Low-Lying States in Odd-Mass Nuclei and the Extended Random Phase Approximation

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Abstract. The Quasi-particle Random Phase Approximation (QRPA) is known to be inadequate for describing the vibrational states in even-even nuclei far from the valley of stability. An extended version of this theory (ERPA) proved successful in removing some deficiencies of the QRPA.

Within the ERPA we derived renormalized interaction strengths between an even-even core and a particle outside of that core in odd-mass nuclei. The problem of investigating the properties of odd-mass nuclei within this approach is essentially reduced to solving a large system of coupled non-linear equations which takes into account the pairing correlations, the multipole-multipole interaction and the core-particle coupling.

1 Introduction

The Quasi-particle Random Phase Approximation presents a successful approach for describing the properties of the excited states of even-even nuclei near the magic numbers. However this theory, based on the quasi-boson approximation, is not applicable to nuclei far away from the valley of stability because the ground states of these nuclei cannot be approximated by the Bogolyubov quasi-particle vacuum state. The problem of extending the QRPA has a long history. We follow the approach proposed in [1] and later developed in [2, 3] and others. A rather complete list of references on this topic can be found in [5] for example. In contrast with the studies in these works where the authors were interested in describing the structure of the low-energy states in even-even nuclei, we focus on the odd-even nuclei.

This study presents a continuation of our ongoing research on the topic of the ground state correlations (GSC) in odd-mass nuclei. In [4] we developed an approach to treat the long-range ground state correlations by including backward-going quasiparticle-phonon vertices using the equation of motion method [10] with explicitly accounting for the Pauli principle.

In the present work, we develop a model, in which the states of the odd-even nuclei are obtained as a result of the interaction between an even-even core, described within the Extended Random Phase Approximation (ERPA) , and a particle outside of the core. The interaction strengths depend on the number of the quasi-particles in the ground state and therefore the core-particle equations cannot be solved independently but become a part of a larger non-linear system of equations including also generalized equations describing the pairing correlations and the excited vibrational states of the core. In the limit case, where the number of the quasi-particles in the ground state is set to zero, this system of equations decouples to reduce to the model obtained in [4] . Our investigation is realized as an extension of the quasi-particle phonon model (QPM) [6–8] framework for low-energy states. The proposed model does not add any new free parameters.

2 Formalism

We use the Hamiltonian

$$H = \sum_{\tau}^{(n,p)} \left\{ \sum_{jm} (E_j - \lambda_\tau) a^\dagger_{jm} a_{jm} - \frac{1}{4} G_\tau^{(0)} : (P_0^\dagger P_0)^\tau : - \right.$$
$$\left. - \frac{1}{2} \sum_{\lambda\mu} \kappa^{(\lambda)} : (M^\dagger_{\lambda\mu} M_{\lambda\mu}) : \right\}. \quad (1)$$

with terms accounting for the average nuclear field, described by the Woods-Saxon potential, the pairing interactions and the isoscalar particle-hole residual forces with the Bohr-Mottelson radial dependence [11] respectively. The single-particle states are specified by the quantum numbers (jm); E_j are the single-particle energies; λ_τ is the chemical potential; $G_\tau^{(0)}$ and $\kappa^{(\lambda)}$ are the strengths in the p-p and in the p-h channel, respectively. The sum goes over protons(p) and neutrons(n) independently and the notation $\tau = \{n, p\}$ is used. The pair creation and the multipole operators entering the normal products in (1) are defined as follows:

$$P_0^+ = \sum_{jm} (-1)^{j-m} a^+_{jm} a^+_{j-m}, \quad (2)$$

$$M^+_{\lambda\mu} = \frac{1}{\sqrt{2\lambda+1}} \sum_{jj'mm'} f^{(\lambda)}_{jj'} \langle jmj'm' \mid \lambda\mu \rangle a^+_{jm} a_{j'm'}, \quad (3)$$

where $f^{(\lambda)}_{jj'}$ are the single particle radial matrix elements of the residual forces.

In what follows we work in quasi-particle representation, defined by the canonical Bogolyubov transformation:

$$a^+_{jm} = u_j \alpha^+_{jm} + (-1)^{j-m} v_j \alpha_{j-m}. \quad (4)$$

We introduce the bifermion quasi-particle operators (and their conjugate ones):

$$B(jj'; \lambda\mu) = \sum_{mm'} (-1)^{j'+m'} \langle jmj'm' \mid \lambda\mu \rangle \alpha^+_{jm} \alpha_{j'-m'}, \quad (5)$$

$$A^+(jj'; \lambda\mu) = \sum_{mm'} \langle jmj'm' \mid \lambda\mu \rangle \alpha^+_{jm} \alpha^+_{j'm'}. \quad (6)$$

The phonon creation operators are defined in the two-quasiparticle space in a standard fashion:

$$Q^{+}_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'}\, A^{+}(jj';\lambda\mu) - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}\, A(jj';\lambda-\mu)\}, \tag{7}$$

where the index $\lambda = 0, 1, 2, 3, ...$ denotes multipolarity and μ is its z-projection in the laboratory system.The normalization of the one-phonon states reads:

$$\langle|[Q_{\lambda\mu i}, Q^{+}_{\lambda'\mu' i'}]|\rangle = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\sum_{jj'}(1-\rho_{jj'})\frac{1}{2}(\psi^{\lambda i}_{jj'}\psi^{\lambda i'}_{jj'} - \phi^{\lambda i}_{jj'}\phi^{\lambda i'}_{jj'}) = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'}. \tag{8}$$

where $|\rangle$ is the ground state of the even-even system, $\rho_{jj'} = \rho_j + \rho_{j'}$ and ρ_j is the normalized quasi-particle distribution in the ground state

$$\rho_j = \frac{1}{\pi_j}\langle|B(jj;00)|\rangle. \tag{9}$$

The Hamiltonian (1) can be expressed using the quasi-particle and phonon operators (see Appendix I). Using the equation of motion method with respect to the operators $Q_{\lambda\mu i}$ and the preconditions (8) and (9) the following system of non-linear equations for the phonon energies, amplitudes and distributions in the ground state is obtained [3]:

$$\frac{G}{4}\sum_{j}\frac{\pi_j^2}{\sqrt{(E_j-\lambda)^2+\varDelta^2}}(1-2\rho_j) = 1 \tag{10}$$

$$\frac{1}{2}\sum_{j}\pi_j^2\left\{1 - \frac{E_j-\lambda}{\sqrt{(E_j-\lambda)^2+\varDelta^2}}(1-2\rho_j)\right\} = n \tag{11}$$

$$\frac{\kappa_\lambda}{\pi_\lambda^2}\sum_{jj'}(1-\rho_{jj'})\frac{(f^{\lambda}_{jj'}u^{+}_{jj'})^2(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2_{\lambda i}} = 1 \tag{12}$$

$$\sum_{jj'}(1-\rho_{jj'})[(\psi^{\lambda i}_{jj'})^2 - (\varphi^{\lambda i}_{jj'})^2] = 2 \tag{13}$$

$$\rho_j = \frac{1}{2}\sum_{\lambda i j'}\frac{\pi_\lambda^2}{\pi_j^2}(1-\rho_{jj'})(\varphi^{\lambda i}_{jj'})^2 \tag{14}$$

We extend this system with equations describing the states in odd-even nuclei, the wave functions of which are obtained as in [9]:

$$\varPsi_\nu(JM) = O^{+}_{JM\nu}|\rangle, \tag{15}$$

where

$$O^{+}_{JM\nu} = C_{J\nu}\alpha^{+}_{JM} + \sum_{i} D^{\lambda i}_{j}(J\nu)P^{+}_{j\lambda i}(JM) - E_{J\nu}\tilde{\alpha}_{JM} - \sum_{i} F^{\lambda i}_{j}(J\nu)\tilde{P}_{j\lambda i}(JM), \tag{16}$$

with

$$P^{+}_{j\lambda i}(JM) = \left[\alpha^{+}_{jm} Q^{+}_{\lambda\mu i}\right]_{JM} \tag{17}$$

and $\tilde{}$ stands for time conjugate according to the convention: $\tilde{P}_{j\lambda i}(JM) = (-1)^{J-M} P_{j\lambda i}(J-M)$.

The normalization condition for the ground state of the odd-even system now reads:

$$\langle|\{O_{JM\nu}, O^{+}_{JM\nu}\}|\rangle = C^2_{j\nu} + E^2_{J\nu} + 2(C_{J\nu}\sum_{j\lambda i} F_{j\lambda i} + E_{J\nu}\sum_{j\lambda i} D_{j\lambda i})\frac{\pi_\lambda}{\pi_J}\rho_j \varphi^{\lambda i}_{Jj} + \\ + \sum_{j\lambda i}(D^2_{j\lambda i} + F^2_{j\lambda i})(1 - \mathcal{L}^{*}_{J}(Jj\lambda i)) = 1. \tag{18}$$

Using the equation of motion method with respect to the operators $O^{+}_{JM\nu}$ and using (18) we obtain the following system of linear equations for each state with quantum numbers JM:

$$\begin{pmatrix} \langle\{[\alpha_{JM}, H], \alpha^{+}_{JM}\}\rangle & V(Jj'\lambda' i') & 0 & -W(Jj'\lambda' i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda' i') & -\langle\{[\alpha_{JM}, H], \alpha^{+}_{JM}\}\rangle & -V(Jj'\lambda' i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda i') \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda' i'} \\ -E_{J\nu} \\ -F_{j'\lambda' i'} \end{pmatrix} =$$

$$= \eta_{J\nu} \begin{pmatrix} 1 & 0 & 0 & -\frac{\pi_\lambda}{\pi_J}\rho_j\varphi^{\lambda i}_{Jj} \\ 0 & (1-\mathcal{L}^{*}(Jj\lambda i)) & -\frac{\pi_\lambda}{\pi_J}\rho_j\varphi^{\lambda i}_{Jj} & 0 \\ 0 & -\frac{\pi_\lambda}{\pi_J}\rho_j\varphi^{\lambda i}_{Jj} & 1 & 0 \\ -\frac{\pi_\lambda}{\pi_J}\rho_j\varphi^{\lambda i}_{Jj} & 0 & 0 & (1-\mathcal{L}^{*}(Jj\lambda i)) \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda' i'} \\ -E_{J\nu} \\ -F_{j'\lambda' i'} \end{pmatrix} \tag{19}$$

The leading terms of the quantities entering the system above are given Appendix II. They depend on the properties of the even-even core and the particle coupled with the core.

There are two features that distinguish the presented equations for the odd-even system from the ones obtained in [4]:

- The presence of the quasi-particle distribution in the ground state ρ_j in all quantities entering the system (19). Therefore the latter system cannot be solved after the system (10)-(14) is solved but both (19) and (10)-(14) form a unified coupled system of equations. Setting ρ_j to 0 we get the case presented in [4].
- The emergence of new terms. Such terms appear in all quantities entering (19). Because of the lengthy expressions we wrote only the terms that we consider not negligible. One complication present the terms on the inverse diagonal in the matrix in the rhs of (19) as they make impossible this system to be reduced to a generalized eigenvalue problem.

3 Numerical Results and Discussion

In this section we present first results of the calculations for the nuclides ^{133}Ba, ^{135}Ba and ^{137}Ba performed within our model. We include quadrupole phonons only. The pairing constants G_τ are fitted in the standard way so as to reproduce the odd-even mass differences in the neighbouring nuclei. The strength parameter $\kappa^{(2)}$ is adjusted so that the low-energy spectrums of the odd-even systems are reasonably close to the experimental values. Doing so we obtain the correct level ordering of the first several levels. As a result the energy of the first 2^+ state in ^{132}Ba, ^{134}Ba and ^{136}Ba lies higher than the experimental value. In that respect the ERPA performs better than RPA as shown in Figure 1.

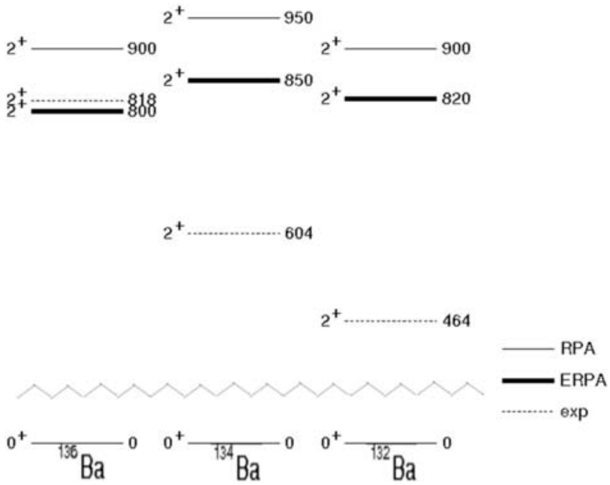

Figure 1.

The reason for that is the decreased values of the collectivity of these states as compared to the model where the GSC beyond RPA are not taken into account. In Table 1 we give typical values of the the protons' and neutrons' quasi-particle distribution in the ground state in ^{132}Ba within ERPA.

Table 1.

^{132}Ba			
$n : 1h_{11/2}$	0.040	$p : 2d_{5/2}$	0.057
$n : 2d_{3/2}$	0.037	$p : 1g_{7/2}$	0.042
$n : 3s_{1/2}$	0.029	$p : 3s_{1/2}$	0.023
$n : 2f_{7/2}$	0.007	$p : 2d_{3/2}$	0.017
$n : 1g_{7/2}$	0.005	$p : 1g_{9/2}$	0.006
$n : 2d_{5/2}$	0.004	$p : 1h_{11/2}$	0.006

The calculations within the presented model give also improved values of the spectroscopic factors for the (d,p)-reaction in the nuclei under consideration.

Appendix I

In terms of quasi-particles and phonons the Hamiltonian is rewritten

$$H = h_0 + h_{pp} + h_{QQ} + h_{QB}, \tag{20}$$

$$h_0 + h_{pp} = \sum_{jm} \varepsilon_j \, \alpha^+_{jm} \, \alpha_{jm}, \tag{21}$$

$$h_{QQ} = -\frac{1}{8} \sum_{\lambda\mu i i'} \mathcal{A}\,(\lambda i i')\,(Q^+_{\lambda\mu i} + (-)^{\lambda-\mu} Q_{\lambda-\mu i})(Q^+_{\lambda-\mu i'} + (-)^{\lambda+\mu} Q_{\lambda\mu i'}), \tag{22}$$

$$h_{QB} = -\frac{1}{2\sqrt{2}} \sum_{\lambda\mu i j j'} \frac{\pi_j}{\pi_\lambda} \Gamma(jj'\lambda i)((-)^{\lambda-\mu} Q^+_{\lambda\mu i} + Q_{\lambda-\mu i}) B(jj^{'};\lambda-\mu) + h.c., \tag{23}$$

where

$$\mathcal{A}\,(\lambda i i') = \frac{X^{\lambda i} + X^{\lambda i^{'}}}{\sqrt{Y^{\lambda i} Y^{\lambda i'}}}, \tag{24}$$

$$\Gamma(jj'\lambda i) = \frac{\pi_\lambda}{\pi_j} \frac{v^{(-)}_{jj'} \, f^{(\lambda)}_{jj'}}{\sqrt{Y^{\lambda i}}}, \tag{25}$$

$$X^{\lambda i} = \sum_{jj'} \frac{(f^{(\lambda)}_{jj'} u^{(+)}_{jj'})^2 \, \varepsilon_{jj'}}{\varepsilon^2_{jj'} - \omega^2_{\lambda i}}, \tag{26}$$

$$Y^{\lambda i} = \sum_{jj'} \frac{(f^{(\lambda)}_{jj'} u^{(+)}_{jj'})^2 \, \varepsilon_{jj'} \, \omega_{\lambda i}}{\left(\varepsilon^2_{jj'} - \omega^2_{\lambda i}\right)^2}, \tag{27}$$

with $v^{(-)}_{jj'} = u_j u_{j'} - v_j v_{j'}$ and $u^{(+)}_{jj'} = u_{j'} v_j + u_j v_{j'}$. The notation $\pi_j = \sqrt{(2j+1)}$ is used.

Appendix II

We give the explicit expressions for the matrix elements $\langle\{\alpha_{JM}, H, \alpha^+_{JM}\}\rangle$, V, W, K and the quantities $\mathcal{L}$.

$$\langle\{\alpha_{JM}, H, \alpha^+_{JM}\}\rangle = \varepsilon_J - \frac{1}{4} \sum_{\lambda_1 i_1 i'_1} \frac{\pi^2_{\lambda_1}}{\pi^2_J} \mathcal{A}(\lambda_1 i_1 i'_1) \sum_{j'} \varphi^{\lambda_1 i_1}_{j'J} \varphi^{\lambda_1 i'_1}_{j'J} (1 - \rho_{j'}) \tag{28}$$

$$\mathcal{L}_J(j\lambda i|j'\lambda' i') = \pi_\lambda \pi_{\lambda'} \sum_{j_1} \psi_{1j}^{\lambda' i'} \psi_{1j'}^{\lambda i} \begin{Bmatrix} j' & j_1 & \lambda \\ j & J & \lambda' \end{Bmatrix} \tag{29}$$

$$\mathcal{L}(Jj\lambda i) = \pi_\lambda^2 \sum_{j'} \left(\psi_{j'j}^{\lambda i}\right)^2 \begin{Bmatrix} j & j' & \lambda \\ j & J & \lambda \end{Bmatrix}, \tag{30}$$

$$\mathcal{L}^*_{J|j'}(j\lambda i|j'\lambda' i') = \pi_\lambda \pi_{\lambda'} \sum_{j_1} (1-\rho_{j_1 j'}) \psi_{1j}^{\lambda' i'} \psi_{1j'}^{\lambda i} \begin{Bmatrix} j' & j_1 & \lambda \\ j & J & \lambda' \end{Bmatrix} \tag{31}$$

$$\mathcal{L}^*(Jj\lambda i) = \pi_\lambda^2 \sum_{j'} (1-\rho_{jj'}) \left(\psi_{j'j}^{\lambda i}\right)^2 \begin{Bmatrix} j & j' & \lambda \\ j & J & \lambda \end{Bmatrix}, \tag{32}$$

$$\begin{aligned} V\left(Jj\lambda i\right) &= \left\langle \left| \left\{ \left[\alpha_{JM}, H\right], P^+_{j\lambda i}\left(JM\right) \right\} \right| \right\rangle = \\ &= -\frac{1}{\sqrt{2}}(1-\rho_j)\mathit{\Gamma}(Jj\lambda i) - \frac{1}{\sqrt{2}} \sum_{j_1\lambda_1 i_1} \mathit{\Gamma}(Jj_1\lambda_1 i_1)\mathcal{L}^*_{J|j_1}(j\lambda i|j_1\lambda_1 i_1) \end{aligned} \tag{33}$$

$$\begin{aligned} W\left(Jj\lambda i\right) &= \left\langle \left| \left\{ \left[\alpha^+_{JM}, H\right], \tilde{P}^+_{j\lambda i}\left(JM\right) \right\} \right| \right\rangle = \\ &= \frac{\pi_\lambda}{\pi_J}\varepsilon_J \rho_j \varphi_{Jj}^{\lambda i} - \frac{1}{4}(1-\rho_j)\frac{\pi_\lambda}{\pi_J} \sum_{i_1} \mathcal{A}(\lambda i i_1)\varphi_{Jj}^{\lambda i_1} \\ &\quad + \frac{1}{4} \sum_{\lambda_1 i_1 i_1'} \frac{\pi_{\lambda_1}}{\pi_J} \mathcal{A}(\lambda_1 i_1 i_1') \sum_{j_1} \varphi_{Jj_1}^{\lambda_1 i_1} \mathcal{L}^*_{J|j_1}(j\lambda i|j_1\lambda_1 i_1') \end{aligned} \tag{34}$$

$$\begin{aligned} I_J(j\lambda i|j'\lambda' i') &= \langle \{P_{j\lambda i}(JM), [H, P^+_{j'\lambda' i'}(JM)]\}\rangle = \\ &= -\frac{1}{4}\delta_{jj'}\delta_{\lambda\lambda'}\delta_{\mu\mu'}(1-\rho_j)\mathcal{A}(\lambda i i') + \varepsilon_j \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'} \\ &\quad + \frac{1}{4}\sum_{i_1} \mathcal{A}(\lambda_1 i_1 i')\mathcal{L}_J(j\lambda i|j'\lambda' i_1) - \frac{1}{4}\sum_{i_2} \mathcal{A}(\lambda i i_2)\mathcal{L}_J(j'\lambda' i'|j\lambda i_2) \end{aligned} \tag{35}$$

$$K_J(j\lambda i|j'\lambda i') = \frac{1}{2}\left[I_J\left(j\lambda i|j'\lambda' i'\right) + I_J\left(j'\lambda' i'|j\lambda i\right)\right] \tag{36}$$

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