the spectrum is perturbed by the Coriolis interaction. For $K \neq 1/2$ (reduced Coriolis interaction) a smooth behavior of the parity splitting is observed.

On the above basis it was suggested that the behaviour of the doublet-splitting allows one to estimate the possible values of the angular momentum projection K on which the parity-doublet structure is built. This is illustrated in Figure 1 for the nucleus ^{223}Ra. It is seen that the experimentally assumed [7] value $K = 3/2$ does not support the staggering behaviour of the parity splitting observed in the experimental data, while if the value $K = 1/2$ is assumed, the staggering behaviour of $\Delta E(I^{\pm})$ is reproduced. In such a way the staggering effect indicates a strong contribution of an intrinsic $K = 1/2$ configuration which is related to the Coriolis coupling interaction. The considered example suggests the possibility to get information about the single-particle orbitals which contribute to the K-configurations associated with the particular collective spectrum of the nucleus.

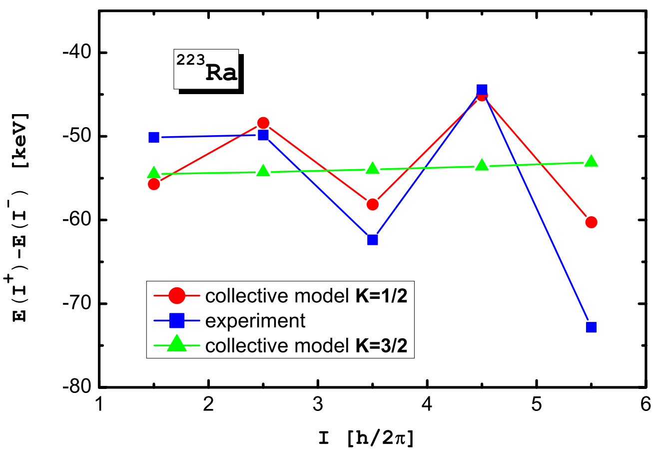

Figure 1. Experimental and theoretical parity-doublet splitting in ^{223}Ra with $K = 3/2$ and $K = 1/2$. Data from [7]

The above result indicates the need of a deeper microscopic analysis of the single-particle motion in odd-mass nuclei with quadrupole-octupole degrees of freedom and its relation to the collective motion of the system. Therefore, in the present work we consider the quadrupole-octupole deformed shell model formalism which is capable to describe the single-particle motion in the field of the reflection asymmetric deformed even-even core of the nucleus. An important task, that will be addressed in the next section, is to estimate the contribution of the Coriolis interaction on the basis of shell model calculations and to compare the result with the estimations suggested by the above collective model approach. As a consequence the question about the unambiguous determination of the quantum number K for the ground state will be clarified on a deeper structure level.

3 Coriolis Interaction by the Quadrupole-Octupole Deformed Shell Model

To examine the Coriolis interaction from the intrinsic point of view, we refer to the shell model analysis. Since the considered nuclei are characterized by shape deformations with a presence of reflection asymmetry, we describe the s.p. state with the quadrupole-octupole deformed shell model for which a numerical code is available [8]. The Hamiltonian for the single particle motion is

$$H_{\text{sp}} = T + V_{\text{ws}} + V_{\text{s.o.}} + \frac{1}{2}(1+\tau_3)V_{\text{Coul}}, \tag{4}$$

where

$$V_{\text{ws}}(r,\theta,\phi) = -V_0\left[1+\exp\left(\frac{r-R(\theta,\phi)}{a(\theta,\phi)}\right)\right]^{-1}$$

is the Woods-Saxon potential with $R(\theta,\varphi) \sim R_0\left(1+\sum_{\lambda=2}^{6}\beta_\lambda Y_{\lambda 0}(\theta,\varphi)\right)$. $V_{\text{s.o.}}$ and V_{Coul} are the spin-orbit and Coulomb terms whose analytic form is given in [8].

By diagonalizing the Hamiltonian (4) in the basis of the axially deformed harmonic-oscillator (ADHO), $|Nn_z\Lambda\Sigma\rangle$ [9], one can obtain the single-particle orbital occupied by the odd particle in the odd-A nucleus. The ADHO basis states can be also denoted as $|Nn_z\Lambda\Omega\rangle$ with $\Omega = \Lambda + \Sigma$. Due to the axial symmetry, Ω is equal to the third projection K of the total angular momentum I. Thus the solution of the deformed shell model problem could provide a microscopic estimation for the quantum number K.

In order to determine unambiguously the s.p. quantum number $\Omega = K$ one has to solve the complete Schrödinger equation which contains the sum of the collective Hamiltonian (1) and the single-particle Hamiltonian (4). However, as a first step it is more simple to consider (1) and (4) separately. Therefore, in the present work we assume constant values for the quadrupole and octupole deformation variables β_2 and β_3 as input values in the shell model Hamiltonian (4). We take the experimentally estimated values of β_2 and β_3, or some testing deformation values in the cases of missing data. Then we obtain the respective shell model solutions which can be used for comparison and inclusion into the collective model problem.

Once the s.p. states are determined one can determine the Coriolis decoupling strength. In the present work we use the following expression for the decoupling parameter a, which is valid for a state with $K = 1/2$ (see for example [9])

$$a = \sum_{Nj} C_{Nj}^2 \left(j + \frac{1}{2} \right) (-1)^{j-\frac{1}{2}}. \tag{5}$$

It corresponds to a diagonalization of the deformed shell model Hamiltonian in the basis of the spherical harmonic oscillator (SHO) with a spin-orbit (SO) interaction, $|Nlj\Omega\rangle$. Here C_{Nj} are the expansion coefficients of the single-particle wave function in the SHO basis

$$\mathcal{F}_\Omega = \sum_{Nj} C_{Nj}^\Omega |Nlj\Omega\rangle, \tag{6}$$

where $l = j \pm 1/2$ is fixed by the parity conservation condition $(-1)^{N-l} = 1$. The expression (5) corresponds to $C_{Nj} = C_{Nj}^{\Omega=1/2}$ with $\Omega = K = 1/2$. To make it useful in the case of ADHO basis one needs to switch to the coefficients in the respective decomposition of the single-particle wave function

$$\mathcal{F}_\Omega = \sum_{Nn_z\Lambda} C_{Nn_z\Lambda}^\Omega |Nn_z\Lambda\Omega\rangle. \tag{7}$$

Note that the major quantum number N has different definitions in both SHO and ADHO basis functions. The SHO eigenenergies (without SO coupling) are $E_{\text{SHO}} = \hbar\omega(N + 3/2)$, while for ADHO one has $E_{\text{ADHO}} = \hbar\omega_z \left(n_z + \frac{1}{2}\right) + \hbar\omega_\perp (2n_\rho + |\Lambda| + 1)$ and $N_{\text{ADHO}} = n_z + 2n_\rho + |\Lambda|$ (see [9] for details). Therefore, any relation between the values of N in SHO and ADHO should be based on a particular physical assumption (see below).

By inserting the completeness condition for the SHO functions $\sum_{Nlj\Omega} |Nlj\Omega\rangle\langle Nlj\Omega| = 1$ in (7) one has

$$\mathcal{F}_\Omega = \sum_{N'lj} \sum_{Nn_z\Lambda} \langle N'lj\Omega|Nn_z\Lambda\Omega\rangle C_{Nn_z\Lambda}^\Omega |N'lj\Omega\rangle, \tag{8}$$

where $\langle N'lj\Omega|Nn_z\Lambda\Omega\rangle$ are the overlap integrals connecting the spherical and ADHO basis functions. Further, we assume that both SHO and ADHO basis states are mainly connected when the respective total numbers of oscillator quanta N' and N are equal. Then by comparing Eqs. (8) and (6) and taking $N = N'$ one finds the relation

$$C_{Nj}^\Omega = \sum_l \sum_{n_z\Lambda} \langle Nlj\Omega|Nn_z\Lambda\Omega\rangle C_{Nn_z\Lambda}^\Omega. \tag{9}$$

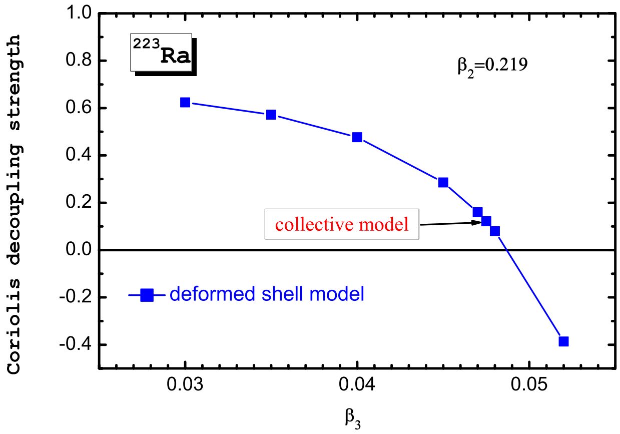

Figure 2. Coriolis decoupling factor a_{dsm} from deformed shell model calculations in ^{223}Ra as a function of several testing β_3-values for $\beta_2 = 0.219$ (also testing).

Since the coefficients $C^{\Omega}_{Nn_z\Lambda}$ are determined in the deformed shell model formalism presented above, the coefficients C^{Ω}_{Nj} can be obtained through Eq. (9) after calculating the overlap integrals $\langle Nlj\Omega|Nn_z\Lambda\Omega\rangle$. We remark that one has the same $C^{\Omega}_{Nn_z\Lambda} = C^{\Omega}_{Nn_z}$ values for the Λ-s corresponding to given N and n_z. Taking into account the spin-orbit coupling in the spherical basis one has

$$\langle Nlj\Omega|Nn_z\Lambda\Omega\rangle = C^{j\,\Omega}_{l\,\Lambda\,\frac{1}{2}\,\Omega-\Lambda}\langle Nl\Lambda|Nn_z\Lambda\rangle, \tag{10}$$

where $C^{j\,\Omega}_{l\,\Lambda\,\frac{1}{2}\,\Omega-\Lambda}$ is a Clebsch-Gordan coefficient.

The transformation brackets $\langle Nl\Lambda|Nn_z\Lambda\rangle$ are calculated by direct numerical integration. For this reason we transform the cylindric and spherical variables of the ADHO and the SHO wave functions, respectively, into Cartesian variables. Then we perform a three dimensional integration in the Cartesian space x, y, z. By calculating the overlap integrals (10) we obtain the SHO decomposition coefficients (9). The Coriolis decoupling factor corresponding to the considered deformed shell model (DSM) is subsequently calculated by inserting the result of (9) into Eq. (5). We denote it by a_{dsm}.

We applied the above algorithm in order to estimate numerically the quantity a_{dsm} in the nuclei ^{223}Ra, ^{237}U and ^{239}Pu. For each nucleus we performed the calculations for several alternative sets of experimental and testing values of deformation parameters β_2 and β_3. The results are given in Table 1. For ^{223}Ra we considered the β_2 deformation value of the even-even core ^{222}Ra, $\beta_2 = 0.192$, known from an experimental reference [10], while for β_3 we took several "testing" values (see Table 1). Also we considered a testing value $\beta_2 = 0.219$ with several additional testing β_3- values. For ^{237}U we took the experimental ^{236}U core value $\beta_2 = 0.282$ [10] with several experimental and testing β_3- values. In ^{239}Pu we considered several

Table 1. Coriolis decoupling factor $a_{\rm dsm}$ from deformed shell model calculations for ^{223}Ra ^{237}U and ^{239}Pu at different sets of experimental and testing deformation parameters. The data taken from the even-even core and the experimental references are respectively notified. The values $a_{\rm qoc}$ of the decoupling factor from the coherent quadrupole-octupole model are given in the last column for comparison. $\Omega_{\rm dsm}$ is the third angular momentum projection of the odd particle obtained by the deformed shell model.

Nucl.	β_2 Ref.	β_3 Ref.	$\Omega_{\rm dsm}$	$a_{\rm dsm}$	$a_{\rm qoc}$
^{223}Ra	0.192 (^{222}Ra) [10]	0.150	3/2	–	0.122
	0.192 (^{222}Ra) [10]	0.100	5/2	–	0.122
	0.192 (^{222}Ra) [10]	0.050	5/2	–	0.122
	0.192 (^{222}Ra) [10]	0.010	1/2	−6.165	0.122
	0.219	0.052	1/2	−0.386	0.122
	0.219	0.0480	1/2	0.080	0.122
	0.219	0.0475	1/2	0.122	0.122
	0.219	0.0470	1/2	0.160	0.122
	0.219	0.045	1/2	0.286	0.122
	0.219	0.040	1/2	0.477	0.122
	0.219	0.035	1/2	0.572	0.122
	0.219	0.030	1/2	0.624	0.122
^{237}U	0.282 (^{236}U) [10]	0.040	1/2	0.355	0.038
	0.282 (^{236}U) [10]	0.045 (^{236}U) [12]	1/2	0.317	0.038
	0.282 (^{236}U) [10]	0.062 (^{236}U) [12]	1/2	0.193	0.038
	0.282 (^{236}U) [10]	0.081 (^{236}U) [12]	1/2	0.090	0.038
	0.282 (^{236}U) [10]	0.095	1/2	0.038	0.038
	0.282 (^{236}U) [10]	0.100	1/2	0.023	0.038
^{239}Pu	0.204 [12]	0.091 (^{238}Pu) [11]	1/2	−0.717	−0.344
	0.213 [12]	0.091 (^{238}Pu) [11]	1/2	−0.536	−0.344
	0.227 [12]	0.091 (^{238}Pu) [11]	1/2	−0.619	−0.344
	0.240	0.091 (^{238}Pu) [11]	1/2	−0.539	−0.344
	0.250	0.091 (^{238}Pu) [11]	1/2	−0.447	−0.344
	0.260	0.091 (^{238}Pu) [11]	1/2	−0.329	−0.344
	0.270	0.091 (^{238}Pu) [11]	1/2	−0.135	−0.344
	0.278	0.091 (^{238}Pu) [11]	1/2	0.016	−0.344
	0.286 [10]	0.091 (^{238}Pu) [11]	1/2	0.149	−0.344
	0.293 [12]	0.091 (^{238}Pu) [11]	1/2	0.231	−0.344
	0.312 [12]	0.091 (^{238}Pu) [11]	5/2	–	−0.344
	0.370 [12]	0.091 (^{238}Pu) [11]	7/2	–	−0.344

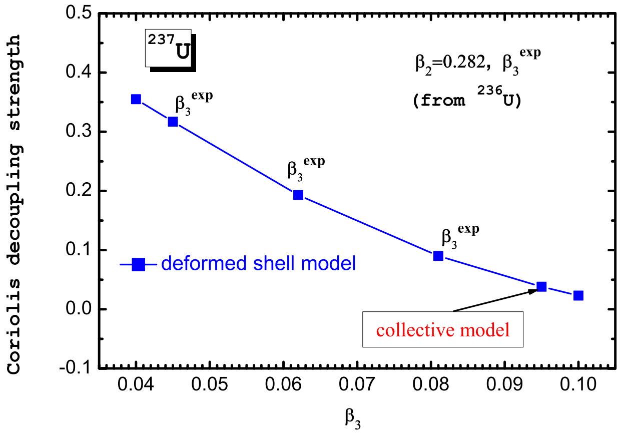

Figure 3. Coriolis decoupling factor $a_{\rm dsm}$ from deformed shell model calculations in ^{237}U as a function of several experimental and testing β_3-values for $\beta_2 = 0.282$ (taken from ^{236}U).

experimental and several testing β_2- values, while β_3 was taken as the experimental value for the even core ^{238}Pu, $\beta_3 = 0.091$. Results for the Coriolis decoupling factor $a_{\rm dsm}$ are given only for the cases when the angular momentum projection $\Omega_{\rm dsm}$ of the odd particle is obtained equal to $1/2$. The respective values of the decoupling parameters from the coherent quadrupole-octupole model of Section 2, denoted by $a_{\rm qoc}$, are also given in Table 1 for comparison. The boxed numbers correspond to the cases of a close consistence between the results of the deformed shell model and the coherent quadrupole–octupole model calculations. We see that in ^{223}Ra the use of the ^{222}Ra core quadrupole deformation $\beta_2 = 0.192$ [10] suggests $\Omega_{\rm dsm} = 5/2$ and $3/2$ at relatively large testing $\beta_3 = 0.05 - 0.15$. At small $\beta_3 \sim 0.1$ one obtains $\Omega_{\rm dsm} = 1/2$, however the respectively calculated $a_{\rm dsm}$ value ~ -6 is more than an order larger in magnitude than the one fitted in the collective model. On the other hand the use of the testing quadrupole deformation $\beta_2 = 0.219$ provides reasonable $a_{\rm dsm}$ values in the range $\beta_3 = 0.03 - 0.05$ of octupole deformations. We remark that the set of deformations $\beta_2 = 0.219$ and $\beta_3 = 0.0475$ reproduces exactly the collective model value $a_{\rm qoc} = a_{\rm dsm} = 0.122$. Also we remark the very fine dependence of $a_{\rm dsm}$ on the octupole deformation. This dependence is illustrated in Figure 2. In ^{237}U the experimental ^{236}U core value $\beta_2 = 0.282$ [10] provides reasonable values for the Coriolis decoupling strength in the region of larger octupole deformations $\beta_3 = 0.08 - 0.10$. The collective model value $a_{\rm qoc} = 0.038$ is reproduced at $\beta_3 = 0.095$. The behaviour of the Coriolis decoupling strength in ^{237}U as a function of β_3 is illustrated in Figure 3. In ^{239}Pu we examine the quantity $a_{\rm dsm}$ for several experimental [10, 12] and several testing values of β_2 at a fixed experimental $\beta_3 = 0.091$ value taken from the even core ^{238}Pu [11]. We see that the collective model value $a_{\rm qoc} = -0.3$ is reproduced at $\beta_2 = 0.260$. The behaviour of the Coriolis decoupling strength in ^{239}Pu as a function of β_2 is shown in Figure 4.

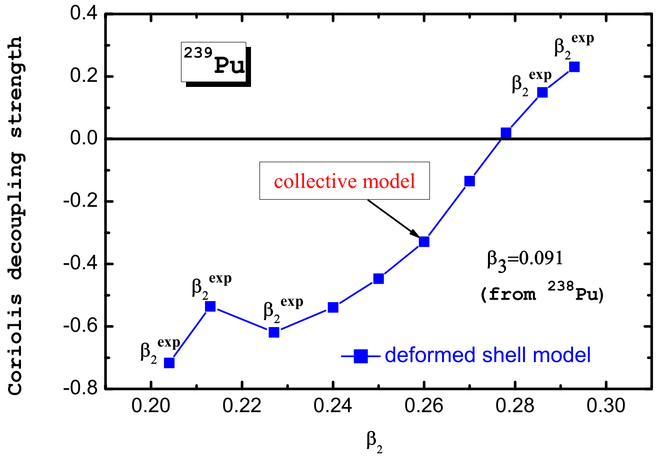

Figure 4. Coriolis decoupling factor $a_{\rm dsm}$ from deformed shell model calculations in ^{239}Pu as a function of several experimental and testing β_2-values for $\beta_3 = 0.091$ (taken from ^{238}Pu).

The implemented calculations show the capability of the proposed approach to identify physically reasonable regions of deformations for the nuclei under consideration. In this respect the present analysis suggests a possible answer of the question about the s.p. orbital on which the parity doublet in ^{223}Ra is built. Thus if one simply takes the quadrupole deformation of the core ^{222}Ra, the result is either $K = 3/2, 5/2$ ($3/2$ suggested in some experimental sources [7]) which contradicts with the observed staggering behaviour of the parity splitting (Figure 1), or $K = 1/2$ which, however provides a rather large magnitude of the Coriolis strength. On the other hand by making a reasonable variation in the quadrupole deformation one finds a deformation region with $K = 1/2$ where the Coriolis decoupling strength fits to the value obtained in the collective model.

The obtained results provide a test for the way in which both partly microscopic and collective models can be applied to a consistent study of the interaction between single-particle and collective motions in odd-mass nuclei. In addition, the supposition of $K = 1/2$ (or $\Omega = 1/2$) in ^{223}Ra demonstrates the advantage of the combined application of the microscopic and collective approach to precisely determine the "points" where the intrinsic and collective nuclear structures meet each other. Such an analysis could be implemented in other nuclei with similar behaviour.

4 Summary

In summary, starting by the analysis of Coriolis interaction effects in odd-mass nuclei within the framework of the collective model of coherent quadrupole-octupole motion, we extend the study by involving the formalism of the reflection asymmetric deformed shell model. We develop an algorithm based on the application of overlap integrals between axially deformed and spherical harmonic oscillator functions allowing us to use the known analytic expression for the Coriolis interaction in the spherical basis. The algorithm was applied to the nuclei ^{237}U and ^{239}Pu whose ground states are recognized [7] as possessing $\Omega = 1/2$, and ^{223}Ra for which the $\Omega = 1/2$ value can be supposed on the basis of the collective spectrum structure. The comparison of the obtained Coriolis strength values with the values obtained in the collective model allows one to identify physically reasonable regions of deformations for the nuclei under consideration. The obtained set of deformation parameters in ^{223}Ra differs from the respective data for the even-even core pointing out the influence of the odd particle on the collective properties of the odd-A nucleus. The results shown in Table 1 demonstrate the way in which both, microscopic and collective approaches can be applied to a consistent study of the interaction between single-particle and collective motions. This appears to be of a special importance in the cases when the intrinsic state on which the collective spectrum is built needs a precise determination.

Acknowledgements

The authors thank Dr. S. Drenska for useful discussions and Dr. D. Kadrev for providing a computer code for multidimensional integration. This work is supported by DFG and by the Bulgarian Scientific Fund under contract F-1502/05.

References

1. A. Bohr and B. R. Mottelson, *Nuclear Structure*, vol. II (New York: Benjamin, 1975).
2. P. A. Butler and W. Nazarewicz, Rev. Mod. Phys. **68**, 349 (1996).
3. G. A. Leander, Y. S. Chen, Phys. Rev. C **35**, 1145 (1987); Phys. Rev. C **37**, 2744 (1988).
4. V. Yu. Denisov and A. Ya. Dzyublik, Nucl. Phys. A **589**, 17 (1995).
5. N. Minkov, S. Drenska, P. Yotov, S. Lalkovski, D. Bonatsos and W. Scheid, Phys. Rev. C, **76**, 034324 (2007).
6. N. Minkov, P. Yotov, S. Drenska, W. Scheid, D. Bonatsos, D. Lenis and D. Petrellis, Phys. Rev. C **73**, 044315 (2006).
7. `http://www.nndc.bnl.gov/ensdf/`.
8. S. Cwiok, J. Dudek, W. Nazarevicz, J. Skalski, T. Werner, Comp. Phys. Comm. **46**, 379 (1987).
9. S. G. Nilsson and I. Ragnarsson, *Shapes and Shells in Nuclear Structure* (Cambridge: Cambridge University Press, 1995).
10. S. Raman, C. W. Nestor, Jr., and P. Tikkanen, At. Data Nucl. Data Tables **78**, 1–128 (2001).
11. T. Kibedi and R. H. Spear, At. Data Nucl. Data Tables **80**, 35–82 (2002).
12. RIPL-2: Reference Input Parameter Library, `http://www-nds.iaea.org/RIPL-2/`.