

Degeneracies in a Non-Integrable Pairing Model

J. Okołowicz¹, M. Płoszajczak², and J. Dukelsky³

¹ Institute of Nuclear Physics, Radzikowskiego 152, PL-31342 Kraków, Poland

² Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DSM – CNRS/IN2P3, BP 5027, F-14076 Caen Cedex 05, France

³ Instituto de Estructura de la Materia, CSIC, Serrano 123, 28006 Madrid, Spain

Abstract. The evolution pattern of exceptional points is studied in a non-integrable limit of the complex-extended 3-level Richardson-Gaudin model. The appearance of a pseudo-diabolic point from the fusion of two exceptional points is demonstrated in the anti-hermitian limit of the model and studied in some details.

1 Introduction

Small quantum systems, whose properties are profoundly affected by environment, *i.e.*, continuum of scattering and decay channels, are intensely studied in various fields of physics (nuclear physics, atomic and molecular physics, nanoscience, quantum optics, *etc.*). These different open quantum systems (OQS), in spite of their specific features, have generic properties, which are common to all weakly bound/unbound systems close to the threshold. An essential part of the motion of short-lived nucleonic matter is in classically forbidden regions, and their properties are impacted by both the continuum and many-body correlations [1, 2].

Resonances are commonly found in various quantum systems, independently of their building blocks. Resonances are genuine intrinsic properties of quantum systems describing preferential decays of unbound states. The effect of resonances and the non-resonant scattering states can be considered in the OQS extension of the shell model (SM), the so-called continuum shell model (CSM) [2]. Two realizations of the CSM have been studied recently: the real-energy CSM [2–4] and the complex-energy CSM [6, 7] based on the Berggren ensemble [5], the so-called Gamow Shell Model (GSM).

For hermitian Hamilton operators, both real-energy CSM [2–4] and complex-energy CSM (GSM) [6, 7] lead naturally to the non-hermitian (complex-symmetric) eigenvalue problem. As a result, OQs exhibit several unintuitive properties, which make them qualitatively different from closed quantum systems (CQS). Among those salient features are: the segregation of time scales in the continuum [11, 12] (see also Ref. [2] for a recent review), the alignment of near-threshold states with decay channels [1, 2, 13], the instability of SM eigenstates at the channel threshold [14, 15], or the resonance crossings [2, 16]. In this lecture, we shall concentrate on the latter phenomenon which will be illustrated on the example of a non-integrable pairing model.

Most studies of degeneracies associated with avoided crossings in quantal spectra, focused on the topological structure of the Hilbert space and the geometric phases [17, 18]. Among these degeneracies, one finds a diabolic point (DP) in hermitian Hamiltonians [17, 19], and an exceptional point (EP) [20–22], which appears in the complex g -plane of a generic Hamiltonian $H(g) = H_0 + gH_1$, where both H_0 and H_1 are hermitian and $[H_0, H_1] \neq 0$. Below, we shall introduce a prototypical OQS, the 3-level Richardson-Gaudin (RG) model, to discuss the appearance of resonance crossings and their evolution with a parameter of a non-integrable perturbation. In particular, we shall show the appearance of a new kind of degeneracy, a pseudo-diabolic point (pseudo-DP) in the *anti-hermitian* limit of this model.

2 The 3-Level Pairing Model

RG models [23] are based on the $SU(2)$ algebra with elements K_l^+ , K_l^- , and K_l^0 , fulfilling the commutation relations: $[K_l^+, K_{l'}^-] = \delta_{ll'} K_l^0$, $[K_l^0, K_{l'}^\pm] = \pm \delta_{ll'} K_l^\pm$, where indices l, l' refer to a particular copy from a set of L , $SU(2)$ algebras. Each $SU(2)$ algebra possesses one quantum degree of freedom. In the following, we shall use the pair representation of $SU(2)$ algebra leading to pairing Hamiltonians. The elementary operators in this representation are the number operators N_j and the pair operators A_j^\dagger, A_j , defined as:

$$N_j = \sum_m a_{jm}^\dagger a_{jm}; \quad A_j^\dagger = \sum_m a_{jm}^\dagger a_{j\bar{m}}^\dagger = (A_j^\dagger)^\dagger \quad (1)$$

where j is the total angular momentum and m is the z -projection. The state $j\bar{m}$ is the time reversal of jm . The relation between the operators of the pair algebra and the generators of the $SU(2)$ algebra is:

$$K_l^0 = \frac{1}{2}N_l - \frac{1}{4}\Omega_l; \quad K_l^+ = (K_l^-)^\dagger = \frac{1}{2}A_l^\dagger \quad (2)$$

where Ω_l is the particle degeneracy of level l . With this correspondence, one can introduce an integrable 3-level pairing Hamiltonian as:

$$H_I(g) = \sum_i \varepsilon_i N_i + g \sum_{ij} A_i^\dagger A_j \quad (3)$$

Below, we shall consider the non-integrable version of the 3-level pairing model [24]:

$$H_{NI}(g) = H_I(g) + g' \sum_i N_i^2, \quad g' = \gamma g \quad (4)$$

where $\gamma < 0$ is a C -number.

3 Level Degeneracies

The position of all possible degeneracies in the complex g -plane are indicated by the roots of the coupled equations:

$$\det [H(g) - EI] = 0; \quad \frac{\partial}{\partial E} \det [H(g) - EI] = 0 \quad (5)$$

By eliminating E from these two equations, we are left with the discriminant $D(g)$, a polynomial in g of degree $M = n(n - 1)$, where n is the number of eigenstates. The discriminant can be written as [21]:

$$D(g) = \prod_{m < m'} [E_m(g) - E_{m'}(g)]^2 \quad (6)$$

where $E_m(g)$, $E_{m'}(g)$ denote the complex eigenvalues of $H(g)$. The eigenvalue degeneracies $E_m(g) = E_{m'}(g)$ at $g = g_\alpha$ ($\alpha = 1, \dots, M$), can be found numerically by looking for sharp minima of the functional $D(g)$. For the non-integrable Hamiltonian, the degenerate eigenvalues are either the single-root (EP) or double-root solutions such as DP or pseudo-DP.

4 Properties of a Pseudo-Diabolic Point in the Anti-Hermitian Limit of a Non-Integrable 3-Level Pairing Model

Let us solve the non-integrable 3-level pairing model for the case of 2 pairs of fermions in a valence space of degeneracy $\Omega_1 = 2, \Omega_2 = 6, \Omega_3 = 2$. The energies of three levels are $\varepsilon_1 = 0, \varepsilon_2 = 1$, and $\varepsilon_3 = 2$. In this model space, there are 4 many-body states.

Figure 1 shows the global pattern of level crossings in the complex g -plane for $g'/g = -1/2$. In the lower half-plane of g , all eigenvalues are either discrete states at the real g -axis or decaying resonances. Complex conjugate degeneracies situated in the upper half-plane ($\text{Im}(g) > 0$) correspond to capturing resonances.

Figure 2 shows a typical pattern associated with the formation/breakup of the pseudo-DP at $g = -i/(4\sqrt{2})$. With an increasing value of g'/g , the two EPs in the complex g -plane approach each other, coalesce at $g'/g = -1/2$ and, subsequently, move along the axis $\text{Re}(g) = 0$ for $g'/g > -1/2$. The coalescence of two EPs at $g'/g = -1/2$, leads to the formation of a double-root singularity, for which the geometric phase associated with a cyclic evolution changes twice faster than for the EP. On the other hand and in contrast to the ordinary DP, both eigenvalues *and* eigenvectors merge, like for an ordinary EP.

The upper part of Figure 3 shows real parts of phase trajectories around the pseudo-DP for all four eigenvectors in the model space. At the pseudo-DP, real part of their eigenenergies are identical. These trajectories in g -plane are described by: $g = -(1/(4\sqrt{2})i) + 0.01e^{i\varphi}$, where φ defines the way of encircling the degeneracy.

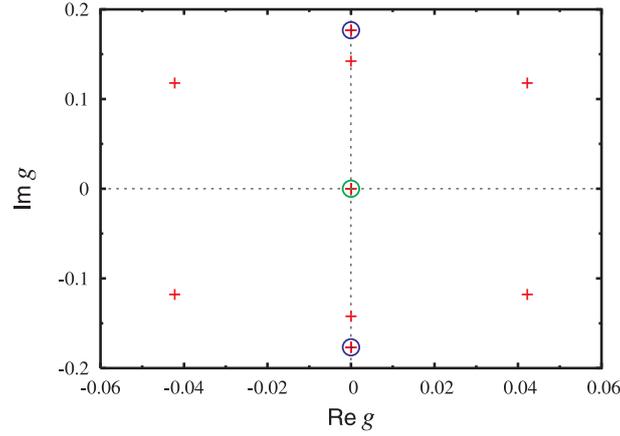


Figure 1. Localization of EPs (the red cross) and pseudo-DPs (the red cross inside a blue circle) in the complex g -plane for the non-integrable 3-level pairing Hamiltonian with $g'/g = -1/2$ (see Eq. (4)). For chosen energies of levels, one finds a trivial degeneracy at $g = 0$ (the non-interacting limit). For more details, see the description in the text.

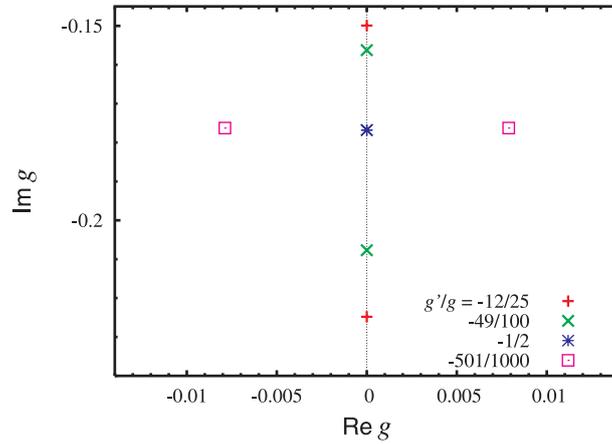


Figure 2. The formation and decay of a pseudo-DP at $g' = -g/2$ (the blue star) in the anti-hermitian limit of the non-integrable 3-level pairing Hamiltonian (4).

The phase of each vector is defined with respect of its value at $\varphi = 0$. The eigenvalues for vectors ‘2’ and ‘3’ are degenerate at the pseudo-DP and change their phases by π at each encircling. Simultaneously, phases of vectors for non-degenerate eigenvalues ‘1’ and ‘4’ remain approximately constant. For a comparison, the lower part of Figure 3 exhibits real parts of phase trajectories of all four vectors around the EP $g = -0.207687i$ at $g'/g = -49/100$ (see Figure 2). These trajectories are specified by: $g = -0.207687i + 0.01e^{i\varphi}$. Eigenvalues for vectors ‘2’ and ‘3’ are degenerate at the EP. After first encircling of the EP they become exchanged. After a second encir-

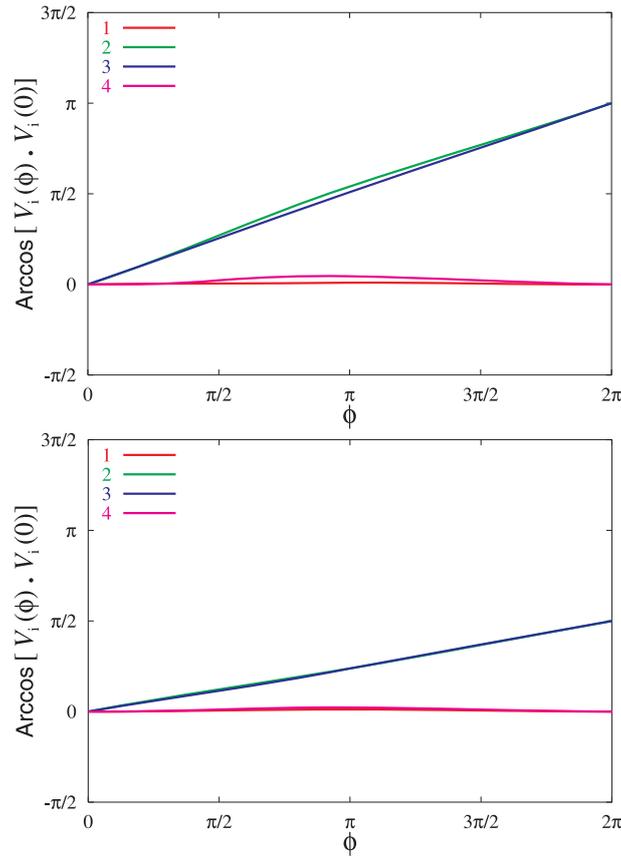


Figure 3. The real parts of phase trajectories for four eigenvectors involved in the pseudo-DP at $g'/g = -1/2$ (the upper part). This degeneracy of two eigenvalues results from a coalescence of two EPs. In the lower part, the real parts of phase trajectories for two eigenvectors which form an EP at $g'/g = -49/100$ (see Figure 2) are shown. The phase of each vector is defined with respect of its reference value at $\varphi = 0$.

cling, the phases of vectors ‘2’ and ‘3’ change by π , and after two more encircling of the EP all phases return to their initial values.

The behavior of eigenenergies in the neighborhood of a pseudo-DP in complex g -plane are shown along the cut $(\mathcal{R}e(g), -1/(4\sqrt{2}))$ (see Figure 4). The fusion of two EPs and the formation of a single pseudo-DP is seen as a sharp crossing of four energies and a coalescence of two widths (the eigenvectors ‘2’ and ‘3’). Even though phases of eigenvectors ‘1’ and ‘4’ are almost constant when encircling the pseudo-DP (see Figure 3), they are essential for the formation of a pseudo-DP degeneracy. It can be shown rigorously, that the pseudo-DP does not result from the Jordan block of 2-level system.

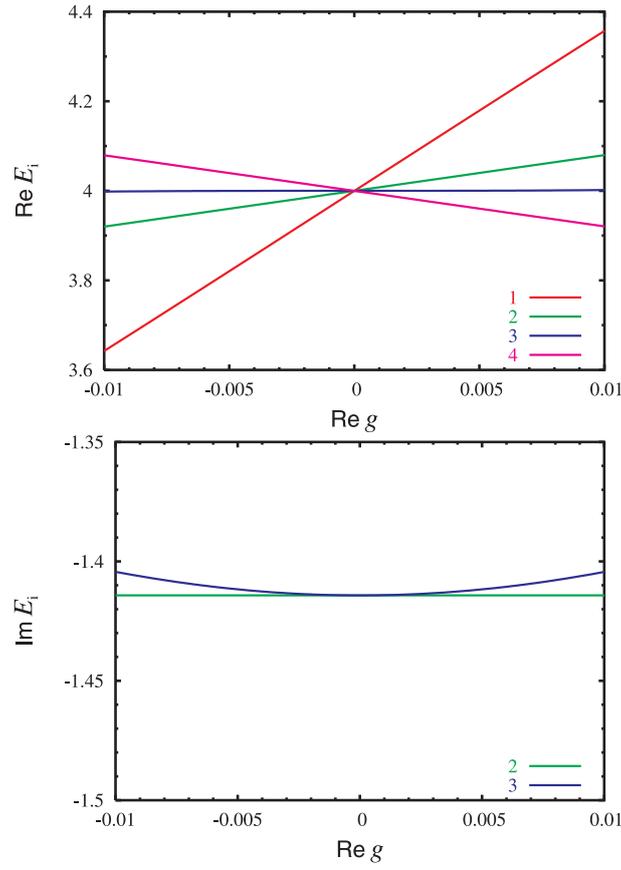


Figure 4. Real and imaginary parts of eigenvalues at around the pseudo-DP $g = -i/(4\sqrt{2})$ are plotted along the cut $(\mathcal{R}e(g), -1/(4\sqrt{2}))$ in g -plane. For more details, see the description in the text.

5 Mixing of Wave Functions at the Pseudo-Diabolic Point: the Pairing Correlation Energy

Salient features of eigenvectors around a pseudo-DP of a 3-level pairing Hamiltonian (4) with $g'/g = -1/2$ can be studied analytically. Let $g = -i(4\sqrt{2})^{-1} + \delta$, where δ is a complex number. The eigenenergies of a Hamiltonian matrix:

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix} = \begin{pmatrix} 4 - 3.79878i + 35.9338\delta + \mathcal{O}(\delta)^2 \\ 4 - \sqrt{2}i + 8\delta + \mathcal{O}(\delta)^2 \\ 4 - \sqrt{2}i + \mathcal{O}(\delta)^2 \\ 4 + 0.263243i - 7.93378\delta + \mathcal{O}(\delta)^2 \end{pmatrix} \quad (7)$$

depend parametrically on the parameter δ . Notice an asymmetric dependence on δ of eigenvalues '2' and '3' which form a pseudo-DP.

The eigenvectors, which are normalized according to the dual metric of RHS [5, 8–10]:

$$\langle \hat{\mathbf{u}}_i | \tilde{\hat{\mathbf{u}}}_j \rangle = \delta_{ij} \quad (8)$$

can be written as:

$$\begin{aligned} \hat{\mathbf{u}}_1 &= \begin{bmatrix} 0.616894 + 0.517406i + (7.05076 - 1.17993i)\delta - (27.8062 + 130.27i)\delta^2 + \mathcal{O}(\delta)^3 \\ 0.731723 - 5.83203i\delta - 127.824\delta^2 + \mathcal{O}(\delta)^3 \\ 0.488757 - 3.21838i\delta - 67.7276\delta^2 + \mathcal{O}(\delta)^3 \\ 0.616894 - 0.517406i - (7.05076 + 1.17993i)\delta - (27.8062 - 130.27i)\delta^2 + \mathcal{O}(\delta)^3 \end{bmatrix}, \\ \hat{\mathbf{u}}_2 &= \begin{bmatrix} -\frac{(1-i)}{4\sqrt{2}\sqrt{2}}\frac{1}{\sqrt{\delta}} - 3\frac{1+i}{4\sqrt{2}}\sqrt{\delta} + \mathcal{O}(\delta)^{\frac{3}{2}} \\ \frac{1+i}{4\sqrt{2}}\frac{1}{\sqrt{\delta}} + \frac{1-i}{2\sqrt{2}\sqrt{2}}\sqrt{\delta} + \mathcal{O}(\delta)^{\frac{3}{2}} \\ \mathcal{O}(\delta)^{\frac{3}{2}} \\ \frac{1-i}{4\sqrt{2}\sqrt{2}}\frac{1}{\sqrt{\delta}} + 3\frac{1+i}{4\sqrt{2}}\sqrt{\delta} + \mathcal{O}(\delta)^{\frac{3}{2}} \end{bmatrix}, \\ \hat{\mathbf{u}}_3 &= \begin{bmatrix} -\frac{1+i}{4\sqrt{2}\sqrt{2}}\frac{1}{\sqrt{\delta}} - \frac{9\sqrt{2}-8-(9\sqrt{2}+8)i}{8\sqrt{2}\sqrt{2}}\sqrt{\delta} + \mathcal{O}(\delta)^{\frac{3}{2}} \\ -\frac{1-i}{42^{1/4}}\frac{1}{\sqrt{\delta}} + \frac{1+i}{4\sqrt{2}\sqrt{2}}\sqrt{\delta} + \mathcal{O}(\delta)^{\frac{3}{2}} \\ \sqrt{3}\sqrt{2}(1+i)\sqrt{\delta} + 11\sqrt{\frac{3\sqrt{2}}{2}}(1-i)\delta^{\frac{3}{2}} + \mathcal{O}(\delta)^{\frac{5}{2}} \\ \frac{1+i}{4\sqrt{2}\sqrt{2}}\frac{1}{\sqrt{\delta}} + \frac{9\sqrt{2}+8-(9\sqrt{2}-8)i}{8\sqrt{2}\sqrt{2}}\sqrt{\delta} + \mathcal{O}(\delta)^{\frac{3}{2}} \end{bmatrix}, \\ \hat{\mathbf{u}}_4 &= \begin{bmatrix} 0.345603 - 0.412057i - (2.62262 - 1.08314i)\delta + (14.2829 + 10.4398i)\delta^2 + \mathcal{O}(\delta)^3 \\ -0.582736 + 0.412499i\delta - 6.21696\delta^2 + \mathcal{O}(\delta)^3 \\ -0.87242 + 3.06004i\delta + 26.4056\delta^2 + \mathcal{O}(\delta)^3 \\ 0.345603 + 0.412057i + (2.62262 + 1.08314i)\delta + (14.2829 - 10.4398i)\delta^2 + \mathcal{O}(\delta)^3 \end{bmatrix} \end{aligned}$$

In the limit $\delta \rightarrow 0$, the eigenvectors $\hat{\mathbf{u}}_2$ and $\hat{\mathbf{u}}_3$ with (identical) eigenvalues E_2, E_3 (see Eq. (7)) have divergent components. This feature of eigenvectors at a pseudo-DP makes it similar to an ordinary EP and leads to a singular behavior of various quantities.

Let us consider for example the pairing operator: $g \sum_{ij} A_i^\dagger A_j$, in the neighborhood of a pseudo-DP for $g'/g = -1/2$:

$$\begin{pmatrix} a_2 i + b_2 \delta + \mathcal{O}(\delta)^2 & a_5/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} & a_6/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} & a_4 i + b_4 \delta + \mathcal{O}(\delta)^2 \\ a_5/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} & a_1/\delta + \mathcal{O}(\delta)^0 & a_1 i/\delta + \mathcal{O}(\delta)^0 & a_7/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} \\ a_6/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} & a_1 i/\delta + \mathcal{O}(\delta)^0 & -a_1/\delta + \mathcal{O}(\delta)^0 & a_8/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} \\ a_4 i + b_4 \delta + \mathcal{O}(\delta)^2 & a_7/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} & a_8/\sqrt{\delta} + \mathcal{O}(\delta)^{1/2} & a_3 i + b_3 \delta + \mathcal{O}(\delta)^2 \end{pmatrix}$$

Coefficients a_i in this expression are:

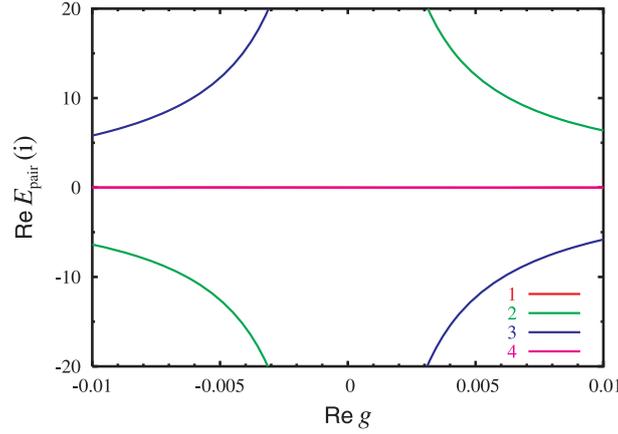


Figure 5. The pairing energy at around the pseudo-DP of the non-integrable 3-level pairing Hamiltonian ($g' = -g/2$).

$$\begin{aligned}
 a_1 &= \frac{1}{16}, \quad a_2 = -7.43796, \quad b_2 = -3.64018, \quad a_3 = 0.455281, \quad b_3 = 0.765176, \\
 a_4 &= 0.603023, \quad b_4 = 1.8219, \quad a_5 = 0.475579(1-i), \quad b_5 = 0.449184(1+i), \\
 a_6 &= 0.475579(1+i), \quad b_6 = 4.59509(1-i), \quad a_7 = 0.118873(1-i), \\
 b_7 &= 1.20969(1+i), \quad a_8 = 0.118873(1+i), \quad b_8 = 0.0511439(1-i)
 \end{aligned}$$

It is readily seen that all matrix elements of the pairing operator involving states ‘2’ and ‘3’ exhibit the square-root divergence if $\delta \rightarrow 0$, *i.e.* when approaching a pseudo-DP. This divergence is cancelled out if one adds diagonal pairing matrix elements for these two states. However, off-diagonal matrix elements between ‘regular’ states (states ‘1’ and ‘4’ in this case) and any combination of states ‘2’ and ‘3’ are divergent because: $a_5 = a_6^*$ and $a_7 = a_8^*$.

Dependence of a real part of the pairing energy on the interaction strength g in the non-integrable pairing Hamiltonian (4) with $g'/g = -1/2$ is plotted along the cut at $(\mathcal{R}e(g), -1/(4\sqrt{2}))$. Pairing energies in strongly mixed eigenstates ‘2’ and ‘3’ have opposite signs and diverge when a pseudo-DP is approached, *i.e.* if $\mathcal{R}e(g) \rightarrow 0$. On the other hand, the sum of pairing energies in states ‘2’ and ‘3’ is both positive and finite and changes smoothly close to a pseudo-DP. These features of pairing energy in the neighborhood of a pseudo-DP are essentially different from those discussed in connection with effects of an ordinary DP on pair-transfer amplitude [25] as the coupling between eigenvectors involved vanishes at the DP.

6 Conclusions

Besides accidental degeneracies, sharp level crossings in the hermitian eigenvalue problem are allowed only between states of different quantum numbers (different symmetries). For the non-hermitian (complex-symmetric) eigenvalue problem,

such as found in OQSs, exact eigenvalue degeneracies appear also for states of the same quantum numbers/symmetries. These are: (i) the DP, when Riemann sheets on which live eigenvalues just touch each other [17], and (ii) the EP, when two Riemann sheets are entangled by the square-root type of singularity [20–22]. We have analyzed here features of degeneracies in the non-hermitian and non-integrable 3-level pairing model (see Eq. (4) which is a schematic representation of the OQS Hamiltonian. In particular, we have shown the appearance of a new kind of degeneracy, the pseudo-DP, in the anti-hermitian limit of this model as a result of the coalescence between two EPs. The resonance eigenfunctions at the pseudo-DP (EP) are entangled and, except for the (complex) eigenenergy, it is impossible to define any physical quantity separately in each of the two resonances involved in the pseudo-DP degeneracy.

This entanglement of wave functions may involve many states of an OQS, leading to strong mixing of wave functions for states of largely different eigenenergies. This mechanism is efficient not only in the close neighborhood of a particular pseudo-DP (EP), *i.e.* for a particular choice of coupling constants in the OQS Hamiltonian, but also for systems with largely different coupling constants. Future studies, using the CSM and realistic effective interactions could provide a first attempt to provide a consistent description of continuum wave functions and related observables in the presence of a pseudo-DP (EP).

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