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Phase Structure of the Interacting Vector Boson Model

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Abstract. The two-fluid Interacting Vector Boson Model (IVBM) with the $U(6)$ as a dynamical group possesses a rich algebraic structure of physical interesting subgroups that define its distinct exactly solvable dynamical limits. The classical images corresponding to different dynamical symmetries are obtained by means of the coherent state method. The phase structure of the IVBM is investigated and the following basic phase shapes, connected to a specific geometric configurations of the ground state, are determined: spherical, $U(3) \otimes U(3)$, $\gamma-$unstable, $O(6)$, axially symmetric deformed, $\overline{SU_+(3)}$, and deformed triaxial, $\overline{SU_-(3)}$. The obtained phase diagram of the IVBM resembles that of IBM-2 and reveals the common physical content that relates the two algebraic models.

1 Introduction

The phase structure of quantum many-body systems has been a subject of great experimental and theoretical interest in the last years. The introduction of the concept of critical point symmetry [1] has recalled the attention of the community to the topic of quantum phase transitions in nuclei. Different models have been used to describe the quantum phase transitions in different many-body systems, such as atomic nuclei [2], molecules [3, 4], atomic clusters [5], and finite polymers. Among these models, those based upon algebraic Hamiltonians play an important role.

There are many approaches which allow the association of a certain geometry to any abstract algebra, but for algebraic models, this can be achieved with the theory of the coherent states [6–9]. The expectation value of the Hamiltonian in the ground coherent state is refereed to as its classical limit. The method of coherent (or intrinsic) states provides a prescription for translating algebraic operators into canonical phase-space coordinates, thereby allowing algebraic models of nuclear structure and dynamics to be interpreted from the perspective of their corresponding classical limits. Of particular relevance for the present study is the construction of the potential energy or potential functions [2, 3].

A nice feature of the algebraic models is the occurrence of phases connected to a specific geometric configurations of the ground state, which arise from the occurrence of different dynamical symmetries. The study of the ground state energy as a function of an appropriately chosen parameter, called control parameter, shows a transition between the different phases. These phase transitions are referred to as ground-state or quantum phase transitions and have been widely investigated in the last years (e.g., a review article [10]). Since these transitions are between different shapes, they are sometimes termed as "shape transitions".

An important aspect of the study of phase transitions is the construction of the phase diagram (structure). In this respect it is interesting to see what is the phase structure of the two-fluid Interacting Vector Boson Model (IVBM). Thus, it is the purpose of the present paper to investigate what are the different phase shapes which might occur within the framework of the IVBM. The first step in the construction of the phase diagram is the identification of all possible dynamical symmetries of the system. In IVBM there are several dynamical symmetries which will be considered in next sections. It should be shown that there exist four distinct shapes corresponding to the four dynamical symmetries of IVBM: (1) spherical shape, $U(3) \otimes U(3)$, (2) $\gamma-$unstable shape, $O(6)$, (3) axially deformed shape, $\overline{SU_+(3)}$, and (4) triaxial shape, $\overline{SU_-(3)}$, which closely resemble the phase structure obtained in the proton-neutron Interacting Boson Model (IBM) (referred also to as IBM-2). An important feature offered by the IBM-2, which is obtained also within the present framework, is the possibility to get triaxial shapes [10–13] besides the axially symmetric ones. This gives rise to the Dieperink tetrahedron [11], which has an extra dimension compared to the Casten triangle [14], and to a new, triaxial shape phase of the model. The latter can not be obtained in the standard IBM-1 with quadratic terms. One needs cubic or higher order terms.

2 The algebraic structure generated by the two vector bosons

The algebraic structure of IVBM [15,16] is realized in terms of creation and annihilation operators of two kinds of vector bosons $u^{\dagger}_m(\alpha)$, $u_m(\alpha)$ ($m = 0, \pm1$), which differ in an additional quantum number $\alpha = \pm1/2$ (or $\alpha = p$ and n)$-$the projection of the $T-$spin (an analogue to the $F-$spin of IBM-2). The number preserving bilinear products of the creation and annihilation operators of the two vector bosons generate the boson representations of the unitary group $U(6)$ [15,16]:

$$A^L_M(\alpha,\beta) = \sum\nolimits_{k,m} C^{LM}_{1k1m} u^{\dagger}_k(\alpha) u_m(\beta), \tag{1}$$

where C^{LM}_{1k1m}, which are the usual Clebsch-Gordan coefficients for $L = 0, 1, 2$ and $M = -L, -L+1, ...L$, define the transformation properties of (1) under rotations. We will use also the notations $u^{\dagger}_m(\alpha = 1/2) = p^{\dagger}_m$ and $u^{\dagger}_m(\alpha = -1/2) = n^{\dagger}_m$.

In the most general case the two-body model Hamiltonian should be expressed in terms of the generators of the group $U(6)$. In some special cases the Hamiltonian can be written in terms of the generators of different subgroups of

$U(6)$. The $U(6)$ group contains the following chains of subgroups [15, 16]:

$$\begin{array}{ccccc} & & U(6) & & \\ & \swarrow & \downarrow & \searrow & \\ U_p(3)\otimes U_n(3) & & O_\pm(6) & & U(3)\otimes U_T(2) \\ \downarrow & \swarrow & \downarrow & & \downarrow \\ SO_p(3)\otimes SO_n(3) & & \overline{SU_\pm(3)}\otimes O(2) & & SU(3)\otimes SO_T(2) \\ & \searrow & \downarrow & \swarrow & \\ & & SO(3) & & \end{array} \tag{2}$$

As can be seen, the IVBM has a rich enough algebraic structure of subgroups. Each of these dynamical symmetries will correspond to a certain possibly different shape phase.

2.1 The $U(3)\otimes U(3)$ Chain

(a) $U(3)\otimes U(3)$ algebra. It consists of two sets of commuting operators:

$$N_p = \sqrt{3}(p^\dagger \times p)^{(0)}, L^p_M = \sqrt{2}(p^\dagger \times p)^{(1)}_M, Q^p_M = \sqrt{2}(p^\dagger \times p)^{(2)}_M \tag{3}$$

and

$$N_n = \sqrt{3}(n^\dagger \times n)^{(0)}, L^n_M = \sqrt{2}(n^\dagger \times n)^{(1)}_M, Q^n_M = \sqrt{2}(n^\dagger \times n)^{(2)}_M. \tag{4}$$

The $SU_\tau(3)$ ($\tau = p, n$) algebra is obtained by excluding the number operator N_τ, whereas the angular momentum algebra $SO_\tau(3)$ is generated by the generator L^τ_M only. The linear Casimir operators are simply $C_1[U_p(3)] = N_p$ and $C_1[U_n(3)] = N_n$.

(b) $O(3)\otimes O(3)$ algebra. This algebra is determined by the operators L^p_M and L^n_M.

2.2 The $SU(3)\otimes U(2)$ Chain

The $SU(3)\otimes U(2)$ algebra also consists of two commuting sets of operators:

(a) $U(2)$ algebra. It is defined by the operator of a number of particles

$$N = N_p + N_n \tag{5}$$

and the “T-spin” operators $T^1_m, (m = 0, \pm 1)$ introduced through

$$T_1 = \sqrt{\tfrac{3}{2}}A^0(p,n), \qquad T_{-1} = -\sqrt{\tfrac{3}{2}}A^0(n,p) \tag{6}$$

$$T_0 = -\tfrac{\sqrt{3}}{2}\left[A^0(p,p) - A^0(n,n)\right]. \tag{7}$$

The above operators $T^1_m(m = 0, \pm 1)$ commute with N. Thus (6) and (7) define the subalgebra $su(2) \subset u(2)$. These operators play an important role in the consideration of the nuclear system as composed by two interacting (proton and neutron) subsystems.

(b) $U(3)$ algebra. It consists of the operators which are a sum of the $p-$ and $n-$boson subsystem operators: the total number of bosons N (5), the angular momentum operator $L_M = L_M^p + L_M^n$ and the components of the "truncated" (or Elliott's) quadrupole operator $\tilde{Q}_M = Q_M^p + Q_M^n$. The operators L_M and $\tilde{Q}_M$ commute with N and define the subalgebra $su(3) \subset u(3)$. The second-order Casimir operator of $U(3)$ is $C_2[U(3)] = \frac{1}{6}\tilde{Q}^2 + \frac{1}{2}L^2 + \frac{1}{3}N^2 = \frac{1}{2}N^2 + 2T^2 + N$, where $\tilde{Q}^2 = 6\sum_{M,\alpha,\beta}(-1)^M A_M^2(\alpha,\alpha)A_M^2(\beta,\beta)$. The $SU(3)$ Casimir is simply

$$K_3 = \frac{1}{6}\tilde{Q}^2 + \frac{1}{2}L^2. \tag{8}$$

2.3 The $O_{\pm}(6)$ Chain

(a) $O(6)$ algebra. The $O_+(6)$ algebra is spanned by the following operators:

$$A_M^1(p,p) \equiv L_M^p,\ A_M^1(n,n) \equiv L_M^n,\ G_{ij}^{(+)} = p_i^\dagger n_j + n_i^\dagger p_j. \tag{9}$$

An alternative $O_-(6)$ algebra can be defined with the generators

$$G_{ij}^{(-)} = \mathrm{i}(p_i^\dagger n_j - n_i^\dagger p_j) \tag{10}$$

instead of $G_{ij}^{(+)}$. Both the $O_+(6)$ and $O_-(6)$ algebras have the same eigenspectrum but differ through phases in the wave functions. They are related by a transformation that is a special case of a wider class of transformations known as inner automorphisms. It is known that representation theory does provide all of the embeddings, but it does not provide all of the dynamical symmetries [17]. Indeed, the inner automorphisms can provide new dynamical symmetry limits, sometimes referred as to "hidden" [17] or "parameter" symmetries [18].

(b) $\overline{SU(3)}$ algebra. There are two distinct $\overline{SU_{\pm}(3)}$ algebras which are generated by a part of the $O(6)$ operators, namely by

$$X_M^{(+)} = A_M^2(p,n) + A_M^2(n,p), \tag{11}$$

$$X_M^{(-)} = \mathrm{i}\left[A_M^2(p,n) - A_M^2(n,p)\right], \tag{12}$$

and $Y_M = -(L_M^p + L_M^n)/\sqrt{2}$. Its second-order Casimir operator is

$$\overline{G_3} = \sum_M (-1)^M (X_M^{(\pm)} X_{-M}^{(\pm)} + Y_M Y_{-M})\ . \tag{13}$$

It will be shown in the next sections that the $\overline{SU_-(3)}$ algebra provides a new physically distinct dynamical symmetry limit of IVBM, which connect the latter with the IBM-2.

3 The Boson Condensate

Usually, the condensate coherent state is defined in terms of the 'condensate boson' creation operator, which is a general linear combination [10, 19]

$$B \equiv \alpha_1 b_1 + \alpha_2 b_2 + \ldots + \alpha_M b_M, \tag{14}$$

where in general the coefficients α_i are complex. Then the (unnormalized) *condensate* coherent state is [10, 19]

$$
\begin{aligned}
\mid N; \alpha_1, \ldots, \alpha_M \rangle &\propto (B^\dagger)^N \mid 0 \rangle \\
&= \left[\sum_k \alpha_k b_k^\dagger \right]^N \mid 0 \rangle,
\end{aligned} \tag{15}
$$

where $\mid 0 \rangle$ is the boson vacuum. The expectation value of a one-body or two-body operator with respect to the condensate (15) can be deduced using arguments based upon formal differentiation [19].

Alternatively, one may use the *Glauber* coherent states [20]

$$
\mid d_1, \ldots, d_M \rangle \propto exp \left[\sum_k d_k b_k^\dagger \right] \mid 0 \rangle, \tag{16}
$$

which do not have a fixed N. Note that the expectation value of the number of bosons $N = \sum_k b_k^\dagger b_k$ between (15) is

$$
\frac{\langle N; \alpha | N | N; \alpha \rangle}{\langle N; \alpha | N; \alpha \rangle} = N \sum_k |\alpha_k|^2. \tag{17}
$$

while with respect to (16) is

$$
\frac{\langle d | N | d \rangle}{\langle d | d \rangle} = \sum_k |d_k|^2. \tag{18}
$$

Thus it is obvious that both Glauber and condensate states yield the same results if taking $\alpha_k \propto \langle N \rangle^{-1/2} d_k$.

Using the fact that the components of the two vector bosons (p_k and n_k, respectively) form a six-dimensional vector, the (unnormalized) CS for IVBM become

$$
\mid N; \xi, \zeta \rangle \propto \left[\sum_k (\xi_k p_k^\dagger + \zeta_k n_k^\dagger) \right]^N \mid 0 \rangle, \tag{19}
$$

where ξ_k and ζ_k are the components of three-dimensional complex vectors. For static problems these variables can be chosen real. The expression (19) defines the so called projective realization of the CS for the fully symmetric representation $[N]_6$ of $SU(6)$. We want to point out that in contrast to the definition of the CS for IBM-2 [11–13], where the numbers of protons, N_π, and neutrons, N_ν, are separately conserved, here only the total number of the two vector bosons $N = N_p + N_n$ is a good quantum number. The parameters ξ_k and ζ_k will determine the deformation of the nucleus in the equilibrium state.

4 Geometry

The standard approach to obtain the geometry of the system is to express the collective variables $\{\xi_k, \zeta_k\}$ in the intrinsic (body-fixed) frame of reference. A convenient parametrization is

$$\xi_0 = r_1 \cos(\chi) \tag{20}$$

$$\xi_{\pm 1} = \mp \frac{1}{\sqrt{2}} r_1 \sin(\chi) e^{\mp i\phi} \tag{21}$$

$$\sum_k |\xi_k|^2 = r_1^2, \tag{22}$$

and

$$\zeta_0 = r_2 \cos(\theta) \tag{23}$$

$$\zeta_{\pm 1} = \mp \frac{1}{\sqrt{2}} r_2 \sin(\theta) e^{\mp i\varphi} \tag{24}$$

$$\sum_k |\zeta_k|^2 = r_2^2 \tag{25}$$

with

$$r_1, r_2 > 0 \tag{26}$$

$$0 \leq \theta, \chi \leq \pi \tag{27}$$

$$0 \leq \varphi, \omega \leq 2\pi. \tag{28}$$

A rotation can orient any one of the vectors ξ and ζ along the quantization axes, but the relative orientations of different vectors are relevant dynamic variables. The most general condensate may be rotated so that ξ is along the z axis, while ζ has polar angles (θ, φ). The angle φ depends on the (arbitrary) choice of the direction of the x axis, and the condensate can always be rotated so that $\varphi = 0$. Thus, in this choice θ plays the role of relative angle between the vectors ξ and ζ, and hence $\xi \cdot \zeta$ =$r_1 r_2 cos\theta$. Then, the two vectors in spherical coordinates become $\xi = (r_1, \chi = 0, \phi = 0)$ and $\zeta = (r_2, \theta, \varphi = 0)$. The condensate in the intrinsic frame takes the form

$$\mid N; r_1, r_2, \theta \rangle = \frac{1}{\sqrt{N!}} (B^\dagger)^N \mid 0 \rangle \tag{29}$$

with

$$B^\dagger = \frac{1}{\sqrt{r_1^2 + r_2^2}} [r_1 p_0^\dagger + r_2 \cos\theta n_0^\dagger + r_2 \sin\theta \frac{1}{\sqrt{2}} (-n_1^\dagger + n_{-1}^\dagger)]. \tag{30}$$

The geometric properties of the ground states of nuclei within the framework of the IVBM can then be studied by considering the energy functional

$$E(N; r_1, r_2, \theta) = \frac{\langle N; r_1, r_2, \theta | H | N; r_1 r_2, \theta \rangle}{\langle N; r_1, r_2, \theta | N; r_1, r_2, \theta \rangle}. \tag{31}$$

The latter, as can be seen, depends on three geometric parameters: the lengths of the two vectors (dipole deformations) r_1 and r_2 and on the angle θ between them (or equivalently on the three rotational invariants r_1^2, r_2^2 and $r_1 \cdot r_2$). By minimizing $E(N; r_1, r_2, \theta)$ (31) with respect to r_1, r_2, and θ , $\partial E/\partial r_1 = \partial E/\partial r_2 = \partial E/\partial \theta = 0$, one can find the equilibrium "shape" corresponding to any boson Hamiltonian, H. This shape is in many cases rigid. However, there are many situations in which the system is rather floppy and undergoes a phase transition between two different shapes.

5 Classical Limit of Different Dynamical Symmetries of IVBM

5.1 The $U(3) \otimes U(3)$ Limit

We consider the Hamiltonian that is linear combination of first order Casimirs of $U_\tau(3)$:

$$H_I = \varepsilon_p N_p + \varepsilon_n N_n. \tag{32}$$

The Hamiltonian (32) can be rewritten in the form

$$H_I = \varepsilon_p N + \varepsilon N_n, \tag{33}$$

where $\varepsilon = \varepsilon_n - \varepsilon_p$. The first term in (33) can be dropped since it does not contribute to the energy surface. Thus, the Hamiltonian determining the properties of the system in the $U(3) \otimes U(3)$ limit is just

$$H_I = \varepsilon N_n. \tag{34}$$

The expectation value of (34) with respect to (29) gives the energy surface

$$\frac{\langle N; r_1, r_2 | H_I | N; r_1, r_2 \rangle}{\langle N; r_1, r_2 | N; r_1, r_2 \rangle} = \varepsilon \frac{N r_2^2}{r_1^2 + r_2^2}. \tag{35}$$

We see that the energy surface depends on the two parameters r_1 and r_2 and is $\theta-$independent. In order to simplify the analysis we introduce a new dynamical variable $\rho = r_2/r_1$ as a measure of "deformation", which together with the parameter θ determine the corresponding "shape". Thus, the expression (35) becomes

$$E(N; \rho) = \varepsilon \frac{N \rho^2}{1 + \rho^2}, \tag{36}$$

which has a minimum at $\rho_0 = 0$. It corresponds to a spherical shape (vibrational limit). The scaled energy $\varepsilon(\rho) = E(N; \rho)/\varepsilon N$ in the $U(3) \otimes U(3)$ limit is given in Figure 1. The inclusion of higher-order terms in N_p and N_n will give rise to an anharmonicity.

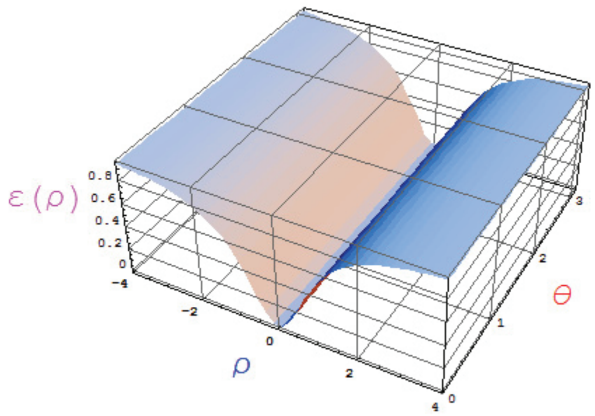

Figure 1. The scaled energy surface $\varepsilon(\rho)$ in the $U(3) \otimes U(3)$ limit.

5.2 The $O(6)$ Limit

The Hamiltonian describing the $O(6)$ (or $\gamma-$unstable) properties can be written down through the $O(6)$ pairing operator $P^{\dagger} = \frac{1}{2}(p^{\dagger} \cdot p^{\dagger} - n^{\dagger} \cdot n^{\dagger})$ in the following form

$$H_{II} = \frac{4k'}{N-1} P^{\dagger} P. \tag{37}$$

Taking the expectation value of (37) one obtains the energy surface

$$E(N;\rho) = k'N \left[\frac{1-\rho^2}{1+\rho^2} \right]^2, \tag{38}$$

which does not depend on θ ($\theta-$unstable) and has a minimum at $\rho_0 \neq 0$ ($\rho_0 = \pm 1$). It corresponds to a deformed "$\gamma-$unstable" (in IBM terms) rotor. As it was mentioned, there are two $O_{\pm}(6)$ algebras that are isomorphic and have the same eigenspectrum but differ through phases in the wave functions resulting into different energy surfaces. The energy surface (38) corresponds to the $O_{-}(6)$ limit. The other, $O_{+}(6)$, limit is not physically important since its energy surface is just a constant. The scaled energy surface $\varepsilon(\rho)$ in the $O_{-}(6)$ limit is given in Figure 2.

5.3 The $\overline{SU_{\pm}(3)}$ Limit

In this case we study the Hamiltonian

$$H_{III} = -\frac{k}{2(N-1)} \overline{G_3}, \tag{39}$$

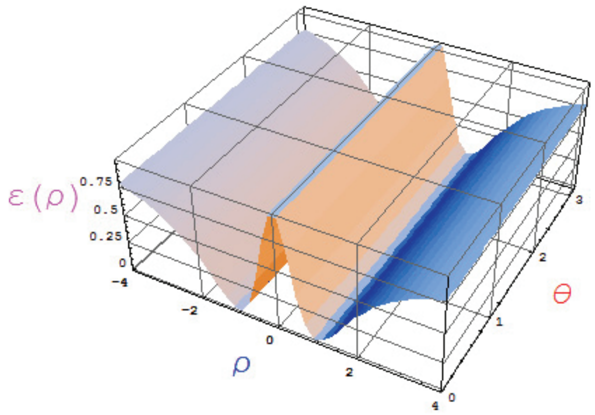

Figure 2. The scaled energy surface $\varepsilon(\rho)$ in the $O(6)$ limit.

where $\overline{G_3}$ is given by (13). Note that the quadrupole operators X^{ω} ($\omega = \pm 1$) entering in $\overline{G_3}$ generate the two distinct algebras $\overline{SU_+(3)}$ and $\overline{SU_-(3)}$, respectively. The energy surface corresponding to the Hamiltonian (39), within to an unimportant constant, is

$$E(N;\rho,\theta) = -kN\left[(\pm)\frac{\rho^2(\cos^2(\theta)\pm 1)}{(1+\rho^2)^2}\right], \tag{40}$$

where the upper (lower) sign corresponds to the $\overline{SU_+(3)}$ ($\overline{SU_-(3)}$) algebra. We plot the scaled energy surfaces corresponding to the two limits under consideration in Figures 3 and 4, respectively. From the figures one can see that for both cases the global minimum occurs at $\rho_0 \neq 0$ ($|\rho_0| = 1$) and $\theta_0 = 0^0$ or $\theta_0 = 90^0$ corresponding to the aligned or perpendicular configurations of the two distinct subsystems. This allows us to interpret the $\overline{SU_-(3)}$ configuration as a triaxial one giving rise to the triaxial shape phase. The fact that the deformation parameter $|\rho| = 1$ means that we have equal deformations of the $p-$ and $n-$boson subsystems which ratio is given by $\rho^2 = \frac{r_2^2}{r_1^2} = \frac{N_n}{N_p}$. Similarly, the equilibrium configurations of the deformed structures in IBM-2 are obtained for $\beta_\pi = \beta_\nu$ [11–13]. Analogously, the energy surface of $\overline{SU_+(3)}$ ($|\rho| = 1$ and $\theta = 0^0$) is interpreted as corresponding to an axially symmetric deformed phase shape. In this regard, the two phase shapes obtained in this subsection closely resemble the ones corresponding to the $SU^*(3)$ and $SU(3)$ of IBM-2 [10–13].

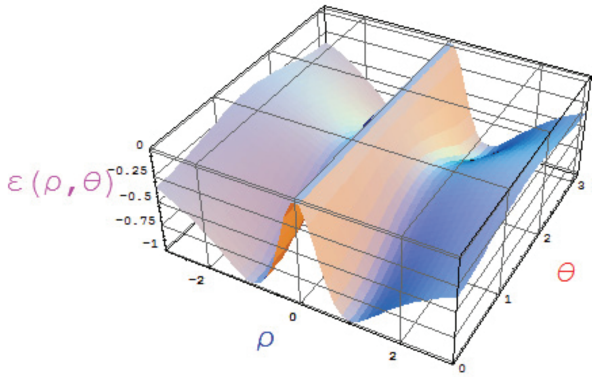

Figure 3. The scaled energy surface $\varepsilon(\rho, \theta)$ in the $\overline{SU_{+}(3)}$ limit.

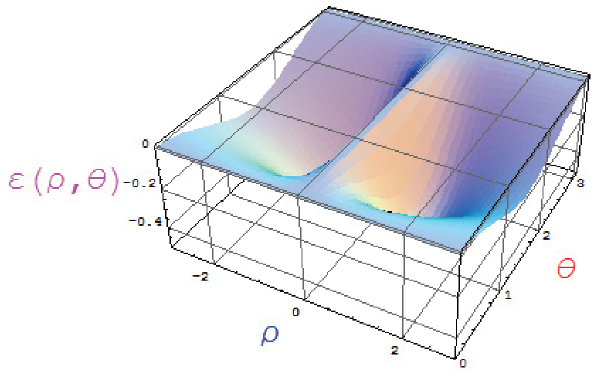

Figure 4. The scaled energy surface $\varepsilon(\rho, \theta)$ in the $\overline{SU_{-}(3)}$ limit.

5.4 The $SU(3) \otimes U(2)$ Limit

This limit can be studied through the Hamiltonian

$$H_{IV} = -\frac{k}{(N-1)} K_3, \tag{41}$$

where the second order Casimir operator K_3 of $SU(3)$ is given by (8). The expectation value of (41) with respect to (31) gives the following energy surface

$$E(N;\rho,\theta) = -kN\left[\frac{(1+2\rho^2\cos^2(\theta)+\rho^4)}{(1+\rho^2)^2}\right]. \tag{42}$$

For positive values of the parameter $k > 0$ one obtains an oscillator in the relative angle θ, which has the equilibrium at $\theta_0 = 0$. To show this one needs to consider a more general semi-classical analysis of the classical limit of H in which the complex coherent state parameters are used.

For negative values of the parameter $k < 0$, the energy surface in the $SU(3)\otimes U(2)$ limit is similar to that of $\overline{SU_-(3)}$ (Figure 4) and hence it also gives rise to a triaxial deformed shape. Indeed, the minimization of the energy surface (42) with respect to θ gives for the equilibrium state the values $\theta_0 = 0^0$ and 90^0, respectively. As one can see, for $\theta_0 = 0^0$ the energy surface is just a constant. At $\theta_0 = 90^0$, the minimum of the energy surface with respect to ρ is obtained for $|\rho_0| = 1$.

6 The Generalized IVBM Hamiltonian and Its Phase Diagram

All physically interesting Hamiltonians can be combined into a single Hamiltonian of the form:

$$H = \eta N_n - \frac{(1-\eta)}{N-1}\left[g\overline{G_3(\omega)} - (1-g)P^\dagger P\right], \tag{43}$$

where the explicit dependence of $\overline{G_3}$ from ω is shown. We have three control parameters: η, g, ω (as much as in IBM-2) resulting in a three dimensional phase diagram. Thus, the shape phase diagram corresponding to the IVBM Hamiltonian (43) can be represented by a pyramid as shown in Figure 5 with each corner denoting a dynamical symmetry. For $\eta = 1$ one obtains the $U(3)\otimes U(3)$ or the *vibrational* limit; for $\eta = 0$ one encounters the three limiting cases of *deformed* shapes discussed above: $g = 0$ ($O(6)-$rotor), $g = 1$, $\omega = (+1)$ ($\overline{SU_+(3)}-$axial rotor), and $g = 1$, $\omega = (-1)$ ($\overline{SU_-(3)}-$triaxial rotor). If one compares this phase diagram with that of IBM-2 [11–13], it can be seen that the only difference in the present case is the corner $U(3)\otimes U(3)$ corresponding to harmonic (or unharmonic) vibrations in a $3-$dimensional phase space (or, alternatively, to two uncoupled oscillators in a $6-$dimensional direct product phase space) instead of the $5-$dimensional one in the IBM-2. These results suggest that only the dynamical symmetries generated by the bosons ("building blocks") are of importance for the description of certain collective motions in nuclei (vibrations, rotations, mixed rotation-vibration modes) rather than the tensorial or fermionic nature of the bosons.

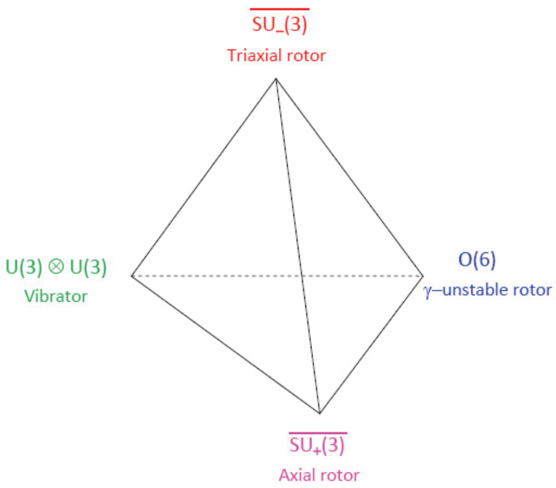

Figure 5. Phase diagram of IVBM. The corners of the pyramid correspond to dynamical symmetries.

7 Conclusions

In the present paper, the geometrical analysis of the different dynamical symmetries of the two-fluid IVBM with the $U(6)$ as a dynamical group is carried out by means of coherent state method. The latter allows the calculation of the classical limit of the Hamiltonians corresponding to different dynamical symmetries in terms of appropriately chosen classical (geometrical) variables representing the boson degrees of freedom. The different dynamical symmetries correspond to qualitatively distinct ground state equilibrium configurations, which constitute the phases of the system.

We have studied the phase structure of IVBM and four shape phases corresponding to its four dynamical limits have been obtained: spherical, $U(3) \otimes U(3)$, $\gamma-$unstable, $O(6)$, axially symmetric deformed, $\overline{SU_+(3)}$, and deformed triaxial, $\overline{SU_-(3)}$. The $\overline{SU_-(3)}$ dynamical symmetry limit which gives rise to the triaxial shape phase is generated by a transformation from the class of inner automorphisms. The inner automorphisms provide new dynamical (hidden) symmetries of the quantum system under consideration. Of the various (infinite in number) automorphisms, only a certain class of seemingly irrelevant ones are of physical importance, as the case considered here.

The obtained phase diagram of IVBM resembles that of IBM-2 and reveals the common physical content that relates the two algebraic models in their description of nuclear two-fluid-like systems with various collective properties. Classically, both IBM-2 and IVBM are able to describe the main collective modes that manifest themselves in nuclear collective motion: $1)$ harmonic (or unharmonic) vibrations; $2)$ rotations of axially or triaxially deformed nuclei; and $3)$ rotation-vibrational properties of transitional ($\gamma-$ unstable) nuclei.

The study of the phase structure, obtained in this paper, can be further extended to the investigation of the quantum phase transitions that can take place between different ground state configurations or "shapes", occurring at zero temperature as a function of the corresponding control parameter.

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