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Adiabatic and Diabatic Dynamics of Fusion in Heavy Ion Collisions

G.G. Adamian[1], **N.V. Antonenko**[1], **W. Scheid**[2]

[1] Bogoliubov Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research, 141980 Dubna, Russia
[2] Institut für Theoretische Physik der Justus-Liebig-Universität, 35392 Giessen, Germany

Abstract. The reaction of fusion of heavy ions to superheavy nuclei is discussed. The mechanism can be described within adiabatic or diabatic approaches, *i.e.*by the melting of the nuclei along the internuclear distance or by the transfer of nucleons between the nuclei, respectively. We find that the diabatic models like the dinuclear system model are advantageous for a correct description of the fusion process.

1 Introduction

The content of this article is a review on the different ideas for the description of fusion of heavy ions to superheavy nuclei. There exist in literature models using adiabatic or diabatic potentials. The fusion is described by two main degrees of freedom, namely by the relative motion of the fusing nuclei with the coordinate R and by the mass and charge transfer between the nuclei expressed by the corresponding asymmetry coordinates.

Adiabatic potentials have the minimum energy of the system for a given set of collective coordinates and a given internuclear distance and smaller barriers for similar projectile and target nuclei. In this case the nuclei melt together reducing the internuclear distance. Adiabatic models usually yield larger fusion cross sections for collisions of equal projectile and target nuclei than those observed in experiments and can often not explain the isotopic trends. Diabatic potentials are strongly repulsive for smaller internuclear distances due to the structural forbiddenness and let develop the fusion process as a nucleon transfer in a touching configuration of the two nuclei what is named the dinuclear system. The cross section decreases with increasing symmetry between projectile and target nuclei which agrees with experimental data. So the dynamics of fusion is basically different in the two descriptions of fusion: In the adiabatic models the nuclei melt together, whereas in the diabatic models the nuclei transfer nucleons between each other up to the instant when the compound nucleus is formed.

2 Adiabatic or Diabatic Potentials between Nuclei

The description of the dynamical way of heavy ion fusion strongly depends on the potential taken between the nuclei. We discriminate between adiabatic and diabatic potentials. Adiabatic potentials have the minimum of energy of the system for a given set of collective coordinates and a given internuclear distance. The potential energy can be calculated with the Strutinsky method,

$$U = U_{LD} + \delta U_{\text{shell}}, \tag{1}$$

where U_{LD} is the energy of the system obtained with a liquid drop model for the shape coordinates and δU_{shell} includes the effects of the shells. The shell effects in nucleus-nucleus collisions are obtained with a two-center shell model which is shortly discussed in the following subsection.

2.1 Two-center Shell Model

In most calculations of potentials one uses the two-center shell model of Maruhn and Greiner [1]. It is based on the two-center oscillator. The parameters of the Maruhn-Greiner model (see Figure 1) are the length ℓ of the system, expressed by the ratio $\lambda = \ell/(2R_0)$ where R_0 is the radius of the spherical compound nucleus, the mass asymmetry η defined by the masses left and right to the plane through the neck, $\eta = (A_1 - A_2)/(A_1 + A_2)$, the neck parameter $\epsilon = E_0/E'$ (ratio of barriers, see Figure 1), and the deformations $\beta_i = a_i/b_i$ with $i = 1$ and 2 (ratio of semiaxes). Recently, Diaz Torres [2] proposed a two-center shell

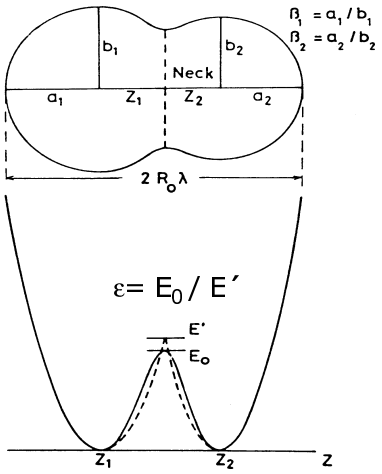

Figure 1. The parameters of the two-center shell model. Here, $2R_0\lambda$ measures the length of the system. The deformation parameters are given by $\beta_i = a_i/b_i$ with $i = 1, 2$. The neck parameter is $\epsilon = E_0/E'$.

model with a greater variability. The potentials are superpositions of two shifted and rotated Woods-Saxon potentials

$$\begin{aligned} V &= \exp(-\vec{R}_1\nabla)\hat{U}(\Omega_1)V_1\hat{U}^{-1}(\Omega_1)\exp(\vec{R}_1\nabla) \\ &+ \exp(-\vec{R}_2\nabla)\hat{U}(\Omega_2)V_2\hat{U}^{-1}(\Omega_2)\exp(\vec{R}_2\nabla) \end{aligned} \quad (2)$$

where V_1 and V_2 are Woods-Saxon potentials.

The centers are positioned at $\vec{R}_1$ and $\vec{R}_2$, the relative coordinate is $\vec{R}$=$\vec{R}_1$-$\vec{R}_2$, and $\hat{U}(\Omega_1)$ and $\hat{U}(\Omega_2)$ are operators for rotation by the Euler angles Ω_1 and Ω_2, respectively. To get the single-particle levels of $T+V$, Diaz Torres used two non-orthogonal sets of oscillator functions around the centers of the two parts of (2) as basis for diagonalizing the Hamiltonian. He considered the two-center potentials of the system consisting of the spherical nuclei ^{16}O and ^{40}Ca for different relative distances and two oblately deformed ^{12}C nuclei with different relative orientations. This two-center shell model is realistic with respect to bound and continuum levels, but difficult to evaluate for heavier systems.

2.2 Calculation of Adiabatic and Diabatic Potentials

In order to calculate adiabatic and diabatic potentials one has to consider both types of single-particle energies for the shell effects. Diabatic TCSM states can be calculated by the maximum overlap method according to Lukasiak *et al.* or by the method of maximum symmetry where one diagonalizes a two-center Hamiltonian with maximum symmetry excluding the neck potential and terms proportional to ℓ_x, ℓ_y and s_x, s_y in the spin-orbit potential. The maximum overlap method and the method of maximum symmetry which is numerically simpler yield nearly the same results for the single-particle energies. The nucleus-nucleus potentials are obtained by the Strutinsky method [3]:

$$\begin{aligned} V_{adiab} &= V_{LD} + \delta U_{shell}, \\ V_{diab} &= V_{adiab} + \sum_\alpha \left(\varepsilon_\alpha^{diab}(R)n_\alpha^{diab}(R) - \varepsilon_\alpha^{adiab}(R)n_\alpha^{adiab}(R)\right). \end{aligned} \quad (3)$$

Diabatic potentials are strongly repulsive and forbid fusion via the internuclear coordinate. They are similar to potentials calculated with double folding methods by using frozen densities. The later type of potentials is also denoted as sudden potentials. Figure 2 shows the diabatic potential for the ^{110}Pd + ^{110}Pd system. One finds that the double folding potential leads to a very similar diabatic potential.

The time-dependence of the transition from a diabatic potential to an adiabatic one can be related to the characteristic relaxation time for the shape degrees of freedom of the system. The potential is given by

$$\begin{aligned} V(\lambda,t) &= V_{adiab}(\lambda) + \Delta V_{diab}(\lambda,t) \qquad \text{with} \\ \Delta V_{diab} &\approx \sum_\alpha \left(\varepsilon_\alpha^{diab}(\lambda,t)n_\alpha^{diab}(\lambda,t) - \varepsilon_\alpha^{adiab}(\lambda)n_\alpha^{adiab}(\lambda)\right). \end{aligned} \quad (4)$$

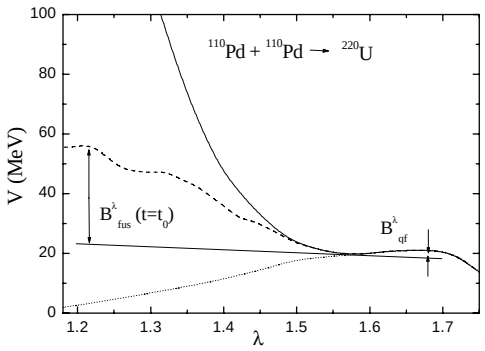

Figure 2. The diabatic (solid curve), the diabatic time-dependent (dashed curve) and the adiabatic (dotted curve) potentials for ^{110}Pd + ^{110}Pd as a function of λ.

Here, λ is the dimensionless internuclear distance parameter of the TCSM (see Figure 1). $n_\alpha^{adiab}(\lambda)$ vary with λ according to a Fermi distribution with temperature $T(\lambda) = \sqrt{E^*(\lambda)/a}$, where $E^*(\lambda)$ is the excitation energy of the system. The diabatic occupation numbers n_α^{diab} follow relaxation equations

$$\frac{dn_\alpha^{diab}(\lambda,t)}{dt} = -\frac{1}{\tau(\lambda,t)}\left(n_\alpha^{diab}(\lambda,t) - n_\alpha^{adiab}(\lambda)\right) \tag{5}$$

with $\tau(\lambda,t) = 2\hbar/\langle\Gamma\rangle \approx 5\times10^{-21}$ s and

$$\langle\Gamma(\lambda,t)\rangle = \sum_\alpha \bar{n}_\alpha^{diab}(\lambda,t)\Gamma_\alpha(\lambda)/\sum_\alpha \bar{n}_\alpha^{diab}(\lambda,t), \tag{6}$$

$$\bar{n}_\alpha^{diab} = n_\alpha^{diab} \qquad \text{for} \quad \varepsilon_\alpha^{diab} > \varepsilon_F,$$
$$\bar{n}_\alpha^{diab} = 1 - n_\alpha^{diab} \qquad \text{for} \quad \varepsilon_\alpha^{diab} \le \varepsilon_F.$$

The dashed potential in Figure 2 lying between the diabatic (solid line) and adiabatic (points) potentials results from the above equations for a time $t_0 = 8\times10^{-21}$ s which is roughly the time for forming the compound nucleus. The astonishing outcome is that a quite high barrier of about 60 MeV remains towards smaller internuclear distances and hinders the direct fusion to ^{220}U along the internuclear coordinate. From these calculations we conclude that in heavier collision systems one has to consider diabatic or modified diabatic potentials with high barriers to the inside hindering a direct fusion process.

With a microscopic approach based on the formalism of irreducible representations of the SU(3) group one finds an influence of the structural forbiddenness on the fusion of heavy nuclei and can estimate the energy thresholds for complete fusion in relative distance and mass asymmetry degrees of freedom [4]. This effects are similar as the shown diabaticity of the internuclear potential.

3 The Motion of the Neck

Here we consider the dynamics of the neck degree of freedom between the touching nuclei [5]. The neck dynamics is described by the neck parameter $\epsilon = E_0/E'$ defined by the ratio of the actual barrier height E_0 to the barrier height E' of the two-center oscillator (see Figure 1). The neck grows with decreasing ϵ. In order to learn about the neck dynamics we calculated the potential energy surface as a function of λ and ϵ for the case of the ^{110}Pd + ^{110}Pd system in the adiabatic approach (see Figure 3) and carried out dynamical, time-dependent calculations which have an adiabatic character because an adiabatic potential energy surface was used. The kinetic energy is written

$$T = \frac{1}{2}\sum_{i,j} B_{i,j}\dot{q}_i\dot{q}_j, \quad i = 1,2, \quad q_1 = \lambda, \quad q_2 = \epsilon, \tag{7}$$

and the dissipative forces are included by use of a Raleigh dissipation function

$$\Phi = \frac{1}{2}\sum_{i,j} \gamma_{i,j}\dot{q}_i\dot{q}_j, \tag{8}$$

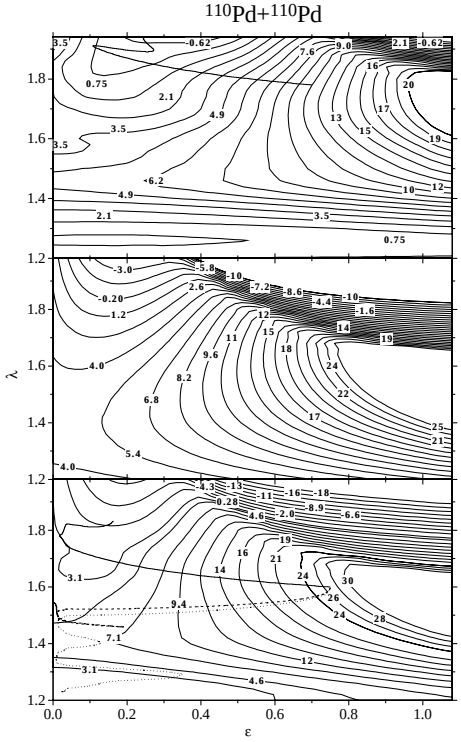

Figure 3. Potential energy surface (units MeV), calculated in the (λ, ϵ)-plane for the reaction ^{110}Pd + ^{110}Pd with shell corrections and $\beta_i = 1$ (lowest part), without shell corrections and $\beta_i = 1$ (middle part), and with shell corrections and $\beta_i = 1.2$ (upper part). The dynamical trajectories in the lowest part starting from the touching configurations and with initial kinetic energies 0, 40 and 60 MeV are presented by solid, dashed and dotted lines, respectively.

where the friction coefficients are calculated with

$$\gamma_{i,j} = 2\Gamma B_{i,j}/\hbar \tag{9}$$

according to the linear response theory and Γ is the average width of single particle states. The classical equations are derived from the Lagrangian $L = T - V$ and Raleigh dissipation function Φ. Starting with $\lambda = 1.59$ and $\epsilon = 0.75$ for ^{110}Pd + ^{110}Pd and with mass parameters obtained within the Werner-Wheeler approximation under the assumption of an incompressible and irrotational flow of the nuclear matter we reached the fission-type valley already after a very short time of $3 - 4 \times 10^{-22}$ s at $\lambda \sim 1.68$ and then found oscillations in this valley in case of a small kinetic energy. The characteristic time of all processes results $\sim 5 \times 10^{-21}$ s. This has as consequence that fusion may occur easier in reactions with heavier isotopes in contradiction to the experimental data. For the system ^{110}Pd + ^{110}Pd we found fusion probabilities $P_{CN} \approx 10^{-1} - 10^{-3}$ in comparison with the experimental value of only about $P_{CN} \approx 10^{-4}$.

With the described method one calculates a wrong dependence of the fusion probability on the isotope composition and of the mass asymmetry of target and projectile [5]. There must exist a hindrance for a fast growth of the neck and the motion to smaller values of λ. We found as an essential hindrance large microscopically calculated mass parameters for the neck motion. We obtained the microscopical mass parameters with the cranking formula [6]. This formula yields (WW= Werner-Wheeler)

$$B^{cr}_{\lambda\lambda} = B^{WW}_{\lambda\lambda}, \quad B^{cr}_{\epsilon\epsilon} \approx 30 \times B^{WW}_{\epsilon\epsilon}, \quad B^{cr}_{\lambda\epsilon} \approx 0.35 \times B^{WW}_{\lambda\epsilon}. \tag{10}$$

The much larger neck mass parameter $B^{cr}_{\epsilon\epsilon}$ has as consequence that the system stays nearly fixed at the entrance configuration, which is the typical dinuclear system configuration, for a sufficient long time. Beside the large neck mass parameter we found also other dynamical restrictions for a fast growth of the neck which are caused by the potential energy surface intermediate between the adiabatic and diabatic limits.

4 Complete Fusion in the Dinuclear Model

4.1 Reaction Models for Fusion with Adiabatic and Diabatic Potentials

Reaction models which use adiabatic potentials describe the nuclear fusion as a melting of the clusters into a compound nucleus. Since the adiabatic potential barrier to the inside is usually smallest for two equal nuclei, such models have the property that the two clusters exchange nucleons in a touching configuration up to the point they are nearly equal (same mass, $\eta \approx 0$) and then they fuse to the compound nucleus along the internuclear distance R. This process yields large cross sections for fusion with similar target and projectile nuclei ($\eta \approx 0$) [5, 7], which contradicts the experimental data in the production of superheavy nuclei.

In contrast to this picture the dinuclear system concept, which is based on the ideas of Volkov [8] and also von Oertzen, makes use of diabatic potentials which are strongly repulsive behind the touching point of the clusters. Therefore, the nuclei can not melt together along the internuclear coordinate [9]. In heavier systems they remain for some time in a touching configuration and form the dinuclear system. Then they start to exchange nucleons up to the point when the smaller nucleus is eaten up by the larger one and the compound nucleus is formed [10]. This process prefers the formation of the compound nucleus between an asymmetric system of two clusters which is in agreement with the experience in the production of heavy and superheavy nuclei.

As one can note, the dynamics of fusion is very different if described by adiabatic or diabatic potentials. The adiabatic potentials prefer the dynamics of fusion in the internuclear coordinate R, whereas the diabatic potentials describe the fusion by the dynamics in the mass asymmetry coordinate η. The question arises which of these very different reaction mechanisms for describing the production of superheavy nuclei is realized in nature. A possible answer would be given for example by a detailed measurement of the quasifission process accompanying the fusion. Quasifission means the direct decay of the dinuclear system without forming the compound nucleus and proceeds always in competition with the exchange of nucleons between the clusters.

It is clear that an adiabatic description with many explicitly treated collective coordinates finally leads to a diabatic description, since the kinetic energy of the relative cluster motion, first available in the system, gets then transferred into other degrees of freedom and the nuclear system will stop its internuclear motion around the touching point. However, this is the starting point of the description with the dinuclear system concept.

In the following we will describe the fusion to superheavy nuclei with the dinuclear system concept. There are different methods like statistical procedures and master equations to calculate production cross sections for superheavy nuclei.

4.2 Fusion to Superheavy Nuclei

The evaporation residue cross section can be written as a sum over partial contributions [11]

$$\sigma_{ER}(E_{c.m.}) = \sum_{J=0}^{J_{max}} \sigma_{cap}(E_{c.m.}, J) P_{CN}(E_{c.m.}, J) W_{sur}(E_{c.m.}, J). \qquad (11)$$

The factors are the partial capture cross section, the fusion and survival probabilities. The contributing angular momenta in σ_{ER} are limited by the survival probability W_{sur} with $J_{max} \approx 10 - 20$ when highly fissile superheavy nuclei are produced with energies above the Coulomb barrier. We approximate the evaporation residue cross section by

$$\sigma_{ER}(E_{c.m.}) = \sigma_{cap}^{eff}(E_{c.m.}) P_{CN}(E_{c.m.}, J = 0) W_{sur}(E_{c.m.}, J = 0) \qquad (12)$$

with an effective capture cross section $\sigma_{cap}^{eff} = (\lambda^2/(4\pi))(J_{max}+1)^2 T(E_{c.m.}, J=0)$. For reactions leading to optimal cross sections for superheavy nuclei, the bombarding energy $E_{c.m.}$ is above the outer Coulomb barrier, and we set $T(E_{c.m.}, J=0) = 0.5$ near this barrier. The effective capture cross section results in the order of a few mb. Whereas the capture cross section and the survival probability are largely similarly formulated in all the models, the fusion probability is treated along very different trajectories through different potential energy surface there. Here we want to present our approach proposed and apply it for the fusion of clusters to superheavy nuclei within the dinuclear system concept.

4.3 Fusion Probability within the DNS Concept

After the system is captured in a DNS configuration, the total relative kinetic energy is transferred into potential and excitation energies. Then the dinuclear system statistically evolves in time by diffusion in the mass asymmetry and relative coordinates. The fusion probability P_{CN} is the probability that the dinuclear system crosses the inner fusion barrier B_{η}^{fus} in η and an excited compound nucleus is formed. This barrier is measured with respect to the potential $U(R_m, \eta_i)$ of the initial dinuclear configuration with the mass fragmentation η_i at the touching radius R_m (see Figure 4). There are different methods to calculate the fusion probability: The diffusion dynamics can be described with Fokker-Planck equations or with the Kramers approximation. Also master equations in the coordinates η and η_Z were used. In the diffusion equations the mean value $\bar{\eta}(t)$ mostly tends to the symmetric fragmentation $\eta = 0$ with an increasing probability for quasifission, determined by the quasifission barrier $B_{qf}^{R}(\eta)$ measured with respect to the minimum of the potential $U(R, \eta)$ at $R = R_m$.

The minima in the potential $U(R_m, \eta)$ play an important role for selecting optimum target and projectile combinations for producing superheavy elements. Sandulescu *et al.* (1976) argued that the nuclei fuse with higher probabilities along the valleys in an adiabatic potential in the R coordinate and pointed to the experimentally successful choice of target-projectile combinations with a Pb nucleus as target as proof for their hypothesis. This idea can simply transferred to the DNS concept. A certain initial system in a minimum of the potential is hindered by the barrier B_{η}^{sym} of the potential in η to move to more symmetric systems which would lead to a fast decay by quasifission. Therefore, an asymmetric DNS in a potential minimum lives a longer time with respect to its decay by quasifission than outside of the minimum and has a larger chance to fuse by diffusion via nucleon transfer into the compound nucleus.

Let us assume that the initial configuration is in a minimum of the driving potential $U(R_m, \eta)$. Then the probability for complete fusion depends on the quasi-stationary rate λ_{η}^{fus} for fusion, on λ_{η}^{sym} for going to a symmetric DNS which is easily decaying into two fragments and on λ_{R}^{qf} for the decay by quasi-

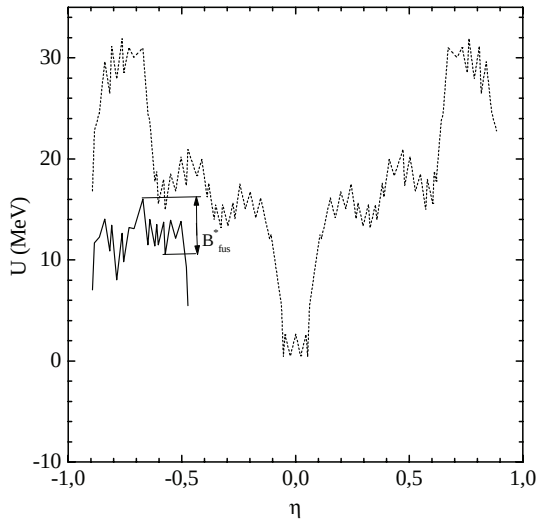

Figure 4. Potential energy of the dinuclear system in the reaction ^{54}Cr + ^{208}Pb $\rightarrow$ ^{262}Sg reaction ($|\eta_i| = 0.59$) as a function of η for $J = 0$. Both curves are obtained with experimental binding energies. The dotted curve is calculated for spherical shapes of the nuclei, the solid curve for deformed shapes in pole-to-pole orientations.

fission of the initial DNS [10].

$$P_{CN} = \lambda_\eta^{fus}/(\lambda_\eta^{fus} + \lambda_\eta^{sym} + \lambda_R^{qf}). \tag{13}$$

The rates can be calculated with two-dimensional Kramers-type formulas falling off exponentially with the fusion barrier B_η^{fus} in η, with the barrier B_η^{sym} in η in the direction to more symmetric configurations and with the quasifission barrier B_{qf}^R, respectively. The probability P_{CN} to overcome B_η^{fus} can be approximately written as

$$P_{CN} \sim \exp\left(-(B_\eta^{fus} - \min[B_\eta^{sym}, B_{qf}^R])/T\right). \tag{14}$$

The barriers, following from the potential $U(R_m, \eta)$, have heights strongly influenced by shell and deformation effects. The temperature T is the local temperature of the initial DNS and obtained from the excitation energy E^*: $T = \sqrt{E^*/a}$ with $a = (A_1 + A_2)/12$ MeV^{-1}.

The main hindrance for complete fusion is the evolution of the initial DNS to more symmetric configurations and the subsequent quasifission. In cold fusion the quasifission mainly arises from the initial DNS because of $B_{qf}^R < B_\eta^{sym}$, whereas in hot fusion reactions which have $B_{qf}^R > B_\eta^{sym}$, the DNS prefers to go to symmetric systems and to decay. Figure 5 shows calculated probabilities P_{CN} for cold (^{A}X + ^{208}Pb) and hot (^{48}Ca + ^{A}Y) fusion reactions.

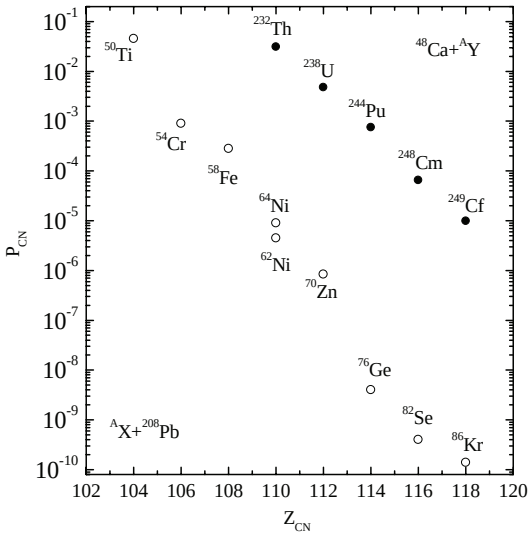

Figure 5. Calculated fusion probabilities P_{CN} for cold fusion reactions (^{A}X + ^{208}Pb) and hot fusion reactions (^{48}Ca + ^{A}Y) as a function of the charge number Z_{CN} of the superheavy compound nucleus.

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