NUCLEAR THEORY, Vol. 29 (2010)
eds. A. Georgieva, N. Minkov, Heron Press, Sofia

Momentum Distributions in Medium and Heavy Exotic Nuclei

M.K. Gaidarov[1], **G.Z. Krumova**[2], **P. Sarriguren**[3], **A.N. Antonov**[1], **M.V. Ivanov**[1], **E. Moya de Guerra**[4]

[1]Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
[2]University of Ruse, Ruse 7017, Bulgaria
[3]Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain
[4]Departamento de Fisica Atómica, Molecular y Nuclear, Facultad de Ciencias Fisicas, Universidad Complutense de Madrid, E-28040 Madrid, Spain

Abstract. Nucleon momentum distributions of even-even isotopes of Ni, Kr, and Sn are studied in the framework of deformed self-consistent mean-field Skyrme HF+BCS method, the theoretical approach based on the light-front dynamics and the theoretical method based on the local density approximation. The isotopic sensitivities of the calculated neutron and proton momentum distributions are investigated together with the effects of pairing and nucleon-nucleon correlations. The role of deformation on the momentum distributions in even-even Kr isotopes is discussed. For comparison, the results for the momentum distribution in nuclear matter are also given.

1 Introduction

Exotic nuclei are known to exhibit qualitatively new phenomena and provide a new testing ground for the understanding of quantum many-body science. It is a challenge for the nuclear theory to study entirely new nuclear topologies comprising, *e.g.* regions of nearly pure neutron matter and exotic nuclear shapes, new types and phases of nucleonic matter and others.

The study of exotic nuclei is inspired by the recent development of radioactive ion beam facilities in GSI (Germany) [1] and RIKEN (Japan) [2, 3] that offer the worldwide unique opportunity to use electrons as probe particles in investigations of the structure of these nuclei. In Ref. [4] we studied charge form factors of light exotic nuclei (^{6,8}He, ^{11}Li, ^{14}Be, ^{17,19}B) using various theoretical predictions of their charge densities. In Ref. [5] our calculations of the charge form factors of exotic nuclei were extended from light (He, Li) to medium and heavy nuclei (Ni, Kr, and Sn). For the Ni, Kr, and Sn isotopes the densities have been obtained in the deformed self-consistent mean-field Hartree-Fock (HF)+BCS method with density-dependent (DD) Skyrme interaction [6–8]. We refer to this mean field approach as DDHF+BCS. A detailed study of the charge radii and neutron skin in Ni, Kr, and Sn nuclei, as well as of the formation of the proton skin, has been performed within the same method in [9].

Another important characteristic of the nuclear ground state is the nucleon momentum distribution (NMD) $n(k)$. The scaling analyses of inclusive electron scattering from a large variety of nuclei (see, e.g. [10–13]) demonstrated the existence of high-momentum components of NMD at momenta $k > 2$ fm^{-1}. It has been shown [14–17] that it is due to the effects of nucleon-nucleon (NN) correlations in nuclei (for a review, see e.g. [18]). It has been pointed out that this specific feature of $n(k)/A$ is similar for all nuclei, and that it is a physical reason for the scaling and superscaling phenomena in nuclei. As known [18, 19], the mean-field approximation (MFA) is unable to describe simultaneously the two important characteristics of the nuclear ground state, the density and momentum distribution. Therefore, a consistent analysis of the effects of the NN correlations on both quantities is required using theoretical methods beyond the MFA in the description of relevant phenomena, e.g. the scaling ones. The self-consistent DDHF approximation has been applied in [7] to calculate NMD in spherical and deformed Nd isotopes, studying the effects of deformation, as well as those of pairing and of dynamical short-range NN correlations.

The main aim of our work (see also [20]) is to calculate the NMD for the same isotopic chains of neutron-rich nuclei (Ni, Kr, and Sn) for which we had studied charge densities, radii, form factors, halo, and skin in our previous works [5] and [9]. The mean-field contributions to $n(k)$ in these nuclei are calculated within the DDHF+BCS approach. The remaining effects of the NN correlations are considered in two ways, namely, within the approach (see [16, 17, 21]) using the light-front dynamics (LFD) method (e.g., [22]) and in that [23] based on the local density approximation (LDA). Several questions are investigated, such as the sensitivity of $n(k)$ to all details of the calculations, e.g.: (i) to different types of Skyrme forces; (ii) to the pairing correlation effects; (iii) to the effects of nuclear deformation; (iv) to the strength of the NN correlations included in the LFD and LDA approaches (respectively, to the values of the correlations strength parameters β and γ). A special attention is paid to the isotopic and isotonic sensitivity of the proton and neutron momentum distributions. The results for $n(k)$ in the exotic nuclei are compared with that in nuclear matter (NM).

2 Theoretical Framework

2.1 Deformed Skyrme HF+BCS Formalism

Some of the results have been obtained from self-consistent deformed Hartree-Fock calculations with density-dependent Skyrme interaction [8] and pairing correlations. Pairing between like nucleons has been included by solving the BCS equations at each iteration either with a fixed pairing gap parameter (determined from the odd-even experimental mass differences) or with a fixed pairing strength parameter. We consider in this paper the Skyrme force SLy4 that gives an appropriate description of bulk properties of spherical and deformed nuclei.

Following the formalism given in Ref. [7] the single-particle Hartree-Fock wave functions in momentum space $\tilde{\Phi}_i(\vec{k},\sigma,q)$ can be expressed as

$$\tilde{\Phi}_i(\vec{k},\sigma,q) = \chi_{q_i}(q) \sum_{\alpha} C_{\alpha}^i \tilde{\phi}_{\alpha}(\vec{k},\sigma). \tag{1}$$

Similarly to the spin-independent proton, neutron and total densities, we define in momentum space the proton, neutron and total momentum distributions by

$$n(\vec{k}) = n(k_{\perp}, k_z) = \sum_i 2v_i^2 n_i(k_{\perp}, k_z)\,, \tag{2}$$

where v_i^2 are the occupation probabilities resulting from the BCS equations and $k_{\perp}, k_z$ are the cylindrical coordinates of $\vec{k}$. The single-particle momentum distributions $n_i(\vec{k})$ are given by [7]

$$n_i(\vec{k}) = n_i(k_{\perp}, k_z) = |\tilde{\Phi}_i^{+}(k_{\perp}, k_z)|^2 + |\tilde{\Phi}_i^{-}(k_{\perp}, k_z)|^2\,. \tag{3}$$

2.2 Methods Going Beyond the MFA: LFD and LDA Approaches

Here the effects of NN correlations accounted for in two correlation methods on the high-momentum contributions to the nucleon momentum distribution are considered.

Using the natural-orbital representation of the one-body density matrix [24], $n(k)$ is written as a sum of the contributions from the hole-states $[\tilde{n}^h(k)]$ (states up to the Fermi level (F.L.)) and the particle-states $[\tilde{n}^p(k)]$ (see also [16]) for protons (Z) and neutrons (N):

$$n_{Z(N)}(k) = \tilde{n}^h_{Z(N)}(k) + \tilde{n}^p_{Z(N)}(k), \tag{4}$$

where

$$\tilde{n}^h_{Z(N)}(k) = \frac{C(k)}{Z(N)} \sum_{nlj}^{F.L.} 2(2j+1)\lambda_{nlj} |\tilde{R}_{nlj}(k)|^2 \tag{5}$$

and

$$\tilde{n}^p_{Z(N)}(k) = \frac{C(k)}{Z(N)} \sum_{F.L.}^{\infty} 2(2j+1)\lambda_{nlj} |\tilde{R}_{nlj}(k)|^2. \tag{6}$$

In our work we substitute $\tilde{n}^h_{Z(N)}(k)$ in (4) and (5) by

$$\tilde{n}^h_{Z(N)}(k) = \frac{C(k)}{Z(N)} \tilde{n}_{Z(N)}(k), \tag{7}$$

where $\tilde{n}_{Z(N)}(k)$ is expressed by the NMD $n^{DDHF}_{Z(N)}(k)$ obtained within the DDHF formalism

$$\tilde{n}_{Z(N)}(k) = \frac{Z(N) n^{DDHF}_{Z(N)}(k)}{\int d\vec{k}' C(k') n^{DDHF}_{Z(N)}(k')}. \tag{8}$$

In (5)-(8) $C(k) = m_N/[(2\pi)^3\sqrt{k^2+m_N^2}]$. According to the assumption made in [16, 17, 21], the particle-state contribution $[\tilde{n}^p_{Z(N)}(k)]$ in (4) and (6) can be substituted by (up to a normalization factor):

$$\tilde{n}^{\mathrm{p}}_{Z(N)}(k) = \beta\big[n_2(k) + n_5(k)\big], \tag{9}$$

where β is a parameter, and $n_2(k)$ and $n_5(k)$ are expressed by angle-averaged corresponding components of the deuteron wave function [21].

Finally, the normalized to unity proton (neutron) momentum distribution has the form:

$$n_{Z(N)}(k) = \frac{C(k)\tilde{n}_{Z(N)}(k) + Z(N)\beta\big[n_2(k) + n_5(k)\big]}{\int d\vec{k}'\Big\{C(k')\tilde{n}_{Z(N)}(k') + Z(N)\beta\big[n_2(k') + n_5(k')\big]\Big\}}. \tag{10}$$

Next, according to Ref. [23], one can introduce proton (neutron) momentum distribution within the local density approximation in the form:

$$n_{Z(N)}(k) = n^{MFA}_{Z(N)}(k) + \delta n_{Z(N)}(k), \tag{11}$$

where $n^{MFA}_{Z(N)}(k)$ is the mean-field contribution, whereas $\delta n_{Z(N)}(k)$ embodies the corrections due to dynamical correlations not included in the MFA. If one applies the LDA to the second term of (11), the nucleon momentum distribution $n_{Z(N)}(k)$ can be written in the form:

$$n_{Z(N)}(k) = n^{MFA}_{Z(N)}(k) + \frac{1}{4\pi^3}\int \delta\nu(k_F^{Z(N)}(r), k)d\vec{r}, \tag{12}$$

where $\delta\nu(k_F^{Z(N)}(r), k)$ corresponds to the occupation probability that is entirely due to the effects of dynamical correlations induced by the NN interaction. The local Fermi momentum $k_F^{Z(N)}(r)$ is related to the proton (neutron) density through the relation

$$k_F^{Z(N)}(r) = \big[3\pi^2\rho_{Z(N)}(r)\big]^{1/3}. \tag{13}$$

By definition of $k_F^{Z(N)}(r)$ one has $\int \delta\nu(k_F^{Z(N)}(r), k)d\vec{k} = 0$. Choosing a correlation function of the form $f(r) = 1 - e^{-\gamma^2 r^2}$ and using the lowest-order cluster approximation an analytical expression for $\delta\nu(k_F^{Z(N)}, k)$ can be obtained [23].

As in the approach based on the LFD method discussed above, in our work we use for $n^{MFA}_{Z(N)}(k)$ the momentum distributions obtained from DDHF calculations $n^{DDHF}_{Z(N)}(k)$. For the densities $\rho_{Z(N)}(r)$ entering (13) we use the HF+BCS proton (neutron) densities [9].

3 Results of Calculations and Discussion

A comparison of the results for the neutron and proton momentum distributions of ^{64}Ni, ^{84}Kr, and ^{120}Sn nuclei obtained in the correlation methods is given in

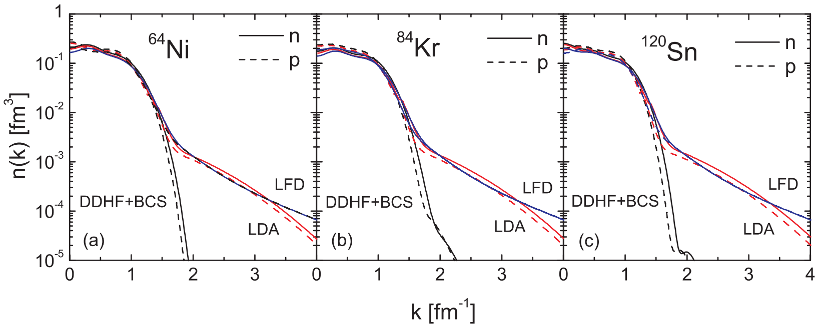

Figure 1. Neutron (solid line) and proton (dashed line) momentum distributions obtained within the DDHF+BCS (black), LFD (blue), and LDA (red) methods for ^{64}Ni (a), ^{84}Kr (b), and ^{120}Sn (c) nuclei. The normalization is: $\int n_{n(p)}(k)d\vec{k} = 1$.

Figure 1 together with the HF momentum distributions. As can be seen, for all nuclei the inclusion of NN correlations strongly affects the high-momentum region of NMD. At $k > 1.5$ fm^{-1} both LFD and LDA momentum distributions start to deviate from the DDHF+BCS case. They behave rather similar in the interval $1.5 < k < 3$ fm^{-1}. At $k > 3$ fm^{-1} the LFD method predicts systematically higher momentum components compared to LDA momentum distributions. This observation can be explained by the different extent to which NN correlations are taken into account in both approaches. Our results for the NMD's in the LFD method for large values of k ($k > 2$ fm^{-1}) are similar to those obtained within the Jastrow correlation method and, thus, the high-momentum tails of $n(k)$ are caused by the short-range NN correlation effects. The LDA approach through the nuclear matter dynamic effects and using the local Fermi momentum $k_F^{Z(N)}(r)$ calculated self-consistently by means of the HF density (13) produces less pronounced high-momentum tail, but still the results are very close to those obtained in the LFD method. As was already shown, at $k > 1.5$ fm^{-1} the DDHF+HF momentum distributions fall off rapidly by several orders of magnitude in contrast to the correlated NMD's. In addition, we observe that: (i) the results shown above are similar for all nuclei in a given isotopic chain and going from Ni to Sn isotopes, as well; (ii) the behavior of $n(k)$ is similar for protons and neutrons; (iii) at high k the proton and neutron NMD's obtained within the LFD method cannot be distinguished from each other because the high-momentum tails in this approach are determined by the high-momentum component of the nucleons in the deuteron [21]; (iv) concerning the NMD's calculated in the LDA approach, some difference between $n(k)$ for protons and neutrons can be observed due to $Z(N)$-dependence of the local Fermi momentum k_F.

In the next Figures 2 and 3 we show the neutron $n_n(k)$ and proton $n_p(k)$ momentum distributions of some selected isotopes in the Ni chain, the same

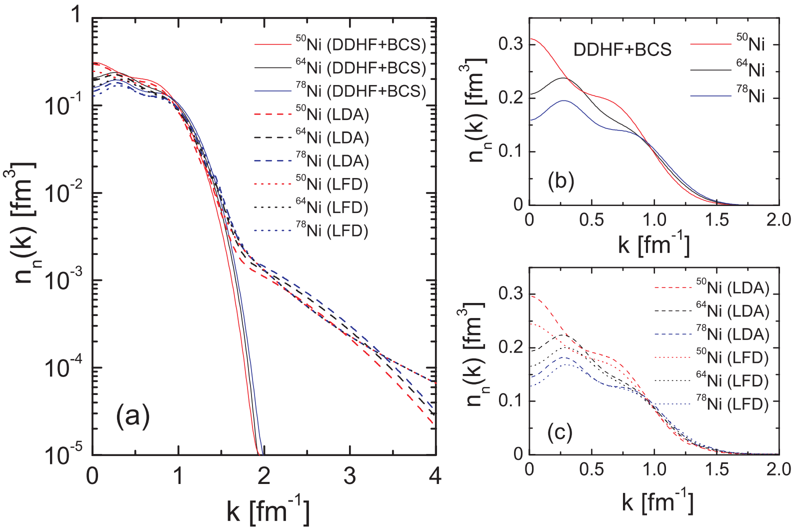

Figure 2. (a) Neutron momentum distributions obtained within the DDHF+BCS (solid line), LFD (dotted line), and LDA (dashed line) methods for ^{50}Ni, ^{64}Ni, and ^{78}Ni isotopes. The normalization is: $\int n_n(k)d\vec{k} = 1$. The DDHF+BCS results, as well as the LFD and LDA results are separately shown in a linear scale in (b) and (c), respectively.

which has been considered in Ref. [9] to calculate important nuclear properties in coordinate space. The results are presented in both logarithmic and linear scales in order to study the isotopic sensitivity of these momentum distributions in the high-momentum region and in the region of small momenta, respectively. In addition, in each of the figures the results for neutron and proton momentum distributions in the DDHF+BCS method and in the correlation LFD and LDA approaches at $k < 2$ fm^{-1} are given separately in panels (b) and (c).

It is seen from the figures that the evolution of the NMD's as we increase the number of neutrons consists of an increase of the high-momentum tails (for $k > 1.5$ fm^{-1}) of $n_n(k)$, while the effect on $n_p(k)$ is opposite. However, the spreading of the tails corresponding to $n_p(k)$ of the considered isotopes is of the same order although the number of protons remains the same. In this respect, the results presented in Figure 3 are challenging because they show how proton momentum distributions "feel" the different number of neutrons in exotic nuclei. We would like also to emphasize that the LFD method does not show this isotopic sensitivity, in contrast to the HF and LDA methods which still demonstrate this trend. Concerning the low-momentum region it can be seen from Figures 2 and 3 that NMD's are very sensitive to the details of the calculations. In this region $n_n(k)$ decreases while, on the contrary, $n_p(k)$ increases with the increase of the number of neutrons N. This is a common feature of the calculated results obtained in all methods. Nevertheless, in this region the spreading is consider-

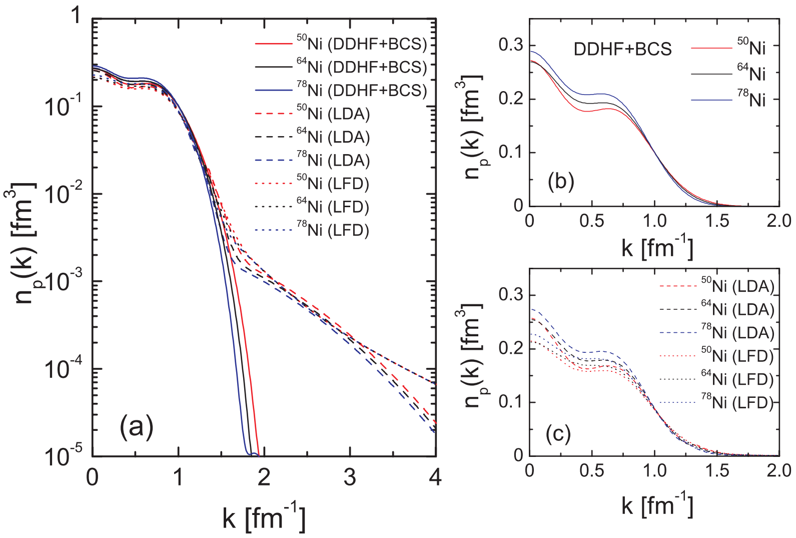

Figure 3. The same as in Figure 2, but for the proton momentum distributions.

ably reduced.

In Figure 4 a comparison of the results for the total momentum distribution $n(k)$ of nuclear matter calculated within the MFA and correlation methods used in our work is shown. The HF momentum distribution is strongly affected by pairing correlations which build up a long tail at high momentum ($k > k_F$). Comparing this result with the result illustrated in Ref. [20] for ^{84}Kr, the different role played by pairing correlations on the DDHF momentum distribution of NM and of finite nuclei becomes clear. Moreover, it is interesting to explore the case when one includes other type of correlations in NM. Stringari *et al.* [23] have already shown in their model based on the LDA the prediction for $n(k)$ in the case of nuclear matter. An enhancement of the high-momentum components of $n(k)$ can be seen from Figure 4 when both LDA and LFD methods are used. Hence, in nuclear matter the effects of short-range and tensor correlations are much stronger than the BCS-correlations taken into account in the DDHF+BCS calculations.

4 Conclusions

The theoretical investigation of the neutron, proton, and total momentum distributions of medium and heavy exotic nuclei, especially of Ni, Kr, and Sn even-even isotopes, was performed on the base of the mean-field method, as well as of two correlation methods taking into account the NN correlations at short distances.

The study of the isotopic sensitivity of various kinds of momentum distri-

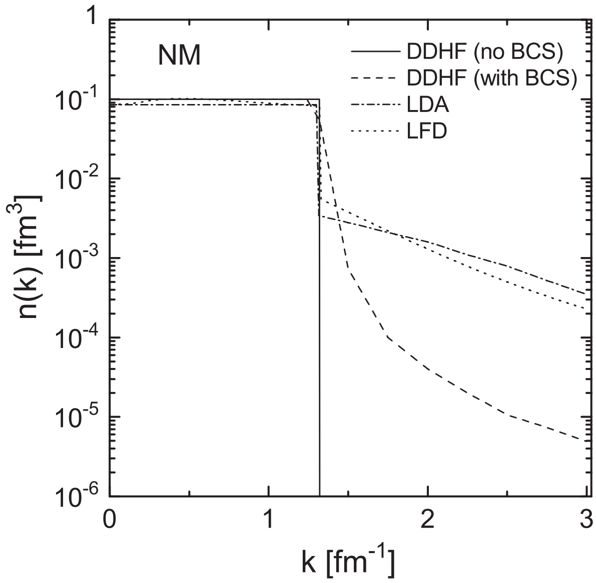

Figure 4. Comparison of DDHF results for the momentum distributions of nuclear matter with (dashed line) and without (solid line) pairing correlations with the results from LFD (dotted line) and LDA (dash-dotted line) methods.

butions shows different trends. For a given isotopic chain, we find that in the high-momentum region ($k > 1.5$ fm^{-1}) the high-momentum tails of the neutron momentum distributions $n_n(k)$ increase with the increase of the number of neutrons N, while the proton momentum distributions $n_p(k)$ exhibit an opposite effect. In the same region the LFD method does not show this isotopic sensitivity, in contrast to the DDHF+BCS and LDA methods. At low momenta $n_n(k)$ decreases while, on the contrary, $n_p(k)$ increases with the increase of N. Additionally to the isotopic sensitivity we studied how the momentum distributions of some isotones are modified keeping the neutron number constant. We find that the total momentum distributions of ^{78}Ni, ^{86}Kr, and ^{100}Sn nuclei (N=50) reveal the same high-momentum tails in all methods used.

Our results for the neutron and proton momentum distributions of ^{98}Kr isotope show small changes in the overall behavior for the oblate and prolate shapes. Although the neutron and proton densities change with deformation [9], the momentum distributions demonstrate a very weak dependence on the character of deformation. The pairing correlations are shown to influence the high-momentum behavior of the neutron $n_n(k)$, proton $n_p(k)$, and total $n(k)$ momentum distributions in the case of ^{84}Kr, but the differences between the results with or without BCS-correlations included in the calculations are very small. The effect of pairing correlations on the HF momentum distribution is much stronger in the case of nuclear matter producing a tail for momenta $k > k_F$.

It turns out that $n_n(k)$ does not change significantly for different values of β, thus showing the strong presence of correlations at short distances within the LFD method not only for the stable, but also for the exotic nuclei. However, a larger sensitivity of $n_n(k)$ on the parameter γ in the LDA approach appears, particularly in the interval $1.5 < k < 3$ fm^{-1}. In our opinion, however, the question for the specific values of the parameters β and γ which determine the strength of the correlations is still open.

We emphasize that, in our work, a possible practical way to make predictions for the momentum distributions of exotic nuclei far from the stability line is proposed that provides a systematic description of $n(k)$ in medium-weight and heavy nuclei. The comparison of the predicted nucleon momentum distributions with the results of possible experiments using a colliding electron-exotic nuclei storage rings would show the effect of the neutron excess in these nuclei and will be also a test of various theoretical models of the structure of exotic nuclei.

Acknowledgements

This work was partly supported by the Bulgarian National Science Fund under Contract No. 02-285, by Ministerio de Ciencia e Innovacion (Spain) under Contract Nos. FIS2008-01301 and FPA2007-62616, and by the Agreement between CSIC (Spain) and the Bulgarian Academy of Sciences (2007BG0011). M.V.I. acknowledges the support from the European Operational Program HRD through Contract BGO051PO001/07/3.3-02/53 with the Bulgarian Ministry of Education, Youth and Science.

References

[1] Electromagnetic Probes and the Structure of Hadrons and Nuclei, *Prog. Part. Nucl. Phys.* **44** (2000) 1-482; G. Shrieder *et al.*, Proposal for GSI RI Beam Factory (2001).

[2] T. Suda, In: *Challenges of Nuclear Structure, Proceedings of 7th International Spring Seminar on Nuclear Physics*, edited by Aldo Covello, World Scientific Publishing Co., Singapore (2002), p. 13; Proposal for RIKEN RI Beam Factory (2001).

[3] T. Katayama, T. Suda, and I. Tanihata, *Physica Scripta* **T104** (2003) 129-143.

[4] A.N. Antonov, M.K. Gaidarov, D.N. Kadrev, P.E. Hodgson, and E. Moya de Guerra, *Int. J. Mod. Phys. E* **13** (2004) 759-772.

[5] A.N. Antonov, D.N. Kadrev, M.K. Gaidarov, E. Moya de Guerra, P. Sarriguren, J.M. Udias, V.K. Lukyanov, E.V. Zemlyanaya, and G.Z. Krumova, *Phys. Rev. C* **72** (2005) 044307.

[6] P. Sarriguren, E. Moya de Guerra, A. Escuderos, and A. C. Carrizo, *Nucl. Phys.* **A635** (1998) 55-85.

[7] E. Moya de Guerra, P. Sarriguren, J.A. Caballero, M. Casas, and D.W.L. Sprung, *Nucl. Phys.* **A529** (1991) 68-94.

[8] D. Vautherin, *Phys. Rev. C* **7** (1973) 296-316.

[9] P. Sarriguren, M.K. Gaidarov, E. Moya de Guerra, and A.N. Antonov, *Phys. Rev. C* **76** (2007) 044322.

[10] G.B. West, *Phys. Rep.* **18** (1975) 263-323; I. Sick, D.B. Day, and J.S. McCarthy, *Phys. Rev. Lett.* **45** (1980) 871-874.

[11] C. Ciofi degli Atti and G. B. West, *Phys. Lett.* **B458** (1999) 447-453 (and references therein).

[12] W.M. Alberico, A. Molinari, T.W. Donnelly, E.L. Kronenberg, and J.W. Van Orden, *Phys. Rev. C* **38** (1988) 1801-1810.

[13] M.B. Barbaro, R. Cenni, A. De Pace, T.W. Donnelly, and A. Molinari, *Nucl. Phys.* **A643** (1998) 137-160.

[14] A.N. Antonov, M.K. Gaidarov, D.N. Kadrev, M.V. Ivanov, E. Moya de Guerra, and J.M. Udias, *Phys. Rev. C* **69** (2004) 044321.

[15] A.N. Antonov, M.V. Ivanov, M.K. Gaidarov, E. Moya de Guerra, P. Sarriguren, and J.M. Udias, *Phys. Rev. C* **73** (2006) 047302; *ibid.* (2006) 059901(E).

[16] A.N. Antonov, M.K. Gaidarov, M.V. Ivanov, D.N. Kadrev, E. Moya de Guerra, P. Sarriguren, and J.M. Udias, *Phys. Rev. C* **71** (2005) 014317.

[17] A.N. Antonov, M.V. Ivanov, M.K. Gaidarov, E. Moya de Guerra, J.A. Caballero, M.B. Barbaro, J.M. Udias, and P. Sarriguren, *Phys. Rev. C* **74** (2006) 054603.

[18] A.N. Antonov, P.E. Hodgson, and I.Zh. Petkov, *Nucleon Momentum and Density Distributions in Nuclei*, Clarendon Press, Oxford (1988); *Nucleon Correlations in Nuclei*, Springer-Verlag, Berlin-Heidelberg-New York (1993).

[19] M. Jaminon, C. Mahaux, and H. Ngô, *Phys. Lett.* **B158** (1985) 103-106; O. Bohigas and S. Stringari, *Nucl. Phys.* **A473** (1987) 509-538; *Phys. Lett.* **B95** (1980) 9-12.

[20] M.K. Gaidarov, G.Z. Krumova, P. Sarriguren, A.N. Antonov, M.V. Ivanov, and E. Moya de Guerra, *Phys. Rev. C* **80** (2009) 054305.

[21] A.N. Antonov, M.K. Gaidarov, M.V. Ivanov, D.N. Kadrev, G.Z. Krumova, P.E. Hodgson, and H.V. von Geramb, *Phys. Rev. C* **65** (2002) 024306.

[22] J. Carbonell and V.A. Karmanov, *Nucl. Phys.* **A581** (1995) 625-653; J. Carbonell, B. Desplanques, V.A. Karmanov, and J.-F. Mathiot, *Phys. Rep.* **300** (1998) 215-347, and references therein.

[23] S. Stringari, M. Traini, and O. Bohigas, *Nucl. Phys.* **A516** (1990) 33-40.

[24] P.-O. Löwdin, *Phys. Rev.* **97** (1955) 1474-1489.