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Multiple Reflection Asymmetric Bands in Actinides

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Abstract.

The low–energy excitation spectra of deformed even–even Th, U and Pu isotopes are analyzed within the dinuclear system model. The model is based on the assumption that the cluster type shapes are produced by the collective motion in the mass–asymmetry coordinate. To describe the reflection asymmetric collective modes characterized by the nonzero values of K, the intrinsic excitations of clusters are taken into account. The observed excitation spectrum and angular momentum dependence of the parity splitting are explained.

1 Introduction

In the even–even isotopes of actinides and also in the heavy Ba and Ce isotopes the low–lying negative parity states are observed together with the usually presented collective positive-parity states combined into rotational or quasirotational ground–state bands. Formation of the positive–parity rotational or quasirotational bands is connected in general to the quadrupole collective motion, while the lowering of the negative-parity states is a signature of the presence of the reflection asymmetric collective mode. There are several approaches to treat the collective motion related to the reflection asymmetric degrees of freedom. One of them is based on the concept of the nuclear mean field [1] which has a static mirror asymmetric deformation or is characterized by a large amplitude of reflection asymmetric vibrations around the equilibrium shape. Another approach is based on the assumption that the reflection asymmetric shape is a consequence of the α–clustering in nuclei [2]. It is also known from the Nilsson–Strutinsky type calculations for light nuclei that nuclear configurations corresponding to the minima of the potential energy contain particular symmetries which are related to certain cluster structures [3,4]. Several calculations performed for heavy nuclei [5–7] have shown that configurations with large equilibrium quadrupole deformations and low–lying collective negative parity states are strongly related to clustering. We mention also a different approach to description of the properties of the alternating parity bands which is based on the idea of the aligned octupole phonons [8, 9].

The main idea of the cluster model developed in [7, 10, 11] is that a dynamics of a reflection asymmetric collective motion can be treated as a collective motion of nucleons between two clusters or as a motion in a mass–asymmetry coordinate. Such collective motion simultaneously creates deformations with even and odd–multipolarities. Within this approach the existing experimental data on the angular momentum dependence of the parity splitting in the excitation spectra and the multipole transition moments (E1,E2,E3) of the low–lying alternating parity states in odd and even actinides $^{220-228}$Ra, ^{223,225,227}Ac, $^{222-224,226,228-232}$Th, ^{231}Pa, $^{232-234,236,238}$U and ^{240,242}Po and the medium mass nuclei ^{144,146,148}Ba, ^{151,153}Pm, ^{146,148}Ce ^{153,155}Eu and ^{146,148}Nd are well described. The good agreement between the results of calculations and the experimental data support a cluster interpretation of the reflection–asymmetric states.

In our previous publications [7, 10, 11] we have considered in the even–even nuclei only the low–lying collective negative parity states with K=0. At the same time, there are experimental data which indicate on a presence of the collective states related to the reflection asymmetric modes characterized by nonzero values of K. To describe in the framework of the cluster approach the properties of these collective modes we should take into account the intrinsic excitations of clusters produced by the motion of the nucleus in mass-asymmetry degrees of freedom. It is the aim of the present investigation to extend our model to take into account such an excitations.

2 Model

The model is based on the assumption that intrinsic wave function of nucleus (A, Z) can be represented as a superposition of different dinuclear system configurations and the mononucleus. As a dinuclear system (DNS) we understand a system of clusters (A_1, Z_1) and (A_2, Z_2), (where $A_1 + A_2 = A$ and $Z_1 + Z_2 = Z$) in touching configuration. The mononucleus represents the part of nuclear wave function which can not be decomposed into dinuclear systems and treated as a compound nucleus without reflection–asymmetric deformation. Since the ground–state wave function is distributed over different configurations, the energy of the mononucleus is not equal to the experimental binding energy.

In our model we suppose that the reflection asymmetric deformation near the ground–state of the nucleus is related to the contribution of asymmetric dinuclear systems. This idea is related to the experimental observation that all actinides nuclei are good α–emitters. Thus there is a significant probability to form an α–cluster on the surface of the considered nucleus. Due to the stable closed–shell structure of the α–cluster, it is reasonable to presume that the ground–state and low–excited states of actinides wave functions can have a significant contribution of $\alpha + (A - 4, Z - 2)$ cluster structures (α–cluster DNS) without appreciable internal excitations of the heavy fragment.

Taking into account the determinative role of the α–cluster DNS in the for-

mulation of the model collective Hamiltonian, we describe a nuclear system produced by the motion of nucleus in mass–asymmetry as consisting of spherical non–excited light cluster (A_2, Z_2) and the heavy deformed fragment (A_1, Z_1) with axially symmetric quadrupole deformation ($\beta = \beta_0$, $\gamma = 0$). We consider only the rotational excitations of the heavy fragment and avoid the consideration of the β– and γ–vibrations. The mononucleus configuration is treated in the same way with light cluster of zero mass.

The relative contribution of the different DNS configurations and the mononucleus are determined by the collective Hamiltonian for the motion in mass–asymmetry (transfer of nucleons between clusters). For convenience, instead of usual definition of mass–asymmetry coordinate $\eta = (A_1 - A_2)/(A_1 + A_2)$ we use positively–defined variable

$$\xi = 2A_2/A = 1 - \eta. \tag{1}$$

The values ξ=0 or ξ=2 correspond to the mononucleus configurations ($A_1 = A, A_2 = 0$) or ($A_1 = 0, A_2 = A$), respectively. For the further use we introduce special notation $\xi_\alpha = 8/A$ for the alpha–particle DNS. The charge asymmetry ξ_Z is not considered as an independent collective variable, and fixed as to minimize the symmetry energy of the DNS for each value of ξ.

The potential energy of the dinuclear system is determined as

$$U(\xi, \epsilon) = B_1(\xi) + B_2(\xi) - B_{12} + V(R = R_m, \xi, \beta_0, \epsilon), \tag{2}$$

where the internuclear distance $R = R_m$ is the touching distance between clusters and is set to be equal to the minimum of the potential in R for a given ξ. The quantities B_1, B_2 are the binding energies of the clusters forming the DNS at a given mass–asymmetry and B_{12} is the binding energy of the compound nucleus. The experimental ground–state masses [12], if available, are used in the calculations. If not, the predictions of [13] are used. Shell effects and pairing correlations are included in the binding energies. Because of the normalization by B_{12}, the energy of the lowest solution must be zero.

The quantity $V(R = R_m, \xi, \beta_0, \epsilon)$ in (2) is the nucleus–nucleus potential which is calculated as a sum of Coulomb U_C and nuclear U_N interactions. The potential U_N is obtained in double–folding procedure with the density dependent nucleon–nucleon interaction. The parameters of nucleon-nucleon interactions are fixed in nuclear structure calculations. The nuclear densities are approximated by the Fermi distributions with the radius parameter r_0=1.15 fm for heavy fragments and r_0=1.00 fm for light (^{4}He, ^{7}Li) fragments. The diffuseness parameter of the density distribution of a light cluster is taken as 0.48 fm. For heavy cluster, we set $a = 0.56\sqrt{B_n^{(0)}/B_n}$, where $B_n^{(0)}$ and B_n are the neutron binding energies of the studied nucleus and of the heaviest isotope of considered element. The details of calculations are presented in [14].

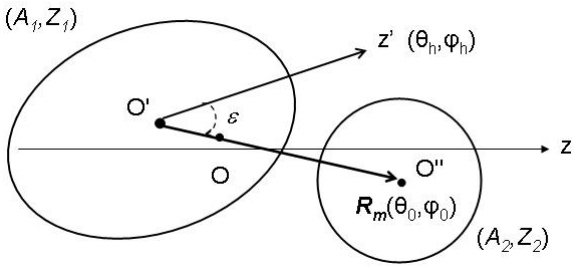

Figure 1. Schematic picture illustrates degrees of freedom used in the model to describe dinuclear system. Orientations of the relative distance vector $\mathbf{R_m}$ and the symmetry axis of the heavy deformed cluster z' with respect to the laboratory frame z are defined by the angles $\Omega_0 = (\theta_0, \phi_0)$ and $\Omega_h = (\theta_h, \phi_h)$, respectively. The angle ϵ is the angle between vector $\mathbf{R_m}$ and z'.

The relative orientation of the clusters in DNS is accounted in (2) by the ϵ (see Figure 1) which is a plain angle between the vector $\mathbf{R_m}$ and the symmetry axis of the deformed heavy nucleus. It can be related to the angles Ω_h=(θ_h, ϕ_h) and $\Omega_\mathbf{0}$=(θ_0, ϕ_0) determining the orientations of symmetry axis of heavy fragment and vector $\mathbf{R_m}$ in the laboratory frame, respectively, by the following expression

$$Y_{l0}(\epsilon, 0) = \sqrt{\frac{4\pi}{2l+1}} \left(Y_2(\Omega_h) \cdot Y_2(\Omega_0)\right). \tag{3}$$

The potential energy as a function of a relative orientation of fragments has a minimum which corresponds to the pole–to–pole orientation (ϵ=0). In order to simplify the calculations we approximate the ϵ–dependence of the potential energy by the second order expansion in the Legendre polynomials which provides the following expression

$$U(\xi, \epsilon) = V(R_m, \xi, \beta_0) + V_0(R_m, \beta_0)\xi \sqrt{\frac{4\pi}{5}} \left(Y_2(\Omega_h) \cdot Y_2(\Omega_0)\right). \tag{4}$$

Since $\xi \ll 1$, we expand the interaction term in (4) leaving only term linear in ξ, that is $V_0(R_m, \xi, \beta_0) \approx V_0(R_m, \beta_0)\xi$. The value of $V_0(R_m, \beta_0)$ is fixed by fitting ϵ–dependence of the potential energy U with expression (4) for the alpha–particle DNS. A specific point in the potential energy is ξ=0 (mononucleus). The potential energy at ξ=0 can not be calculated by means of eq. (2) so it is considered as a parameter of the model.

In the following, we treat mass–asymmetry variable ξ as a continuous variable. In order to solve the Schrödinger equation in ξ, a smooth parametrization

of the potential $V(\xi)$ is used

$$V(\xi) = V(\xi = 0) + \sum_{k=1}^{k=3} a_{2k}\xi^{2l}, \tag{5}$$

where parameters a_{2k} are determined by the experimental ground–state energy, calculated potential energies for $\xi = \xi_\alpha$ and $\xi = \xi_{Li}$ and by the requirement that the potential (5) has minimum for alpha-particle DNS if $V(\xi_\alpha) < 0$. If the minimum is located at $\xi = 0$, only two parameters, a_2 and a_4 in (5) are necessary. The potential energy at the mononucleus $V(\xi = 0)$ is fixed so as to reproduce the experimental binding energy of the corresponded nucleus as the lowest 0^+ solution of the eigenvalue problem with the collective Hamiltonian of the model.

The kinetic energy can be constructed from the parts describing the kinetic energy of motion in mass–asymmetry ξ, the rotation of the DNS as a whole and the rotational excitations of the heavy fragment. The β– and γ–vibrations of the heavy fragment, which are responsible for the appearance of low-lying rotational bands build on the 0^+ state and 2^+, are not considered in the model. While the γ-vibrations does not affect much the angular modes, the change of the deformation parameter caused by the β-vibrations have an effect on the angular dependence of the potential. We account for this effect by taking the dynamical deformations of the fragments of DNS from [15] instead of the ground-state deformations.

The Hamiltonian of the model can be presented in the form

$$\begin{aligned}
&\hat{H} = \hat{H}_0 + \hat{V}_{int} \\
&\hat{H}_0 = -\frac{\hbar^2}{2B_\xi}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + \frac{\hbar^2}{2\Im_h}\hat{l}_h + \frac{\hbar^2}{2\mu R_m^2}\hat{l}_0 + V(\xi, R_m) \\
&\hat{V}_{int} = \frac{C_0\xi}{2}\sum_\mu Y^*_{2\mu}(\Omega_h)Y_{2\mu}(\Omega_0),
\end{aligned} \tag{6}$$

where

$$\hat{l}_i = -\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} - \frac{1}{\sin^2\theta_i^2}\frac{\partial^2}{\partial\phi_i^2}, \qquad (i = 0, h), \tag{7}$$

$\mu = m_0\frac{A_1A_2}{A}$ is the reduced mass of the DNS and $\Im_h$ is the moment of inertia of the heavy fragment. We set $\Im_h = c_0\Im_h^{(r)}$, where $\Im_h^{(r)}$ is the rigid body moment of inertia of the heavy fragment, and c_0 is a scaling parameter, which is fixed to describe the energy of first 2^+ state as the lowest 2^+ solution of the model Hamiltonian.

In eq. (6), B_ξ is the mass parameter for the motion in mass–asymmetry. The procedure of calculation of B_ξ is given in [16]. Our calculation show that B_ξ is a smooth function of mass number A. As as consequence we take nearly the

same value of $B_\xi = 20 \times 10^5\ m_0\text{fm}^2$ for all considered nuclei with a variation of 10%.

The Hamiltonian (6) is diagonalized on the set of basis functions

$$\Phi^{l_1,l_2,n}_{LM,p} = F_n(\xi)\left[Y_{l_1}(\Omega_h) \otimes Y_{l_2}(\Omega_{\mathbf{R}})\right]_{(L,M)}, \tag{8}$$

with $p = (-1)^{l_1+l_2}$, n=0,1,2,..., l_1=0,2,4,..., l_2=0,1,2... Here, $F_n(\xi)$ satisfy the condition $L'_n(0) = 0$ for a mononucleus configuration (for example, radial wave-functions for the two-dimension oscillator). The angular part of the wave function (8) is given by the bipolar spherical harmonics, which provides the proper transformation with respect to the rotation and space inversion. Since we assumed that the heavy fragment is the axially–symmetric quadrupole rotator the quantum number l_1 can take only even values.

3 Results of Calculation

The excitation spectra obtained by solving the Schrödinger equation with Hamiltonian (6) are presented on Figure 2 together with available experimental data for ^{230}Th,^{232}U, ^{238}U, and ^{240}Pu nuclei. These are representative for all considered nuclei. The experimental energies and spin and parity assignments are taken from [17]. One can see that the structure of the calculated spectra are very similar for all considered nuclei. To elucidate the peculiarities of this structure, the spectrum provided by the model can be approximately interpreted by assuming that mass–asymmetry motion can be roughly separated from the angular motion and examining what function from the basis set (8) contribute most to the wave functions of states of a given rotational band.

For the ground–state rotational band the angular part of the wave–function of the state with angular momentum L can be approximately given by the function $[Y_{l_1=L}(\Omega_h) \otimes Y_{l_2=0}(\Omega_0)]_{(L,M)}$. Since $p = (-1)^{l_1+l_2}$ and l_1 can take only even values, the ground–state band contains only states of positive parity and even angular momentum. The wave functions of the lowest negative parity states have the dominant contribution from the component $[Y_{l_1=L}(\Omega_h) \otimes Y_{l_2=1}(\Omega_0)]_{(L+1,M)}$. Such a solutions form a rotational band containing the state of odd angular momentum and negative parity. This band can be interpreted as $K^\pi = 0^-$ band. One should note, however, that each eigenfunction of Hamiltonian (6) is supperposition of states with different K values, thus K can be considered as a quantum number only approximately. The next lowest band of negative parity is formed from the states with main contribution of $[Y_{l_1=L}(\Omega_h) \otimes Y_{l_2=1}(\Omega_0)]_{(L-1,M)}$ component for odd angular momentum and $[Y_{l_1=L}(\Omega_h) \otimes Y_{l_2=1}(\Omega_0)]_{(L,M)}$ component for even angular momentum. This band can be approximately interpreted as $K^\pi = 1^-$. In this case the angular momentum created by the vibration is taken away from the total angular momentum thus increasing the energy of this rotational band with respect to the states of 0^- band.

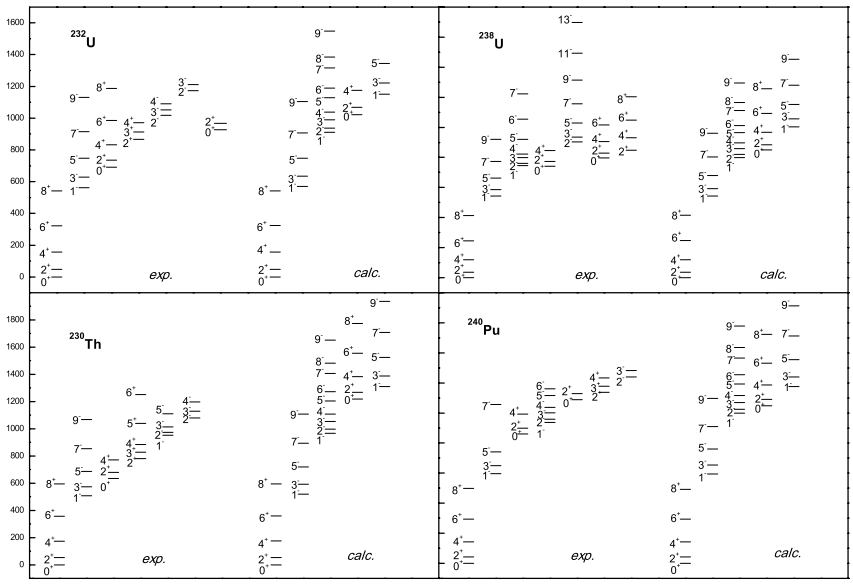

Figure 2. Calculated and experimental low–energy level schemes of ^{230}Th, ^{232}U, ^{238}U, ^{240}Pu. Experimental energies, spin and parity assignments are taken from [17].

The first excited 0^+ state is obtained as a lowest excitation in the mass-asymmetry coordinate. Note that $\beta-$ and $\gamma-$ vibrations are not included in the calculations. Thus we expect that that the lowest spectra of actinides will contain at least two 0^+ states at the energy of around 1 MeV. The sequence of rotational bands discussed above is repeated again with excited 0^+ playing a role of the ground state.

One can see from the Figure 2 that the overall agreement between the calculated and experimental spectra are rather good. The ground–state and first negative–parity band and the first excited 0^+ band are reproduced quite well for most considered cases. However, beyond the lowest negative–parity band the situation is not so clear. In ^{232}U and ^{230}Th the next lowest band is claimed to be $K^\pi = 2^-$, while in some cases (not presented here) is claimed to be $K^\pi = 3^-$ as for ^{236}U and ^{242}Pu. In contrast, in our calculations, the next lowest rotational band is always built upon 1^- state. If the model is correct this suggests that those $K^\pi = 2^-$ and $K^\pi = 3^-$ bands are in fact starts with 1^-, with lowest states experimentally missing.

The interesting quantity is the parity splitting defined as the energy shift between the states of 0^- and 0^+ rotational bands. Dependence of the experimental and calculated values of a parity splitting in the ground state and the first negative parity bands treated as a unified alternating parity band, on angular momentum

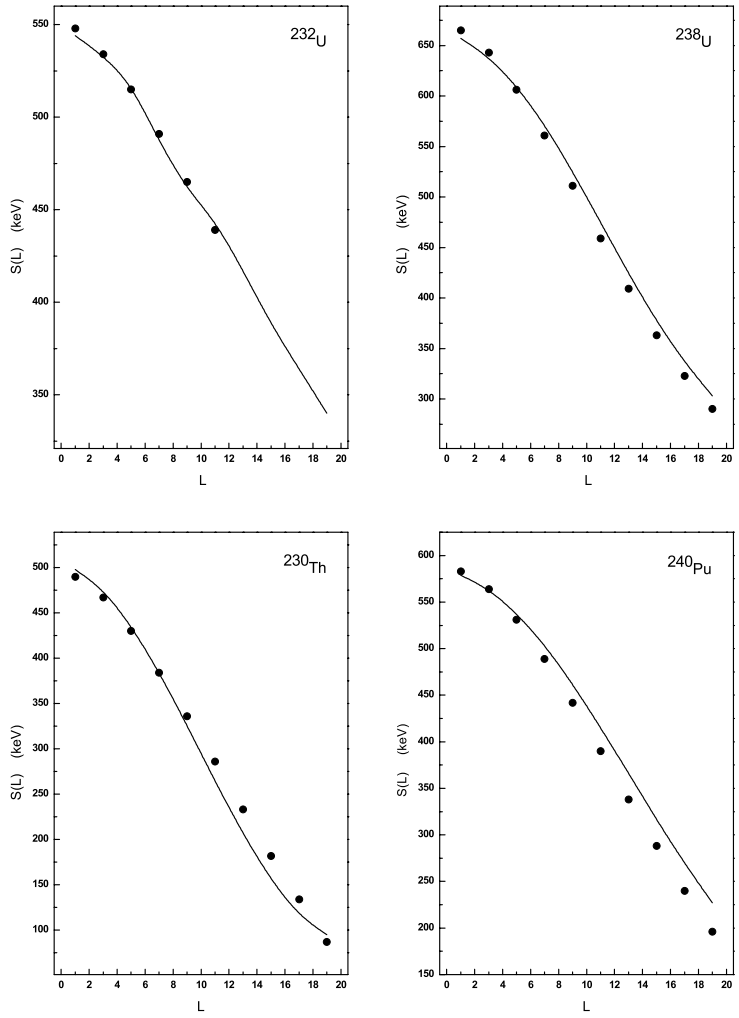

Figure 3. Calculated (lines) and experimental (solid circles) values of parity splitting (see Eq.(9)). Experimental values are taken from [17].

is illustrated in Figure 3. The parity splitting is defined by the expression

$$S(I^-) = E(I^-) - \frac{(I+1)E^+_{(I-1)} + IE^+_{(I+1)}}{2I+1}, \qquad (9)$$

which provides zero value of parity splitting for the rotational band of nucleus with rigid octupole deformation.

Having maximum value in the beginning of the band, the parity splitting tends to decrease with increase of angular momentum. In the proposed model

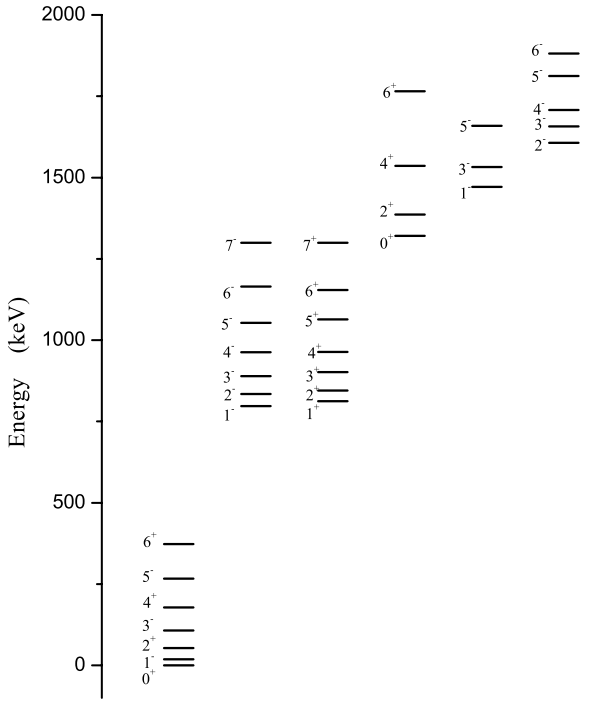

Figure 4. Calculated level scheme of dinuclear system α+^{234}Th.

the value of parity splitting is related to the weight of the α–cluster dinuclear system component in the intrinsic wave function of the nucleus. To show this, on Figure 4 the spectrum for the pure α–cluster DNS is presented. In this imaginary case the weight of mononucleus configuration is zero. One can see that in this case the ground–state band has the form of unperturbed alternative parity rotational band as it is expected for the stable reflection asymmetric shape. Thus, for α–particle DNS the parity splitting is close to zero for all values of angular momentum. It is also clear that in opposite case, when the weight of DNS configurations in the wave function is zero we deal with quadrupole–deformed nucleus and the energies of the negative parity states become infinite granting infinite value of parity splitting. Thus, the parity splitting is determined by the relative weights of the cluster components and the mononucleus in the wave function. Due to the larger moment of inertia than that of the mononucleus, the weight of α-particle DNS is increasing with angular momentum providing decrease of parity splitting.

4 Conclusion

We have suggested a cluster interpretation of the properties of the multiple negative parity bands in deformed even–even actinides. The collective motion related to the cluster degree of freedom leads to the admixture of the very asymmetric cluster configurations to the intrinsic nucleus wave function. To take into account the reflection asymmetric modes with nonzero values of K, the rotational excitations of the heavy cluster is taken into account. The resulting energy spectrum consists of the ground state band, the excited 0^+ band and several negative parity rotational bands which can be approximately interpreted as $K^\pi = 0^-$ and $K^\pi = 1^-$ bands. The angular momentum dependence of the parity splitting is described. The results of calculations are in overall agreement with the experimental data. The model suggests the interpretation of second negative parity band in the spectra of the even-even deformed actinides as $K^\pi = 1^-$.

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