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Thermal Effects on Neutrino-Nucleus Inelastic Scattering in Stellar Environment

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Abstract. Thermal effects on an inelastic neutrino-nucleus scattering off even-even nuclei in the iron region are studied. It is assumed that allowed GT_0 transitions in the neutral channel dominate the inelastic cross section $\sigma_{\nu A}$. To obtain the GT_0 strength distribution at finite temperature, the thermal quasiparticle random phase approximation in the context of thermo field dynamics is employed. It is found that a temperature growth increases the fraction of low- and negative-energy transitions in the GT_0 strength distribution. The neutral-current neutrino-nucleus inelastic cross section is calculated for relevant temperatures during the supernova core collapse. In agreement with the earlier studies within the shell-model approach a temperature increase leads to a considerable increase in $\sigma_{\nu A}$ for E_ν lower than the energy of GT_0 resonance.

1 Introduction

Neutrinos play an exceptional role in the evolution of the core collapse of a massive star towards a supernova explosion. Suffice it to mention that according to the standard models of supernovae ∼99% of the collapse energy is radiated in neutrinos. Until the iron core of the collapsing star reaches densities $\rho \approx 4 \times 10^{11}$ g·cm^{-3}, almost all of the energy of the collapse is transported by the neutrinos. At higher densities of the core elastic neutrino-nucleus and inelastic neutrino-electron scattering come into play as the sources for neutrino trapping and neutrino thermalization.

At the end of 1980th it was pointed out by W. C. Haxton that inelastic neutrino-nucleus scattering (INNS) mediated by the neutral-current can be as important as other processes of neutrino down-scattering [1]. For example, the INNS might contribute to the neutrino opacities and thermalization during the collapse phase, to the revival of the stalled shock wave in the delayed explosion mechanism, and to explosive nucleosynthesis.

The estimates by W. C. Haxton were based on nuclei in their ground states, i.e. on ”cold” nuclei. Later, it was realized that the INNS occurs in hot stellar environment ($T \geq 0.8$ MeV) and due to thermal population of nuclear excited states sizeable changes of the INNS cross section are to be expected. The effect was firstly analyzed in [2] and then in [3] on the basis of large-scale shell-model (LSSM) calculations. In Refs. [3, 4], it was revealed that the INNS cross section

$\sigma_{\nu A}(E_\nu)$ noticeably increases at $T \neq 0$ and neutrino energies $E_\nu \lesssim 10$ MeV especially for neutrino scattering off even-even nuclides.

However, in the subsequent core-collapse supernova simulations [5] with inclusion of several dozens of nuclides it was demonstrated that on the whole the inclusion of the INNS process has no significant effect on the collapse dynamics and the shock propagation. However, it has a significant effect on the spectrum of neutrinos generated in the ν_e burst.

Here, we present the other approach in treating the thermal effects on the cross section of INNS. Essentially, our approach is based on the thermal quasiparticle random phase approximation. However, we apply it in the context of the thermo field dynamics which enables a transparent treatment of excitation and de-excitation processes in a hot many-body system like a nucleus.

2 Cross Section of Inelastic Neutrino-Nucleus Scattering

We consider a process

$$(A, Z) + \nu_e \to (A, Z)^* + \nu_e'$$

Up to moderate neutrino energies E_ν, this neutral-current process is dominated by GT transitions mediated by the operator

$$D = \left(\frac{g_{_A}}{g_{_V}}\right)(\vec{\sigma}\tau_0), \tag{1}$$

where $g_{_A}$ and $g_{_V}$ are the axial and vector weak coupling constants, $\vec{\sigma}$ is the spin operator and τ_0 is the zero-component of the isospin operator.

At $T \neq 0$ the INNS cross section $\sigma_{\nu A}(E_\nu)$ is given by

$$\sigma_{\nu A}(E_\nu) = \frac{G_F^2}{4\pi} \sum_{i\,f} [E_\nu + (E_i - E_f)]^2 B_{if}(GT_0)\, g_i(T), \tag{2}$$

where indices i, f run over initial and final states of the nucleus; $B_{if}(GT_0) = |\langle i||D||f\rangle|^2$, and G_F is the Fermi constant; $g_i(T)$ is the Boltzmann thermal occupation factor of the i-th nuclear initial state.

Within the LSSM approach an explicit calculation of $\sigma_{\nu A}(E_\nu)$ appeared to be impracticable. To overcome this difficulty, the approximate procedure was adopted in [3]. The total cross section was split into two parts — the down-scattering part $\sigma^d_{\nu A}(E_\nu)$ and the up-scattering part $\sigma^{up}_{\nu A}(E_\nu)$. The term $\sigma^d_{\nu A}(E_\nu)$ includes transitions when the scattered neutrino loses its energy $(E_f > E_i)$ whereas the term $\sigma^{up}_{\nu A}(E_\nu)$ includes the transitions when a scattered neutrino gets an energy from a hot nucleus $(E_f < E_i)$. Assuming the validity of the Brink hypothesis for the GT_0 resonance the down-scattering term was transformed to the weighted sum over only those final nuclear states which are coupled by the direct GT_0 transition with the nuclear ground state. As a result, $\sigma^d_{\nu A}(E_\nu)$ appeared to be independent of T.

Thus, within the LSSM approach the equation for $\sigma_{\nu A}(E_\nu)$ has the form

$$\sigma_{\nu A}(E_\nu) = \sigma^{d}_{\nu A}(E_\nu) + \sigma^{up}_{\nu A}(E_\nu) = \frac{G_F^2}{4\pi}\left[\sum_f (E_\nu - E_f)^2\, B_{0f}(GT_0) + \sum_{E_i > E_f} \left[E_\nu + (E_i - E_f)\right]^2 B_{if}(GT_0)\, g_i(T)\right]. \quad (3)$$

3 The Formalism

The thermo field dynamics (TFD) [6–8] is the real time formalism to treat thermal effects in quantum field and many-body theories.

Within the TFD the grand canonical average of a given operator A is calculated as the expectation value in a specially constructed, temperature-dependent state $|0(T)\rangle$ which is termed the thermal vacuum. In this sense, the thermal vacuum describes the thermal equilibrium of the system.

Such a "vacuum state" cannot be constructed as long as one stays in the Hilbert space of the original many-body system. This aim can be achieved by a formal doubling of the system degrees of freedom. That is, for the system governed by the Hamiltonian H the fictitious "tilde"-system identical with the original one is introduced. The quantities associated with the fictitious system are marked by the tilde.

The essential ingredients of the TFD are the thermal Hamiltonian $\mathcal{H} = H - \widetilde{H}$ and the thermal vacuum state $|0(T)\rangle$. An excitation spectrum of a hot nucleus is obtained by diagonalization of $\mathcal{H}$. At the same time, its thermal behavior is controlled by the thermal vacuum state which is the eigenstate of $\mathcal{H}$ with the zero eigenvalue. The thermal vacuum has to satisfy the thermal state condition [6–8]

$$A|0(T)\rangle = \sigma\, \mathrm{e}^{\mathcal{H}/2T}\, \widetilde{A}^\dagger |0(T)\rangle, \quad (4)$$

where $\sigma = 1$ for bosonic A and $\sigma = -i$ for fermionic A.

Applying the TFD formalism to a hot nucleus we follow Refs. [9, 10].

3.1 The Model Thermal Hamiltonian

We employ the model Hamiltonian of the Quasiparticle-Phonon Model (QPM) H_{QPM} [11] which consists of proton and neutron mean fields H_{sp}, the BCS pairing interactions H_{pair} and isoscalar and isovector separable particle-hole interactions. Since the process under study involves nuclear excitations of abnormal parity, the separable spin-multipole interactions H^{ph}_{SM} are used in the particle-hole channel.

$$H_{QPM} = H_{sp} + H_{pair} + H^{ph}_{SM}. \quad (5)$$

The three terms of H_{QPM} read

$$H_{sp} = \sum_{\tau=p,n} \sum_{jm}^{\tau} (E_j - \lambda_\tau) a^\dagger_{jm} a_{jm} \,,$$

$$H_{pair} = -\frac{1}{4} \sum_{\tau=p,n} G_\tau \sum_{\substack{jm \\ j'm'}}^{\tau} a^\dagger_{jm} a^\dagger_{\overline{jm}} a_{\overline{j'm'}} a_{j'm'},$$

$$H^{ph}_{SM} = -\frac{1}{2} \sum_{L\lambda} (\kappa_0^{(L\lambda)} + \kappa_1^{(L\lambda)} \vec{\tau}_1 \vec{\tau}_2) \sum_{\mu} S^\dagger_{L\lambda\mu} S_{L\lambda\mu},$$

where

$$S^\dagger_{L\lambda\mu} = \sum_{\tau=p,n} \sum_{\substack{jm \\ j'm'}}^{\tau} \langle jm | i^L R_L(r) [Y_L \vec{\sigma}]^\lambda_\mu | j'm' \rangle a^\dagger_{jm} a_{j'm'} \,.$$

The quantum numbers j, m actually represent the complete set of single-particle quantum numbers n, l, j, m, τ ($\tau = n, p$ is the isotopic index) and $a_{\overline{jm}} = (-1)^{j-m} a_{j-m}$. The notation $\sum^\tau$ implies a summation over neutron ($\tau = n$) or proton ($\tau = p$) single-particle states only.

To determine a thermal behavior of a nucleus governed by the Hamiltonian (5), we should diagonalize the thermal Hamiltonian $\mathcal{H}_{QPM} = H_{QPM} - \widetilde{H}_{QPM}$ and find the corresponding thermal vacuum state. This will be made in two steps.

3.2 Thermal Quasiparticles

At the first step, the sum of single-particle and pairing terms

$$\mathcal{H}_{BCS} = H_{sp} + H_{pair} - \widetilde{H}_{sp} - \widetilde{H}_{pair}$$

is diagonalized. To this aim, the two subsequent unitary transformations of the original $a^\dagger_{jm}, a_{jm}$ and tilde $\widetilde{a}^\dagger_{jm}, \widetilde{a}_{jm}$ single-particle operators are made. The first one is the usual Bogoliubov transformation to the Bogoliubov quasiparticles $\alpha^\dagger_{jm}, \alpha_{jm}$ and their tilde counterparts $\widetilde{\alpha}^\dagger_{jm}, \widetilde{\alpha}_{jm}$

$$\begin{aligned} \alpha^\dagger_{jm} &= u_j a^\dagger_{jm} - v_j a_{\overline{jm}} \\ \widetilde{\alpha}^\dagger_{jm} &= u_j \widetilde{a}^\dagger_{jm} - v_j \widetilde{a}_{\overline{jm}}. \end{aligned} \tag{6}$$

The second transformation mixes the original and tilde degrees of freedom

$$\begin{aligned} \beta^\dagger_{jm} &= x_j \alpha^\dagger_{jm} - i y_j \widetilde{\alpha}_{jm} \\ \widetilde{\beta}^\dagger_{jm} &= x_j \widetilde{\alpha}^\dagger_{jm} + i y_j \alpha_{jm}. \end{aligned} \tag{7}$$

The coefficients x_j, y_j are temperature dependent. The transformation (7) is termed the thermal Bogoliubov transformation and the operators $\beta^\dagger_{jm}, \beta_{jm}$ are named the thermal quasiparticle operators. Since both the transformations are the unitary ones, the following constraints should be fulfilled: $u_j^2+v_j^2=1$ and $x_j^2+y_j^2=1$.

Applying the principle of compensation of dangerous diagrams (see, e.g., [11]) one gets the coefficients u_j, v_j

$$\begin{pmatrix} u_j \\ v_j \end{pmatrix} = \frac{1}{\sqrt{2}}\left(1\pm\frac{E_j-\lambda_\tau}{\sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}}\right)^{1/2} \tag{8}$$

In Eq. (8), the energy gap Δ_τ and the chemical potential λ_τ are the functions of the coefficients x_j, y_j and obey the following equations:

$$\Delta_\tau = \frac{G_\tau}{2}\sum_j{}^\tau (2j+1)(x_j^2-y_j^2)u_jv_j \tag{9}$$

$$N_\tau = \sum_j{}^\tau (2j+1)(v_j^2x_j^2+u_j^2y_j^2). \tag{10}$$

At this stage the thermal BCS Hamiltonian $\mathcal{H}_{BCS}$ appears to be diagonal

$$\mathcal{H}_{BCS} = \mathcal{H}_{sp}+\mathcal{H}_{pair} \simeq \sum_\tau \sum_{jm}{}^\tau \varepsilon_j(T)(\beta^\dagger_{jm}\beta_{jm}-\widetilde{\beta}^\dagger_{jm}\widetilde{\beta}_{jm}),$$

where $\varepsilon_j(T)$ is the quasiparticle energy, $\varepsilon_j(T)=\sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}$. However, the coefficients x_j, y_j are still unknown.

To find x_j, y_j and to determine the thermal vacuum $|0(T);qp\rangle$ in the thermal BCS approximation the thermal state condition (4) is used in the form

$$a_{jm}|0(T);qp\rangle = i\,\exp\left(\frac{\mathcal{H}_{BCS}}{2T}\right)\widetilde{a}^\dagger_{jm}|0(T);qp\rangle.$$

As a result one gets

$$y_j = \left[1+\exp\left(\frac{\varepsilon_j}{T}\right)\right]^{-1/2}, \quad x_j = \left(1-y_j^2\right)^{1/2}. \tag{11}$$

Moreover, $\beta_{jm}|0(T);qp\rangle = \widetilde{\beta}_{jm}|0(T);qp\rangle = 0$.

Thus, the coefficients x_j, y_j are connected with the thermal occupation numbers of the Bogoliubov quasiparticles. Equations (8-11) are the equations of the thermal BCS approximation.

3.3 Thermal Phonons

Now the thermal Hamiltonian reads

$$\mathcal{H}_{QPM} = \sum_\tau \sum_{jm}{}^\tau \varepsilon_j(\beta^\dagger_{jm}\beta_{jm}-\widetilde{\beta}^\dagger_{jm}\widetilde{\beta}_{jm}) - \tag{12}$$

$$-\frac{1}{2}\sum_{L\lambda\mu}\sum_{\tau,\rho=\pm1}(\kappa_0^{(L\lambda)}+\rho\kappa_1^{(L\lambda)})\left\{S^\dagger_{L\lambda\mu}(\tau)S_{L\lambda\mu}(\rho\tau)-\widetilde{S}^\dagger_{L\lambda\mu}(\tau)\widetilde{S}_{L\lambda\mu}(\rho\tau)\right\}.$$

At the second step, we diagonalize a part of the Hamiltonian (12) in a basis of thermal phonon operators

$$Q^{\dagger}_{\lambda\mu i}=\frac{1}{2}\sum_{\tau}\sum_{j_1j_2}^{\tau}\Bigl(\psi^{\lambda i}_{j_1j_2}[\beta^{\dagger}_{j_1}\beta^{\dagger}_{j_2}]^{\lambda}_{\mu}+\widetilde{\psi}^{\lambda i}_{j_1j_2}[\widetilde{\beta}^{\dagger}_{\overline{j_1}}\widetilde{\beta}^{\dagger}_{\overline{j_2}}]^{\lambda}_{\mu}+2i\,\eta^{\lambda i}_{j_1j_2}[\beta^{\dagger}_{j_1}\widetilde{\beta}^{\dagger}_{\overline{j_2}}]^{\lambda}_{\mu}\Bigr)$$
$$+\Bigl(\phi^{\lambda i}_{j_1j_2}[\beta_{\overline{j_1}}\beta_{\overline{j_2}}]^{\lambda}_{\mu}+\widetilde{\phi}^{\lambda i}_{j_1j_2}[\widetilde{\beta}_{j_1}\widetilde{\beta}_{j_2}]^{\lambda}_{\mu}+2i\,\xi^{\lambda i}_{j_1j_2}[\beta_{\overline{j_1}}\widetilde{\beta}_{j_2}]^{\lambda}_{\mu}\Bigr)\,.$$

Moreover, we define new thermal vacuum $|0(T);ph\rangle$ as the vacuum for thermal phonons: $Q_{\lambda\mu i}|0(T);ph\rangle = 0$, $\widetilde{Q}_{\lambda\mu i}|0(T);ph\rangle = 0$.

Thermal phonon operators are treated as bosonic ones, i.e.

$$[Q_{\lambda'\mu'k'},\,Q^{\dagger}_{\lambda\mu k}]=\delta_{\lambda'\lambda}\delta_{\mu'\mu}\delta_{k'k},\quad [\widetilde{Q}_{\lambda'\mu'k'},\,Q^{\dagger}_{\lambda\mu k}]=0,\quad \textit{etc.}$$

This imposes the following constraint on the amplitudes ψ, $\widetilde{\psi}$, η, *etc.*:

$$\frac{1}{2}\sum_{\tau}\sum_{j_1j_2}^{\tau}(\psi^{\lambda k'}_{j_1j_2}\psi^{\lambda k}_{j_1j_2}+\widetilde{\psi}^{\lambda k'}_{j_1j_2}\widetilde{\psi}^{\lambda k}_{j_1j_2}+2\eta^{\lambda k'}_{j_1j_2}\eta^{\lambda k}_{j_1j_2}$$
$$-\phi^{\lambda k'}_{j_1k_2}\phi^{\lambda k}_{j_1j_2}-\widetilde{\phi}^{\lambda k'}_{j_1k_2}\widetilde{\phi}^{\lambda k}_{j_1j_2}-2\xi^{\lambda' k'}_{j_1j_2}\xi^{\lambda k}_{j_1j_2})=\delta_{k'k}. \tag{13}$$

The energies of thermal phonons can be found, e.g., applying the variational principle under the constraint (13). The corresponding secular equation for thermal phonon energies reads

$$\frac{2\lambda+1}{\kappa_1^{(L,\lambda)}}=\sum_{\tau}\sum_{j_1\,j_2}^{\tau}(f^{(L,\lambda)}_{j_1j_2})^2\left[\frac{(u^{(-)}_{j_1j_2})^2\varepsilon^{(+)}_{j_1j_2}(1-y^2_{j_1}-y^2_{j_2})}{(\varepsilon^{(+)}_{j_1j_2})^2-\omega^2}\right.$$
$$\left.-\frac{(v^{(+)}_{j_1j_2})^2\varepsilon^{(-)}_{j_1j_2}(y^2_{j_1}-y^2_{j_2})}{(\varepsilon^{(-)}_{j_1j_2})^2-\omega^2}\right], \tag{14}$$

where $\varepsilon^{(\pm)}_{j_1j_2}=\varepsilon_{j_1}\pm\varepsilon_{j_2}$, $u^{(-)}_{j_1j_2}=u_{j_1}v_{j_2}-v_{j_1}u_{j_2}$, $v^{(+)}_{j_1j_2}=u_{j_1}u_{j_2}+v_{j_1}v_{j_2}$, and $f^{(L,\lambda)}_{j_1j_2}$ is the reduced single-particle matrix element of the spin-multipole operator $S_{L\lambda\mu}$. Equation (14) is the equation of the thermal quasiparticle random phase approximation[1].

Then the Hamiltonian (12) approximately takes a diagonal form

$$\mathcal{H}_{TQRPA}=\sum_{\lambda\mu k}\omega_{\lambda k}(T)\left(Q^{\dagger}_{\lambda\mu k}Q_{\lambda\mu k}-\widetilde{Q}^{\dagger}_{\lambda\mu k}\widetilde{Q}_{\lambda\mu k}\right). \tag{15}$$

However, since $\mathcal{H}_{TQRPA}$ (15) is invariant with respect to the unitary transformation

$$Q^{\dagger}_{\lambda\mu i}\to X_{\lambda i}Q^{\dagger}_{\lambda\mu i}-Y_{\lambda i}\widetilde{Q}_{\lambda\mu i},\quad \widetilde{Q}^{\dagger}_{\lambda\mu i}\to X_{\lambda i}\widetilde{Q}^{\dagger}_{\lambda\mu i}-Y_{\lambda i}Q_{\lambda\mu i},$$

[1] Strictly speaking, Eq. (14) is valid when all the particle-hole coupling constants except one $\kappa_1^{(L,\lambda)}$ vanish in the term H^{ph}_{SM} of (5) and (12).

where $X^2_{\lambda i} - Y^2_{\lambda i} = 1$, the phonon amplitudes ψ, $\widetilde{\psi}$, η, *etc.* as well as the phonon vacuum $|0(T);ph\rangle$ are not determined unambiguously yet (see Refs. [9, 10] for more details).

The "true" thermal vacuum of the TQRPA is found by meeting the thermal state condition in the form

$$S_{L\lambda\mu}|0(T);ph\rangle = \exp\left(\frac{\mathcal{H}_{QRPA}}{2T}\right)\widetilde{S}^{\dagger}_{L\lambda\mu}|0(T);ph\rangle.$$

Then one gets for $X_{\lambda i}$, $Y_{\lambda i}$

$$Y_{\lambda i} = \left[\exp\left(\frac{\omega_{\lambda i}}{T}\right) - 1\right]^{-1/2}; \quad X_{\lambda i} = [1 + Y^2_{1i}]^{1/2} \tag{16}$$

and phonon amplitudes $\psi, \widetilde{\psi}, \eta$, *etc.* appear to be dependent on both the types of the thermal occupation numbers — the quasiparticle ones x_i, y_i (11) and the phonon ones $X_{\lambda i}, Y_{\lambda i}$ (16). The expressions for the phonon amplitudes can be found in [9].

3.4 Transition Probabilities and the INNS Cross Section

Considering neutrino-nucleus inelastic scattering in stellar environments we assume that the nucleus is in a thermal equilibrium state treated as the thermal (phonon) vacuum. The scattering leads to transition from the thermal vacuum to thermal one-phonon states. In the case of a neutrino down-scattering, a one-phonon state $Q^{\dagger}_{\lambda i}|0(T);ph\rangle$ is excited. For a neutrino up-scattering the nucleus is de-excited. This process is treated as the transition to a tilde one-phonon state $\widetilde{Q}^{\dagger}_{\lambda i}|0(T);ph\rangle$ lying lower than the thermal vacuum.

The corresponding transition matrix elements are

$$\begin{aligned} \text{Excitation :} \qquad & \Phi(\omega_{\lambda i}) = \left|\langle Q_{\lambda i}\|D_{\lambda i}\|0(T);ph\rangle\right|^2 \\ \text{De-excitation :} \qquad & \widetilde{\Phi}(-\omega_{\lambda i}) = \left|\langle \widetilde{Q}_{\lambda i}\|D_{\lambda i}\|0(T);ph\rangle\right|^2 \end{aligned}$$

where $D_{\lambda i}$ is given in Eq. (1).

Thus, within the present approach the INNS cross section reads

$$\begin{aligned} \sigma_{\nu A}(E_\nu) &= \sigma^{d}_{\nu A}(E_\nu) + \sigma^{up}_{\nu A}(E_\nu) = \\ &\frac{G_F^2}{4\pi}\sum_i (E_\nu - \omega_{\lambda i})^2 \Phi_{\lambda i} + \frac{G_F^2}{4\pi}\sum_i (E_\nu + \omega_{\lambda i})^2 \widetilde{\Phi}_{\lambda i} = \\ &\frac{G_F^2}{4\pi}\sum_i \Phi_{\lambda i}\left\{(E_\nu - \omega_{\lambda i})^2 + \exp\left(-\frac{\omega_{\lambda i}}{T}\right)(E_\nu + \omega_{\lambda i})^2\right\}. \end{aligned} \tag{17}$$

Transforming the second line in the above equation to the third one we take into account that $\widetilde{\Phi}_{\lambda i} = \exp(-\omega_{\lambda i}/T)\Phi_{\lambda i}$ (see Ref. [9]).

In contrast with Eq. (3) the down-scattering term $\sigma^{d}_{\nu A}(E_\nu)$ in our approach depends on temperature T.

4 Calculations for the Hot Nucleus ^{54}Fe

Numerical calculations are performed for the ^{54}Fe nucleus.

Single-particle wave functions and energies were calculated in the spherically symmetric Woods-Saxon potential. The constants of pairing interaction were determined to reproduce experimental pairing energies in the BCS approximation. All these parameters are the same as in our previous calculations [12–14] of electron capture rates on the same nuclide in stellar environment.

The GT$_0$ resonance in a nucleus is generated by the spin-dependent part of the effective nuclear interaction. In interactions of nuclei with electromagnetic probes the GT$_0$ resonance reveals itself as the $M1$-resonance. In the framework of the QRPA the structure and the energy of the $M1$-resonance are determined by the spin-monopole (or spin-spin) and spin-quadrupole isoscalar and isovector terms of the Hamiltonian (5). To determine the corresponding four coupling constants $\kappa_{0,1}^{(01)}$ and $\kappa_{0,1}^{(21)}$, we take advantage of the old QPM results for magnetic resonances [11, 15, 16].

Spin-quadrupole forces do not affect noticeably one-phonon 1^+ states with energies $\omega_{1i} \leq 15$ MeV and, therefore, the corresponding terms in (5) can be omitted, i.e. $\kappa_{0,1}^{(21)} = 0$. The radial formfactor $R_0(r)$ of the spin-spin forces is taken to be equal to 1 (no radial dependence). According to estimations in Refs. [16, 17], the isoscalar spin-spin interaction is very weak in comparison with the isovector one. Therefore, we actually need to determine only one coupling constant — $\kappa_1^{(01)}$. This is done by fitting the theoretical position of $M1$ resonance to its experimental value [18]. It is worth mentioning that the obtained value of $\kappa_1^{(01)}$ is in appropriate agreement with that obtained from the position of charge-exchange GT$_\pm$ resonances built on ^{54}Fe [13, 14].

The effective axial coupling constant was taken the same as in the shell-model calculations [3] $(g_A/g_V)_{eff} = 0.74\,(g_A/g_V)_{bare}$.

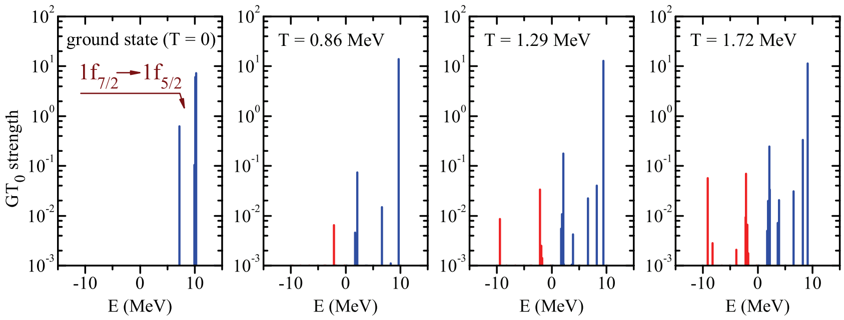

Figure 1. GT$_0$ strength distributions in the ^{54}Fe nucleus at different temperatures. Transition energy is shown along the abscissa axis.

Figure 1 shows distributions of GT_0 transition strengths for the ground state of ^{54}Fe and for three values of T occurring at different collapse stages: $T = 0.86$ MeV corresponds to the condition in the core of a presupernova model for a $15M_{\odot}$ star; $T = 1.29$ MeV and $T = 1.72$ MeV relate approximately to neutrino trapping and neutrino thermalization stages, respectively.

At $T = 0$, the transition strength is concentrated mostly in two one-phonon 1^+ states forming the GT_0 resonance near $E_x \approx 10$ MeV. The main contribution to phonon structures comes from the proton and neutron single-particle transitions $1f_{7/2} \to 1f_{5/2}$, and the proton transition $2p_{3/2} \to 2p_{1/2}$ which is due to pairing correlations in the proton subsystem. With temperature increase the fraction of low-energy transitions in the GT_0 strength distribution increases. The reasons are weakening and the subsequent collapse of pairing correlations (at $T \approx 0.8$ MeV) and appearance of low-energy particle-particle and hole-hole transitions due to thermal smearing of neutron and proton Fermi surfaces. Moreover, at finite temperature the "negative energy" transitions appear. As a result the resonance energy centroid is shifted down by 1.1 MeV at $T = 1.72$ MeV. This indicates a violation of the Brink hypothesis in the present approach.

The strength distributions displayed in Figure 1 are used to calculate the cross section of INNS off ^{54}Fe. The results are shown in Figure 2. As in the LSSM calculations [3], at $T = 0$ the cross section $\sigma_{\nu A}(E_\nu)$ is equal to zero when the neutrino energy is less than the energy of the lowest 1^+ state in ^{54}Fe. Within the QRPA, the lowest 1^+ state in ^{54}Fe has the excitation energy $E_x(1^+) \sim 7$ MeV (see Figure 1).

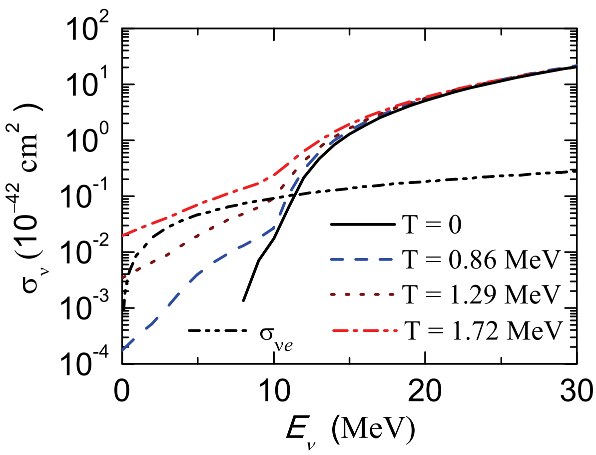

Figure 2. Cross section of inelastic neutrino scattering off ^{54}Fe as a function of neutrino energy E_ν at $T = 0$ (solid line), $T = 0.86$ MeV (dashed line), $T = 1.29$ MeV (dotted line), and $T = 1.72$ MeV (dashed–dotted line). The cross section $\sigma_{\nu e}$ of neutrino scattering off an electron is shown by the dashed-double-dotted line.

At $T \neq 0$ the INNS cross section does not vanish at any neutrino energy and increases quite rapidly with temperature for neutrinos with energies $E_\nu < 10$ MeV. The values of $\sigma_{\nu A}(E_\nu)$ presented in Figure 2 agree well with the results of the LSSM calculations [3].

When the neutrino energy exceeds the energy of the GT_0 resonance ($E_\nu \geq 10$ MeV) the dependence of the cross section $\sigma_{\nu A}(E_\nu)$ on E_ν becomes smoother. At these neutrino energies temperature does not affect $\sigma_{\nu A}(E_\nu)$ any more. Due to thermal effects $\sigma_{\nu A}(E_\nu)$ becomes larger than the cross section of inelastic neutrino-electron scattering at $T \sim 1.6 - 1.7$ MeV even at low neutrino energies $E_\nu \lesssim 10$ MeV. These features were noted in [3] as well.

5 Conclusions

We have investigated the temperature dependence of the cross section of inelastic neutrino-nucleus scattering off the hot nucleus ^{54}Fe. Thermal effects were treated within the thermal quasiparticle random phase approximation in the context of the formalism of the thermo field dynamics. These studies are relevant for supernova simulations.

The present studies are based on the formalism which seems to be quite different from that of the large-scale shell-model approach [3,4]. Indeed, within the LSSM approach a hot nucleus is considered in the canonical ensemble whereas the TFD formalism treats a hot nucleus in the grand canonical ensemble. In contrast to the LSSM approach [3, 4] we do not assume the Brink hypothesis when treating the down-scattering component of the cross section $\sigma_{\nu A}(E_\nu)$ and the corresponding term $\sigma^d_{\nu A}(E_\nu)$ appears to be dependent on T in our calculations.

Despite of the above-mentioned differences, our calculations have revealed the same thermal effects as were found within the LSSM studies [3,4]: The temperature growth leads to a considerable increase in the INNS cross section for E_ν lower than the G_0 resonance energy, and at certain T the INNS cross section becomes larger than the neutrino-electron scattering cross section at $E_\nu < 10$ MeV. What is more, our values of $\sigma_{\nu A}(E_\nu)$ are very close to the LSSM values given in [4].

Thus, the results of our study show that the present approach provides a valuable tool for the evaluation of the inelastic neutrino-nucleus cross sections under stellar conditions.

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