Nuclear and Nucleon Contributions to the Parity-Violating Electron Scattering

O. Moreno[1], P. Sarriguren[1], E. Moya de Guerra[1,2], J. M. Udias[2], T. W. Donnelly[3], and I. Sick[4]

[1] Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain.
[2] Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, E-28040 Madrid, Spain.
[3] Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.
[4] Departement für Physik, Universität Basel, CH-4056 Basel, Switzerland.

Abstract. The parity-violating (PV) elastic electron scattering is studied here for the even-even, $N = Z$ nuclei ^{12}C, ^{24}Mg, ^{28}Si and ^{32}S, whose ground states have been obtained from a self-consistent axially-symmetric mean field with effective Skyrme forces and including pairing interactions within BCS approximation. Various contributions to the PV asymmetry have been analyzed, in particular nuclear isospin mixing, nucleon strange content and Coulomb distortion of the incoming electron wave function. In addition, the influence of the ingredients of the formalism (pairing gaps, quadrupole deformation) has been studied. Kinematic ranges where the PV asymmetry is most suitable for measurement are proposed.

1 Theoretical Formalism

1.1 Parity-Violation Asymmetry

The electromagnetic and the weak interactions contribute to the charged-lepton elastic scattering off nuclei. It is a semileptonic process where both the lepton and the quark inside the nucleon carry electromagnetic and weak charge. The weak current taking part in the process is the neutral one (WNC) mediated by the exchange of the neutral vector boson Z^0. Although at small momentum transfers the contribution of the weak interaction is much smaller than the electromagnetic one, just its presence leads to the violation of parity in the process, as is well known since the seminal experiments by Wu and collaborators [1]. The degree of parity violation can be measured using the parity-violating asymmetry,

$$\mathcal{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}\,, \tag{1}$$

which is proportional to the difference between the cross-section of longitudinally-polarized electrons parallel and antiparallel to their momentum.

Within plane wave Born approximation (PWBA), the parity-violation asymmetry of a lepton scattering can be written as:

$$\mathcal{A} = \frac{\mathcal{G}}{2\pi\alpha\sqrt{2}}|Q^2|\frac{W^{PV}}{W^{PC}}\,, \tag{2}$$

where $\mathcal{G}$ and α are the Fermi and fine-structure coupling constants, respectively, W^{PV}, W^{PC} are the parity-violating and parity-conserving responses, and Q is the four-momentum transfer in the scattering process. The asymmetry is thus decomposed in a factor depending on Standard Model constants and a factor related to the nuclear structure (the ratio of PV to PC responses), apart from the Q^2 dependence.

If we restrict ourselves to $J^\pi = 0^+$ nuclear targets, the responses ratio becomes:

$$\frac{W^{PV}}{W^{PC}} = \frac{a_A F_{C\,0}(q)\tilde{F}_{C\,0}(q)}{F^2_{C\,0}(q)} = \frac{a_A \tilde{F}_{C\,0}(q)}{F_{C\,0}(q)}\,, \tag{3}$$

where the tilde form factors $\tilde{F}_{C\,0}$ correspond to the WNC, in contrast to the usual, EM form factors $F_{C\,0}$. Both can be decomposed into proton and neutron contributions and expressed in terms of Coulomb operators acting on the nuclear ground state:

$$\begin{aligned} F_{C\,0}(q) &= \langle 0||M^C_{0\,(\pi)}(q)||0\rangle + \langle 0||M^C_{0\,(\nu)}(q)||0\rangle \\ \tilde{F}_{C\,0}(q) &= \langle 0||\tilde{M}^C_{0\,(\pi)}(q)||0\rangle + \langle 0||\tilde{M}^C_{0\,(\nu)}(q)||0\rangle \end{aligned} \tag{4}$$

The tilde operators $\tilde{M}^C_0$ differ from the usual (EM) Coulomb operators in the structure of the nucleon form factors that they include, as will be shown later on.

When considering an $N = Z$ nuclear target with pure isospin $T = 0$ in its ground state, only the isoscalar part of the form factors is nonzero, the EM and WNC form factors becoming thus proportional and giving rise to the following parity-violating asymmetry:

$$\mathcal{A} = \mathcal{A}^0 \equiv \left[\frac{G\,|Q^2|}{2\pi\alpha\sqrt{2}}\right] a_A \beta^{(0)}_V \cong 3.22 \times 10^{-6}|Q^2| \tag{5}$$

with the squared momentum transfer measured in fm^{-1}. Starting from this reference value one can establish deviations proportional to the reference value itself,

$$\mathcal{A} = \mathcal{A}^0(1 + \Gamma) \tag{6}$$

The relative error of the asymmetry can be written [2] as

$$\frac{\delta\mathcal{A}}{\mathcal{A}} = \frac{1}{\sqrt{\mathcal{F}\,X_0}} \tag{7}$$

where the relevant experimental setting features have been gathered in X_0 (including beam luminosity, running time and solid angle of detection) and the remaining quantities (depending on lepton and nuclear properties as well as on the specific scattering kinematic conditions) are contained in the so called figure-of-merit (FOM),

$$\mathcal{F} = \frac{d\sigma}{d\Omega}\,\mathcal{A}^2 \tag{8}$$

The larger this quantity, the smaller the relative error in the measurement of the asymmetry. Since the asymmetry increases with $|Q^2|$ (Eq. (2)) and the cross-section decreases with it, a compromise must be found to get the largest possible values of the figure-of-merit.

1.2 Coulomb Multipole Operators and Nucleon Form Factors

The Coulomb multipole operators appearing in Eq. (4) can be written in terms of the contributions of order 0 and 1 in the incoming electron momentum, p/m_N [3–5]:

$$M_J^{C\,M_J}(q\mathbf{x}) = \frac{\kappa}{\sqrt{\tau}}\left[G_E(\tau)\mathcal{M}_J^{M_J}(q\mathbf{x}) + \tau\left[2G_M(\tau) - G_E(\tau)\right]\Theta_J^{M_J}(q\mathbf{x})\right], \tag{9}$$

where the kinematic factors κ, τ are proportional to q^2 and Q^2 respectively. The first term in the right-hand side is the standard (zeroth-order) Coulomb form factor,

$$\mathcal{M}_J^{M_J}(q\mathbf{x}) = j_J(q\mathrm{x})Y_J^{M_J}(\hat{\mathbf{x}})\,, \tag{10}$$

whereas the second term (first-order in p/m_N) is the spin-orbit correction term,

$$\Theta_J^{M_J}(q\mathbf{x}) = -\frac{i}{q^2}\vec{\sigma}\cdot\left[\left(\vec{\nabla}\mathcal{M}_J^{M_J}(q\mathbf{x})\times\vec{\nabla}\right)\right] \tag{11}$$

This structure of the Coulomb multipole operator is valid for both protons and neutrons in the EM and WNC (with tilde) versions appearing in Eq. (4); the only change is in the functional form of the factors G_E, G_M, which are the electric (E) and magnetic (M) nucleon form factors. Their momentum transfer dependence follows in our work the Höhler parametrization [6]. The WNC electric form factors for protons (π) and neutrons (ν) are built from the EM ones as

$$\begin{aligned}\tilde{G}_{E_\pi} &= \beta_V^\pi G_E^\pi + \beta_V^\nu G_E^\nu + \frac{1}{2}\beta_V^{(s)}G_E^{(s)}\\ \tilde{G}_{E_\nu} &= \beta_V^\nu G_E^\pi + \beta_V^\pi G_E^\nu + \frac{1}{2}\beta_V^{(s)}G_E^{(s)}\end{aligned} \tag{12}$$

where the relative weights of the proton, neutron and strangeness form factors take the Standard Model values $\beta_V^\pi = 0.04$, $\beta_V^\nu = -0.5$ and $\beta_V^{(s)} = -1$ respectively. Equivalent relations hold for the WNC magnetic form factors by replacing the subscripts E by M.

For the strangeness form factor a dipole-type expression has been used with an upper limit value of the electric form factor, whereas the magnetic form factor has been neglected.

1.3 Nuclear Target Ground State Structure

The structure of the nuclear ground state $|0\rangle$ used as target in the scattering process has been obtained in our work from an axially-deformed Hartree-Fock (HF) mean field using SLy4 Skyrme nucleon-nucleon effective interactions. Pairing correlations have been added using BCS equations after each iteration of the HF procedure. The resulting single-particle wave functions, which are naturally expressed in terms of an axially-deformed harmonic oscillator basis, can also be written in terms of a spherical harmonic oscillator (s.h.o.) basis:

$$\Phi^i_{HF}(\vec{r}) = \sum_{nlj} c^i_{nlj}\, \phi_{nljm_j}(\vec{r}) \tag{13}$$

The Coulomb monopole operator matrix elements in the nuclear ground state can also be decomposed in this s.h.o. basis as

$$\langle 0||M^C_{0\,(\pi,\nu)}(q)||0\rangle = \sum_{nn'lj} f^{\pi,\nu}_{nn'lj}(q)\, \rho^{\pi,\nu}_{nn'lj}\,, \tag{14}$$

where

$$\begin{aligned} f^{\pi,\nu}_{nn'lj}(q) = {} & \frac{\kappa}{\sqrt{\tau}} G_{E_{\pi,\nu}}(\tau) \langle \phi_{n'lj}|\mathcal{M}^0_0(q\mathbf{x})|\phi_{nlj}\rangle + \\ & + \kappa\sqrt{\tau}\left[2G_{M_{\pi,\nu}}(\tau) - G_{E_{\pi,\nu}}(\tau)\right] \langle \phi_{n'lj}|\Theta^0_0(q\mathbf{x})|\phi_{nlj}\rangle \end{aligned} \tag{15}$$

is the matrix element of the Coulomb monopole operator between two s.h.o. states, following the definition given in Eq. (9). It is clearly momentum-dependent, vanishes unless both basis functions have the same l, j quantum numbers, and the j-dependence is due only to the spin-orbit term. The other factor is

$$\rho_{nn'lj} = \sum_i 2v_i^2\, c^i_{nlj} c^i_{n'lj}\ \ , \tag{16}$$

containing the expansion coefficients in Eq. (13) and all the single-particle states occupation probabilities v_i^2 resulting from the pairing interaction. The nuclear structure is contained in this quantity, which is the spherical part of the density matrix. Distinction between EM and WNC matrix elements affects only the f factor (not ρ) through the nucleon form factors which can be of G type or of $\tilde{G}$ type (defined in Eq. (12)).

1.4 Contributions to the PV Asymmetry

The PV asymmetry deviation can be separated into an isospin-mixing term and a strangeness term, $\Gamma = \Gamma^{(I)} + \Gamma^{(s)}$. The isospin mixing term is computed considering $G_E^{(s)} = 0$ and $G_M^{(s)} = 0$ in the WNC form factors, whereas the strangeness term is computed using $G_{E_\pi} = 0$, $G_{E_\nu} = 0$, $G_{M_\pi} = 0$ and $G_{M_\nu} = 0$ in the WNC

form factors of Eqs. (12). The strangeness term cannot be completely isolated unless further approximations are performed on its expression, namely inserting the same neutron and proton densities (ie., ignoring isospin mixing), neglecting the small value of G_{E_ν}, and not taking into account the small spin-orbit contribution (so that $G_E^{(s)}$ enters into the expression but not $G_M^{(s)}$). This simplified strangeness term takes thus the form:

$$\Gamma^{(s)} = \frac{\beta_V^{(s)}}{\beta_V^{(0)}} \frac{G_E^{(s)}}{G_E^{(0)}} \,. \tag{17}$$

and it is independent of the nuclear structure. As for the isospin-mixing part, neglecting the spin-orbit terms in the nucleon form factors and the small electric neutron form factor G_{E_ν}, one arrives at:

$$\Gamma^{(I)} = \frac{\beta_V^{\nu}}{\beta_V^{(0)}} \left(\frac{\langle 0||M_{0\,(\nu)}(q)||0\rangle - \langle 0||M_{0\,(\pi)}(q)||0\rangle}{\langle 0||M_{0\,(\pi)}(q)||0\rangle} \right) \tag{18}$$

which is a more simple and intuitive expression accounting for the isospin mixing contribution to the asymmetry, consisting basically on the difference between the proton and neutron ground-state radial densities ρ_π, ρ_ν.

1.5 Coulomb Distortion Effects

Performing a calculation that fully takes into account Coulomb distortion is important here since, although the effect is expected to be small for the relatively light target nuclei under consideration, the other effects to be analyzed are also very small. The standard treatment of Coulomb distortion is applied for elastic PV electron scattering within a partial wave formalism. The Dirac equation is solved for massless electrons in the Coulomb potential generated by the nucleus [7–9] to obtain the distorted wave (DW) results.

The main effect of including the distortion of the electron wave function is the smoothing of the PWBA divergences that result from the EM and WNC form factors reaching their diffraction minima at slightly different transfer momenta when isospin mixing is included. Except for in these divergent regions of the PWBA results, the asymmetry deviations are very similar for PW and DW calculations.

Even when no isospin-mixing contributions are taken into account, the distorting potential introduces by itself asymmetry deviations. To prevent this influence from hiding other contributions to the deviation (isospin-mixing, strangeness), we already include in the reference value of the asymmetry the effects of Coulomb distortion. In other words, instead of using $\mathcal{A}^0$ of Eq. (5) as the reference value to which deviations are referred, in the DW calculations we use $\mathcal{A}^0_{DW}$ which includes distortion and which can be easily computed for each target nucleus in a model independent way using experimental charge distributions. PW calculations, however, still use $\mathcal{A}^0$ as reference value. Comparisons between PW and DW calculations will be shown later on.

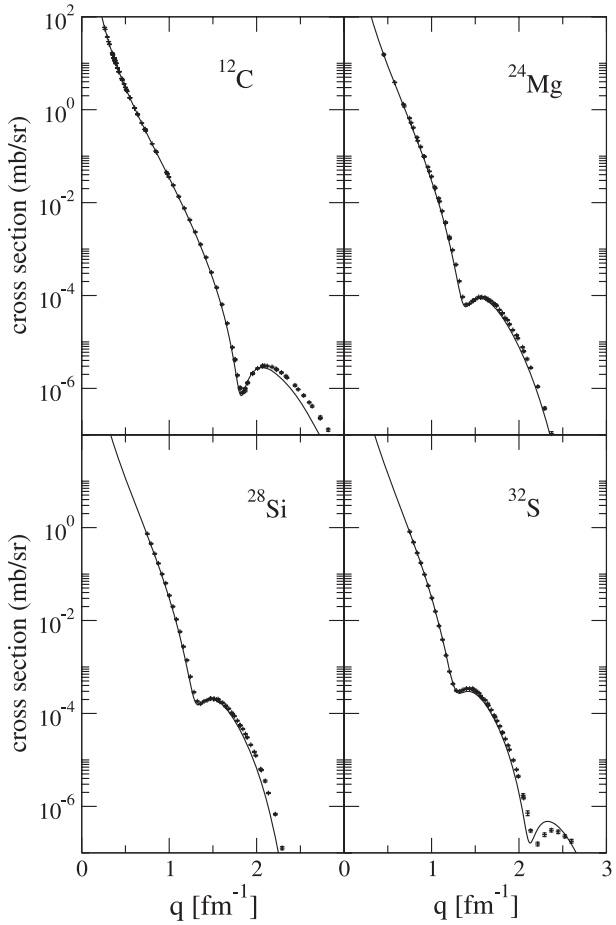

Figure 1. Theoretical (solid line) and experimental data on electron-nucleus cross section. Nuclear densities come from a HF+BCS calculation and distortion for 1 GeV incoming electron wave functions have been considered.

2 Results

PV asymmetry deviations from various contributions have been analyzed in four even-even, $N = Z$ isotopes, namely ^{12}C, ^{24}Mg, ^{28}Si and ^{32}S, which are the most abundant isotopes of the corresponding element (79% abundance for ^{24}Mg, and higher than 90% for the other isotopes). Furthermore, all targets have a large excitation energy of the first (2^+) excited state, and all but sulfur are suitable in elemental form for high-current electron scattering experiments, as required for a measurement of the PV asymmetry.

The electron-nucleus cross-section obtained with our distorted wave formalism (for 400 MeV electrons) is shown in Figure 1 for the four isotopes under study,

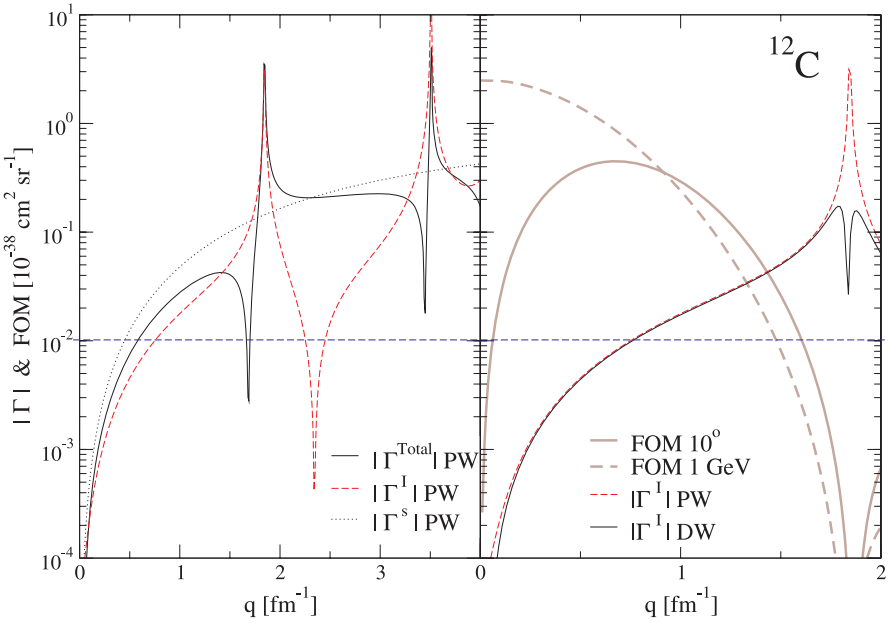

Figure 2. Left panel: Contributions to the PV asymmetry deviation for plane-wave electron scattering off ^{12}C target. Right panel: Plane wave (PW) and distorted wave (DW) calculations of the isospin contribution to the asymmetry deviation, together with FOM for fixed scattering angle and fixed electron energy.

together with experimental data with error bars ([10]- [13] and references therein). The agreement is apparent up to a momentum transfer of around 2 fm^{-1}. Above this momentum, the effect of the residual interactions not taken into account in the nuclear target may become important.

PV asymmetry deviations and FOM are shown for ^{12}C, the lightest of our isotopes, in Figure 2. In the left panel the isospin and the strangeness contributions to the asymmetry deviation appear in absolute value together with their combination (with different sign) to get the total contribution. The nuclear ground state has been obtained from a HF(SLy4)+BCS($\Delta_{\pi,\nu}$= 1 MeV) calculation and the electron scattering is treated within PWBA. In the right panel, the isospin contribution to the asymmetry deviation is shown again, but this time in comparison with the DW treatment for 1 GeV incoming electrons. Both results are very similar given the small charge of the target, except for in the divergent regions of the PW calculation, which disappear in the DWBA calculation. Two FOM are also shown in this panel, one of them for a fixed scattering angle of 10^o and the other one for a fixed energy of 1 GeV for the incoming electron. Finally, in both panels the dashed horizontal line at 10^{-2} allows a fast identification of the momentum transfer regions where the asymmetry deviates more than 1% from the Standard Model value. It happens in ^{12}C for momentum transfers above 0.74 fm^{-1}, which, up to around 1.4 fm^{-1}, can

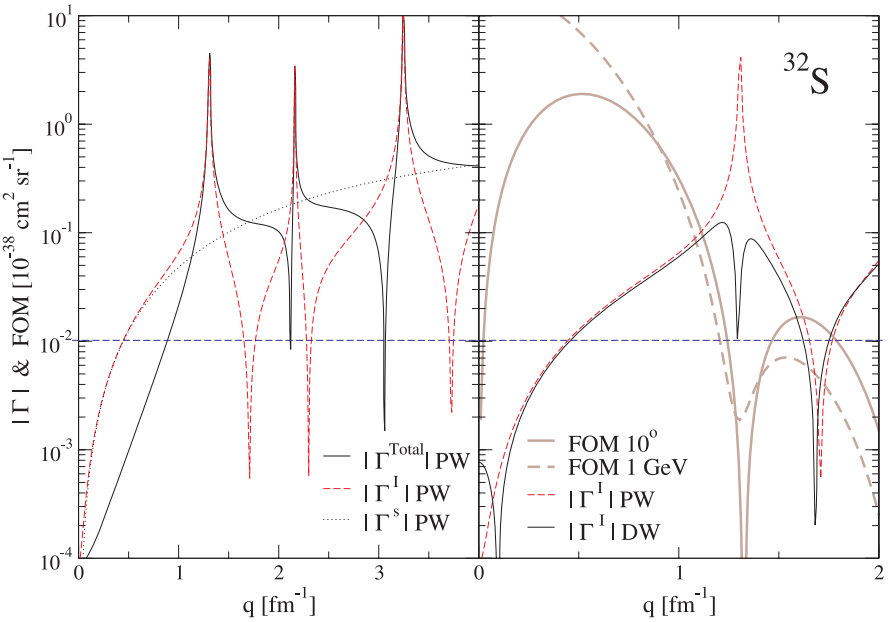

Figure 3. Same as in Figure 2, but for ^{32}S nuclear target.

be considered to lie in the region of the maximum value of the FOM. This region is our kinematical choice for the measurement of the PV electron scattering off ^{12}C.

The same results but for our heaviest nucleus, ^{32}S, appear in Figure 3. In this case the absolute value of the isospin and strangeness contributions are very similar, and therefore the addition of them, which have different signs, gives a much lower value of the total deviation in the low-momentum region, as shown in the left panel. The effect of a distorted treatment, shown in the right panel, is larger than in the ^{12}C case but again not crucial, except for in the regions where the PW calculation diverges. The most suitable momentum transfer region to measure the isospin contribution to the deviation lies between 0.45 and 1.05 fm^{-1}, as deduced by taking into account the value of the deviation itself (above the 10^{-2} line) and the value of the FOM.

For the four nuclei under study here, the optimal kinematic choice for the measurement of PV asymmetry deviations lies, in general, between 0.5 and 1 fm^{-1}. The isospin contribution in this region is shown in Figure 4, in a non-logarithmic scale this time, for a distorted wave calculation for 1 GeV electrons. As the atomic number increases, so does the isospin-mixing contribution. The strangeness contribution, not appearing in this figure, is in fact independent of the nuclear target, at least in the approximation used in all these plots, Eq. (17).

More specifically, Table 1 contains the suggested kinematic ranges to measure the total and the isospin contribution to the PV asymmetry deviation in the nuclei under study. The deviation in these ranges is more than 1% of the reference value

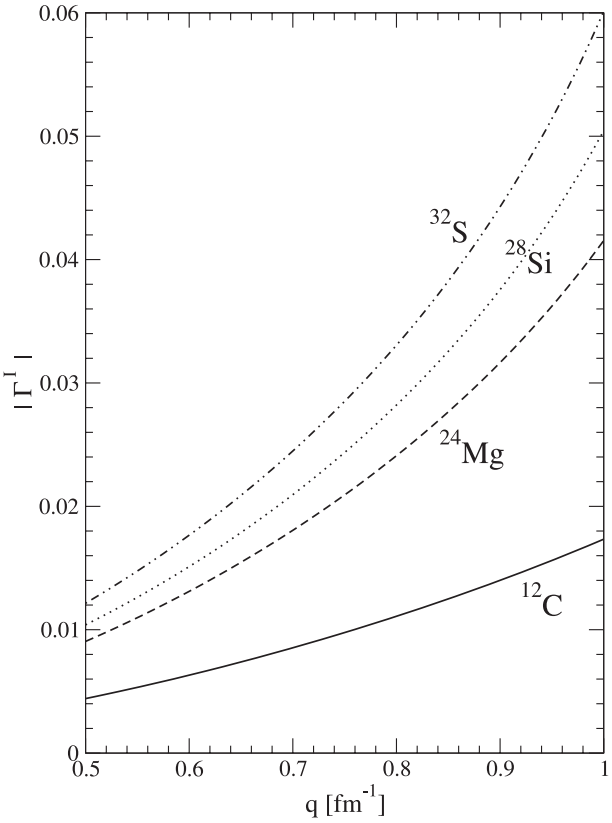

Figure 4. Isospin mixing contribution to the PV asymmetry deviation Γ^{I} for the four nuclei under study from a HF(SLy4)+BCS($\Delta_{\pi,\nu}$= 1 MeV) calculation in DW for 1 GeV electrons, in the momentum transfer region of most experimental interest.

and, at the same time, the fixed-angle FOM value is at most one order of magnitude lower than the maximum value reached at the peak. It is worth noting that we are predicting deviations of 1% in already small values of the total asymmetry, which in these regions are between 10^{-5} and 10^{-6}.

Table 1. Momentum transfer intervals (in fm^{-1}) most suitable to measure the isospin contribution and the total PV asymmetry deviation.

Isotope	for Γ^{I}	for Γ^{Total}
^{12}C	$0.74 \leq \mathrm{q} \leq 1.42$	$0.57 \leq \mathrm{q} \leq 1.42$
^{24}Mg	$0.51 \leq \mathrm{q} \leq 1.10$	–
^{28}Si	$0.48 \leq \mathrm{q} \leq 1.08$	$1.00 \leq \mathrm{q} \leq 1.08$
^{32}S	$0.45 \leq \mathrm{q} \leq 1.05$	$0.88 \leq \mathrm{q} \leq 1.05$

The influence of some of the ingredients in the formalism has been also analyzed. For example, different values of pairing energy gaps have been used to obtain the HF+BCS ground states of the target nuclei. It has been found that the results are very similar in a wide range of momentum transfers when similar pairing gaps are used for protons and neutrons, although for very different gap values the isospin mixing contribution to the asymmetry deviation increases. The influence of the quadrupole deformation of the target nucleus can be also analyzed in our axially deformed mean field formalism by forcing different nuclear shapes, resulting in no significant changes found between them.

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