The Effect of Tensor Term on the Single-Particle Spectrum

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Abstract. An existing experimental work [2, 3] has predicted a reduction in the proton $\pi 1d_{3/2} - 1d_{5/2}$ spin-orbit splitting for the $Z = 14$ isotopes, when one goes towards the neutron rich nuclei. In a shell model analysis this reduction was attributed to a tensor force. In this work, the chain has been analyzed within the SHF+BCS framework, using the SLy5 parameterization with tensor interaction explicitly added. We have shown that indeed the tensor term is the one responsible for the observed shrinkage.

Since there was a general discussion if the $2p_{3/2} - 1f_{7/2}$ gap persists in ^{28}Si we have examined the $N = 28$ isotonic chain. For this chain, we have performed similar analysis. We have observed a substantial reduction in the splitting due to proton-neutron symmetry effects. Still a gap is obtained with a small but finite size in the ^{28}Si. We have shown that this reduction, previously ascribed to a tensor force is not strongly affected by a tensor term in our considerations. We claim indeed that the tensor has virtually no effect on the reduction of the $2p_{3/2} - 1f_{7/2}$ gap.

1 Introduction

The single-particle nuclear levels undergo modifications when the neutron number N or proton number Z changes. With the progress in experimental studies it has become clear that the single-particle levels towards the drip lines are very difficult to be described within the frameworks of the existing models. From the experimental side, the studies of nuclei far from the valley of β-stability are a difficult task, because the nuclei involved are generally short-lived and their production rate can be low. However, there are several regions where new experimental data have been obtained. In the analysis of these results it was proven that the shell evolution cannot be adequately described by the existing microscopic models. The problem appears more clearly when one goes further towards the drip-lines. The necessity of a tensor term to describe the experimental results becomes prominent [1].

The recent experiment at GANIL (Grand Accélérateur National d'Ions Lourds) done by Bastin *et al.* [2] measured the first 2^+ excitation in ^{42}Si at 770 keV. The low energy of this state is due to the disappearance of both $Z = 14$ and $N = 28$ shell closures (according to the authors). In addition they have also observed the level schemes of ^{41,43}P and the observed levels confirmed their conclusion. According to the same paper, this effect (disappearing of the shell closure) is ascribed to a tensor force. With a shell model calculation the authors have obtained a reduction of the neutron gap from ^{48}Ca to ^{42}Si of 1.94 MeV. They claim that a realistic gap for ^{42}Si is 5.94 MeV. On the contrary, in the work of Fridmann *et al.* [4] on two-proton knockout reactions, it is found that the $Z = 14$ shell closure persists at $N = 28$.

As far as ^{42}Si is a nucleus very close to the neutron-drip line, it can provide a good playground for checking how accurate are the models utilized, and it can prove that the tensor term should not be neglected in further investigations on exotic nuclei.

The $N = 28$ is the first magic shell due to the spin-orbit force, this is why it can be quite a fruitful region for examinations of the spin orbit-force. A lot of experimental efforts are concentrated on this chain. The question whether the neutron gap $\nu(1f_{7/2} - 2p_{3/2})$ persists when one goes towards the more neutron rich nuclei still exists. At SPIRAL (Systeme de Production d'Ions Radioactifs en ligne) in GANIL [3] a reduction of this gap has been observed when one goes from ^{48}Ca to ^{46}Ar by $330(90)$ keV, and this has been attributed almost entirely to the tensor force. In a subsequent paper [2], it is claimed that the gap is reduced from ^{48}Ca to ^{42}Si by 1 MeV, in other words the reduction is more than 25%. This effect was also interpreted as an effect due to a tensor force [2, 5]. However we show [6] that the tensor contribution in this case is not the only cause of the reduction, in both relativistic Hartree-Fock calculations and non-relativistic mean field approach.

This paper is organized as follows. In Section 2 we briefly give the main ingredients of the model used in this analysis, namely the non-relativistic HF-BCS model with a Skyrme type interaction. In Section 3 the results obtained for the $Z = 14$ isotopic chain and the $N = 28$ isotonic chain of nuclei are discussed. Conclusions are drawn in Section 4.

2 The Model

We assume a spherical symmetry description of the nuclei studied here. Although this is a strong assumption for some nuclei of the isotopic and isotonic chains that we consider, it allows an easier insight into the role of separate parts of the mean field such as the central and spin-orbit potentials.

The Skyrme-HF model is widely used and we refer to Ref. [7] for the notations of the present work as well as for the effective interaction SLy5 used here for generating the self-consistent mean field. In addition to the usual central and spin-orbit components of the Skyrme interactions (represented here by the SLy5 parametrization) we want to study the possible effects of a tensor component of the Skyrme force. This component was introduced in earlier versions of the Skyrme force [8–10] but it is only recently that attention was focused on its effects on single-particle spectra in spin-unsaturated nuclei.

Along the chain of $N = 28$ isotones ($Z = 14$ isotopes) the proton-proton (neutron-neutron) pairing correlations will play a role, and they are treated in BCS approximation [12] using a zero-range, density-dependent pairing interaction of the form:

$$V^{(\mathrm{n\ or\ p})} = V_0^{(\mathrm{n\ or\ p})} \left(1 - \frac{\rho(\boldsymbol{R})}{\rho_0}\right) \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) , \tag{1}$$

where $\boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$, $\rho_0 = 0.16$ fm^{-3} is the nuclear matter saturation density and $V_0^{(\mathrm{n\ or\ p})}$ is fitted for each chain to reproduce the empirical two-neutron (two-proton) separation energies. The pairing window contains the orbitals of the $1p$, $2s$-$1d$ and $2p$-$1f$ shells.

The effect of a tensor component in the Skyrme interaction is very simply described by a modification of the coefficients α and β which multiply neutron or proton spin densities as explained, *e.g.*, in Ref. [11]. This modification affects the single-particle spin-orbit potentials, especially in the case of spin-unsaturated subshells which contribute importantly to the spin densities. One has:

$$\begin{aligned} \alpha &= \alpha_C + \alpha_T , \\ \beta &= \beta_C + \beta_T , \end{aligned} \tag{2}$$

where α_C, β_C come from the central, velocity-dependent part of the Skyrme force whereas α_T, β_T come from the tensor component. Using the Skyrme force SLy5 the values of α_T and β_T were determined [11] to be $\alpha_T = -170$ MeV fm^5, $\beta_T = 100$ MeV fm^5, and we shall adopt these values.

In the non relativistic Skyrme-HF model the radial HF equations can be expressed in terms of an energy-dependent equivalent potential V_{eq}^{lj}:

$$\frac{\hbar^2}{2m}\left[-\frac{d^2}{dr^2}\psi(r) + \frac{l(l+1)}{r^2}\psi(r)\right] + V_{\mathrm{eq}}^{lj}(r,\epsilon)\psi(r) = \epsilon\psi(r) , \tag{3}$$

where

$$V_{\mathrm{eq}}^{lj}(r,\epsilon) = \frac{m^*(r)}{m}U_0(r) + \frac{m^*(r)}{m}U_{\mathrm{so}}^{lj}(r) + V_{\mathrm{eq}}^{\mathrm{m}*} . \tag{4}$$

Here, $U_{\mathrm{so}}^{lj}(r) = U_{\mathrm{so}}(r) \times [j(j+1) - l(l+1) - 3/4]$, $U_{\mathrm{so}}(r)$ is the spin-orbit HF potential,

$$V_{\mathrm{eq}}^{\mathrm{m}*} = \left[1 - \frac{m^*(r)}{m}\right]\epsilon - \frac{m^{*2}(r)}{2m\hbar^2}\left(\frac{\hbar^2}{2m^*(r)}\right)^{\prime 2} + \frac{m^*(r)}{2m}\left(\frac{\hbar^2}{2m^*(r)}\right)^{\prime\prime} , \tag{5}$$

and $U_0(r)$ and $m^*(r)$ are the HF central potential and effective mass, respectively [7]. The spin orbit potential can be explicitly presented as:

$$U_{\mathrm{so}}^q(r) = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right) \tag{6}$$

The first term is from the spin-orbit interaction, $\alpha = \alpha_T + \alpha_C$ and $\beta = \beta_T + \beta_C$; q stands for protons or neutrons while q' is for neutrons or protons. The spin-density is defined as:

$$J_q(r) = \frac{1}{4\pi r^3}\sum_{nlj}\left[j_i(j_i+1) - l_i(l_i+1) - \frac{3}{4}\right] n_i^q u_i^2(r) \tag{7}$$

The Coulomb potential is included in U_0, in the case of protons.

With the help of Eqs. (3)–(5) we can write ϵ as a sum of kinetic, central, spin-orbit and non-locality (or m^*-dependent) contributions:

$$\epsilon = \epsilon_{\text{kin}} + \epsilon_{\text{cen.}} + \epsilon_{\text{s.o.}} + \epsilon_{m*} \; . \tag{8}$$

In the above expression the three last terms correspond to the three terms of Eq. (4). This decomposition is useful for understanding the evolution of single-particle energies when N or Z is changing [13], and it will be used in the discussion of results in Section 3.

3 Results: Comparison and Analysis

3.1 The $Z = 14$ Chain

First we will discuss the evolution of the proton single particle spectra of the $2s-1d$ shell in the isotopic chain $Z = 14$ when the neutron number increases from $N = 20$ (^{34}Si, $\nu 1f_{7/2}$ empty) to $N = 28$ (^{42}Si, $\nu 1f_{7/2}$ filled). The question of reduction of the proton $1d_{3/2} - 1d_{5/2}$ spin-orbit splitting when going towards more neutron rich nuclei is of particular interest. In shell model analysis it was found [2] that this spin-orbit splitting is reduced by about 1.94 MeV if one fills the $1f_{7/2}$ neutron sub-shell. We have performed SHF+BCS calculations to examine the behavior of the proton $1d_{3/2} - 1d_{5/2}$ spin-orbit splitting with increasing neutron excess. In the results presented on Figure 1 there is an obvious reduction of the energies of all single particle states with the mass increase. This can easily be described as an effect due to a proton-neutron symmetry potential. However no decreasing of the spin-orbit splitting of the d orbitals can be observed. When a more detailed analysis is

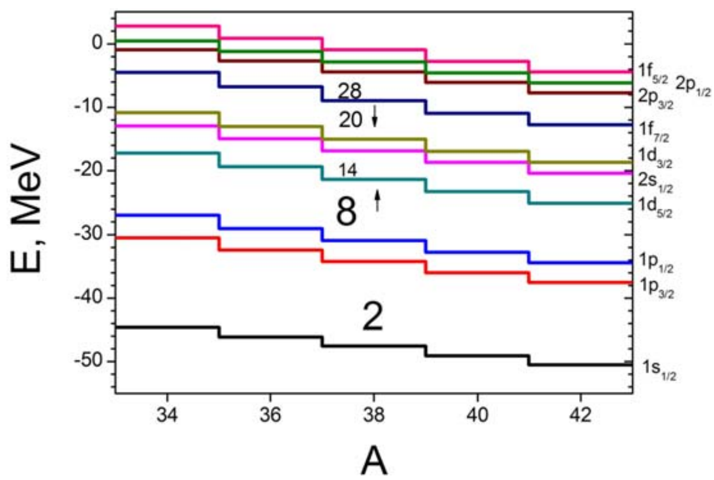

Figure 1. Proton s.p. states for the $Z = 14$ isotopic chain. No tensor term included.

done, one can see that the splitting remains practically unchanged – from 6.38 MeV for ^{34}Si to 6.36 MeV for ^{42}Si. The result should not be a surprise, the contribution to the energy of a state, due to the spin-orbit term in the Hamiltonian, may be written using Eq. (3) as:

$$\epsilon_{s.o.} = \int dr\psi(r)U_{s.o.}(r)\psi(r) \times [j(j+1) - l(l+1) - 3/4] \tag{9}$$

We can assume that the wave-functions of the two states of interest – $\pi 1d_{3/2}, 1d_{5/2}$ change very slowly when one fills the neutron $\nu 1f_{7/2}$ subshell. This assumption should hold, as the $\nu 1f_{7/2}$ neutron state does not interact directly with the proton states, and the only changes in the wave functions are due to self-consistent effects. The energy splitting can be rewritten in the following form:

$$\epsilon_{split} = \int drU_{s.o.}(r) \times C(r) \tag{10}$$

Here $C(r)$ is a function of the wave-functions of the states. As far as $C(r)$ will change slowly with the increasing of the mass, the only way to introduce the gap reduction is through a change in the spin-orbit potential. In the case of no tensor term the s.o. potential is an algebraic function of density derivatives, so it is not strongly affected by the filling of $\nu 1f_{7/2}$, as seen from Figure 2. In fact, here the

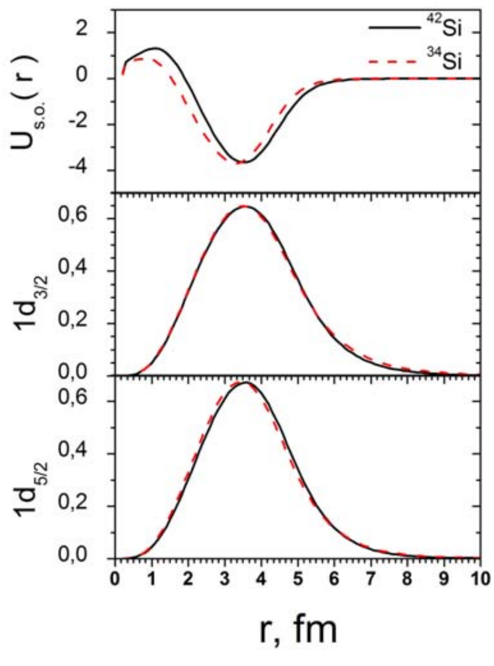

Figure 2. $\pi 1d_{3/2}, 1d_{5/2}$ wave functions, calculated for ^{34}Si and ^{42}Si. The spin-orbit potential for these two nuclei, in case of no tensor contribution is presented in the upper panel.

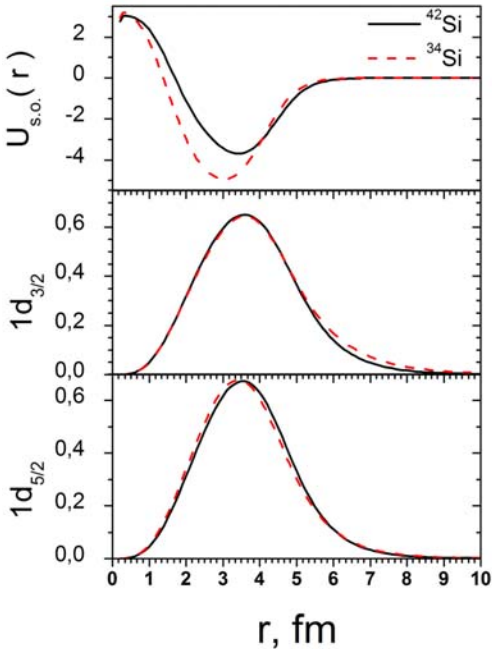

Figure 3. Same as Figure 2. Here the tensor term is taken into account.

calculations were done with a Sly5 parameterization, which already has taken into account the J^2 term due to a spin-gradient coupling, still its effect is very small. Obtaining a decrease in the spin-orbit splitting, with magnitude of more than 30% is impossible in this case. To describe properly the observed reduction one should include the tensor term explicitly Eq. (6). As one can see from Figure 3, the spin-orbit potential is decreased when one goes to more neutron rich nuclei, this leads to an overall reduction of the $\pi 1d_{3/2} - 1d_{5/2}$ splitting. In this case the splitting is diminished from 8.62 to 7.10 MeV. This effect is easy to be described within the HF-framework taking into account the properties of the tensor interaction. Firstly, the neutron spin density J_n becomes larger when one fills the neutron orbital $1f_{7/2}$ according Eq. (7). Since the term proportional to J_n is with opposite sign ($\beta < 0$), with respect to the overall spin-density potential Eq. (6), the increasing J_n leads to a decrease in the overall potential. If we perform a decomposition of the splitting, according to Eq. (8), we can obtain information on the importance of all the terms playing a role in the formation of the splitting. In Figure 4 is presented the result, obtained from the application of such a decomposition. The importance of the tensor term is prominent. Still in the data an overall increase of the gap in comparison to the case of no tensor force is observed. This is due to the term proportional to J_p in the tensor force. As it remains virtually constant throughout the chain, its effect is simply summed to the spin-orbit splitting and does not produce any change in the slope of the $\pi 1d_{3/2} - 1d_{5/2}$ splitting.

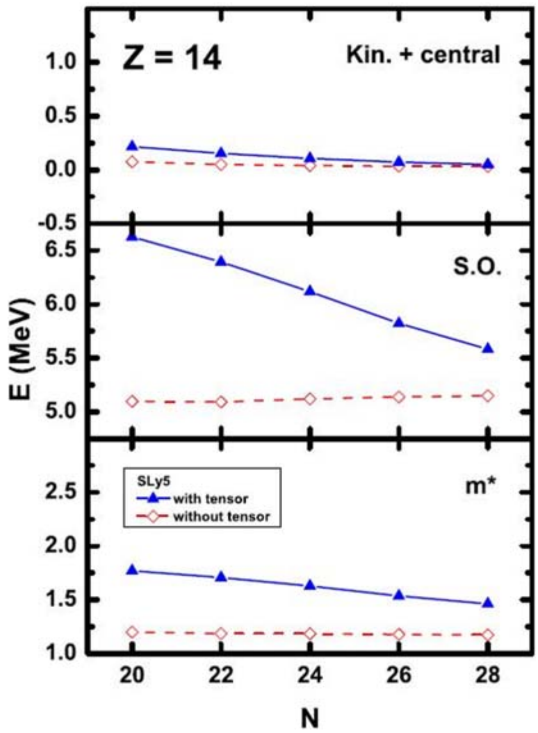

Figure 4. Decomposition of the $\pi 1d_{3/2} - 1d_{5/2}$ splitting according to Eq. (8). Dashed line is used for the no tensor term case, in solid line is given the case with this term included.

We have obtained a result that is close to the one obtained by Bastin *et al.*, 1.52 MeV decrease in our case in comparison to 1.94 MeV given by them [2]. The results are in agreement with the data obtained within a relativistic Hartree-Fock model [14]. A detailed comparison between the results obtained with both models may be found in [6]. Still we should not treat this result as final. We have not included the particle-vibration coupling effects [15], and it is predicted, that this effect should be sizable. Thus a direct comparison to the experimental single particle spectrum is still impracticable.

3.2 The $N = 28$ Chain

For this chain we will be interested in the $2p - 1f$ neutron shell. The changes in the single particle energies of states within this shell, when one goes towards the proton drip-line, will be analyzed. We will restrict ourselves to the region ^{48}Ca–^{42}Si. Experimental observations, combined with shell model analysis [2, 3] lead to the conclusion that a compression of the $1f - 2p$ shell occurs when one removes protons from the proton $\pi 1d_{3/2}$ and $\pi 2s_{1/2}$ levels keeping the neutron number fixed. This compression was predicted to affect the $1f_{7/2}$ - $2p_{3/2}$ gap by around 1 MeV,

Table 1. The $2p_{3/2}$–$1f_{7/2}$ gap for the isotonic chain $N = 28$; The units are MeV.

(N,Z)	(28,14)	(28,16)	(28,18)	(28,20)
SLy5	2.55	3.09	3.49	3.75
SLy5+T	3.79	4.08	4.66	5.15

from 4.8 MeV in ^{48}Ca to less than 3.8 MeV in ^{42}Si. In Table 1 we present the calculated gap reduction with and without tensor term included. The inclusion of a tensor term leads to an overall increase in the size of the gap, but this effect is due to the term containing α_T Eq. (6). As this term stays fairly constant and relatively small, it cannot affect the gap reduction. The determination of the absolute values of the gaps is an aim above our possibilities now, as far as the model does not include substantial effects, like the PVC (particle-vibration coupling, already mentioned). Still we should be able to describe the relative reductions observed in this shell.

The astonishing result is that both mean field descriptions – with and without tensor force, predict almost the same reductions around $1,2$ MeV. A shrinkage of the same magnitude, as the one described here, was already interpreted, as due to a tensor term in the works of Bastin and Gaudefroy – $[2, 3]$; in the work of Zou *et al.* [5] the authors investigate the reduction in the same gap between ^{48}Ca and ^{46}Ar isotones, claiming that the tensor force has a leading role in it. This is why we need a further investigation of our result. If a decomposition of the type of Eq. (8)

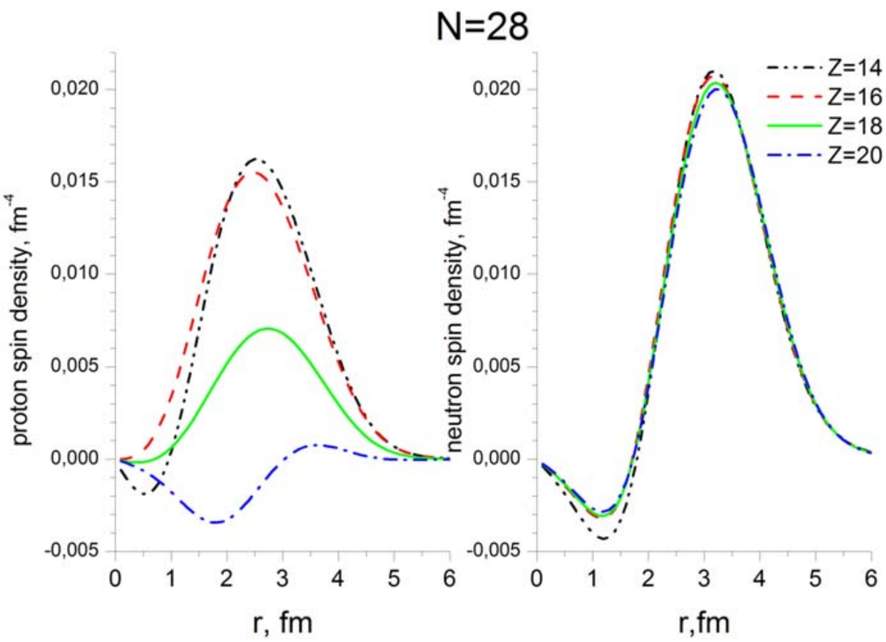

Figure 5. Proton and neutron spin densities for the $N = 28$ isotonic chain

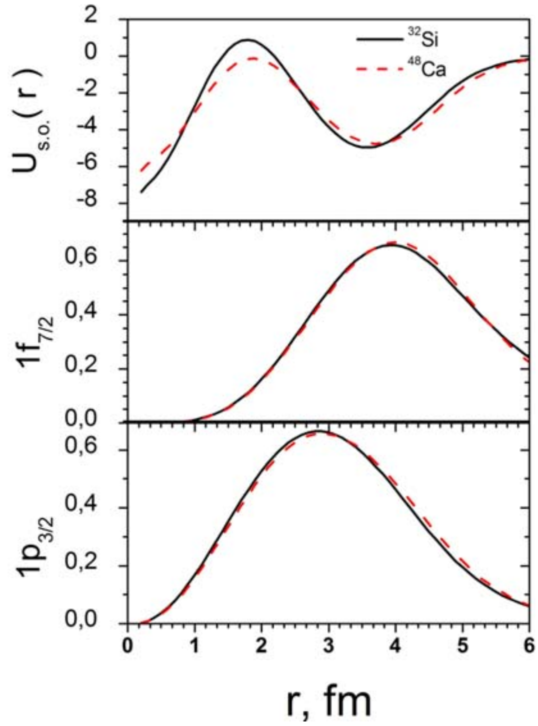

Figure 6. $2p_{3/2}$ and $1f_{7/2}$ neutron wave functions, calculated for ^{42}Si and ^{48}Ca. The spin-orbit potential for these two nuclei, with tensor term included is presented in the upper panel.

is done one can see that the spin-orbit term remains fairly constant for both models (with and without tensor force) as presented on Figure 5. The whole reduction of the gap is practically due to the two other terms. This fact makes us conclude that the observed reduction is due to the proton-neutron symmetry potential effects, and not due to the tensor interaction. Still an interesting question is why the tensor effect is so small for this region. We will try to follow the considerations in the previous subsection, to answer this question. Here we expect the wave-functions of the $2p_{3/2}$ and $1f_{7/2}$ states to remain almost unchanged when one deplete the protons $2s - 1d$ shell. So, again we expect the variations in the splittings of the p and f states to be due to a change in the spin-orbit potential. As far as $2p_{3/2}$ is a spin up state ($\uparrow$) and $f_{7/2}$ is also a spin up state ($\uparrow$) we expect that the tensor term will move the two levels in same directions, thus it will lead to a small effect on the shell gap. Still the only way to influence the relative splittings within a doublet, is through a change in the spin-orbit potential. Such a deviation can be introduced by the variations in the proton spin densities in agreement with Eq. (6). On Figure 5 we present the neutron and proton spin densities. As expected the neutron spin density is relatively big, but stays constant for the whole chain, while the proton spin-density changes notably. Still we would like to mention that this affects a limited region in space. We construct the corresponding spin-orbit potential, which is shown in the upper panel of Figure 6. From the last is seen that even smeared, the changes in the potential due to the alterations in the proton spin density still exist in the region below 4 fm. Still the wave function of the $f_{7/2}$ state is concentrated a bit further Figure 6. This is why the changes in the spin-orbit potential have no strong effect on the splitting of the f doublet. For the $2p_{3/2}$ state the matrix elements are even smaller so the effect of the tensor force is not notable. Thus the relative spin-orbit splitting does not change with the mass number.

We want to stress again that the obtained results on the $1f - 2p$ compression, show that the shrinkage effect in the $1f-2p$ shell is observed, still the interpretation of this effect is that it is induced by the proton-neutron symmetry potential itself, and can be properly taken into account within a SHF framework.

4 Conclusions

In this work we have studied the mechanisms which lead to a reduction of the proton $1d_{3/2}$-$1d_{5/2}$ spin-orbit splitting $\Delta_{\rm so}(N)$ with decreasing the mass number in the Si isotopic chain. We also investigate a quenching of the neutron $2p_{3/2}$-$1f_{7/2}$ gap $\Delta_{\rm gap}(Z)$ as one goes from ^{48}Ca towards ^{42}Si. We have used the self-consistent mean field approach in its non-relativistic version with the SLy5 parametrization.

One of the goals of this study is to determine to which extent a tensor component in the effective nucleon-nucleon interaction can affect the values of $\Delta_{\rm so}(N)$ and $\Delta_{\rm gap}(Z)$. Our main conclusions are: 1) the reduction of $\Delta_{\rm so}(N)$ when going from ^{34}Si to ^{42}Si is mainly due to the presence of a tensor component in the effective interaction. The magnitude of the change in $\Delta_{\rm so}(N)$ is consistent with the empirical observations. 2) On the other hand, the evolution of $\Delta_{\rm gap}(Z)$ when Z decreases from 20 to 14 does not depend strongly on a tensor component in the interaction.

We have performed a detailed analysis of the origin of the evolution of $\Delta_{\rm gap}(Z)$ and $\Delta_{\rm so}(N)$. The reduction of the former can be traced back to changes occurring in the symmetry part of the central potential, to the m^*-term of the Skyrme-HF model, and only to a small extent to the spin-orbit potential.

It would be important to extend in future studies this type of analysis to include effects such as particle-vibration coupling which are beyond the present mean field models and which are known to affect single-particle spectra.

Acknowledgments

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