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Order Amidst Chaos: The Alhassid-Whelan Arc of Regularity

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Abstract. A Quasi Dynamical Symmetry (QDS) is a situation where a symmetry is broken, however observable quantities such as energies or transition strenghts behave as if the symmetry were present. We study a QDS situated in the interior of the symmetry triangle of the Interacting Boson Model, based on SU(3) symmetry, and its locus. This QDS lies close to the Alhassid–Whelan arc of regularity, a narrow region inside the triangle where spectra exhibit a high degree of order.

1 Introduction

The study of the persistence of a symmetry, in the context of the Interacting Boson Model (IBM) [1], in spite of strong symmetry-breaking interactions, has been recently a subject of great interest [2–5]. The study of the interior of the symmetry triangle of the IBM [6] has led to some puzzling results. It was the original work of Alhassid and Whelan [7–9] which brought to the surface a region inside the triangle where regularity is emerging among chaos.

This study is related to work done in Ref. [10]. Before proceeding we have to discern the differences between the present study and the work reported in [10]. In the previous work the arc of regularity has been studied by considering only the lowest part of the spectrum. In the present study the full spectrum will be considered. The difference from the original work done by Alhassid and Whelan is that a much larger number of bosons will be used.

In Ref. [10] one of the measures of deviation from SU(3) used, was based on the existence of degeneracies in the presence of a symmetry. However, at higher energies the degeneracies break down, while the symmetry is still present. For higher energies statistical tools have to be used. In order to determine the fluctuation properties of the unfolded energy levels [11, 12], the nearest- neighbour level spacing distribution, P(S), is used, which is the probability that two adjacent energies differ by an amount of S.

In Section 2 we describe the Hamiltonian of the model, in Section 3 the statistical measure of chaos is introduced and in Section 4 numerical results are illustrated.

2 The Hamiltonian

We work in the context of the Interacting Boson Model (IBM). Low lying collective states in nuclei can be described in terms of a monopole boson called s, with spin 0 and a quadrupole boson, called d, with spin 2. The 36 bilinear combinations $(s^\dagger s, s^\dagger \tilde{d}_\mu, d^\dagger_\mu s, d^\dagger_\mu \tilde{d}_\nu)$ form a U(6) dynamical algebra. The IBM has 3 dynamical symmetries, U(5), appropriate for spherical vibrational nuclei, SU(3), suitable for prolate deformed nuclei and O(6), proper for certain axially asymmetric nuclei. These dynamical symmetries are placed at the vertices of the symmetry triangle (Casten's triangle) [6], shown in Figure 1, which is the parameter space of the model.

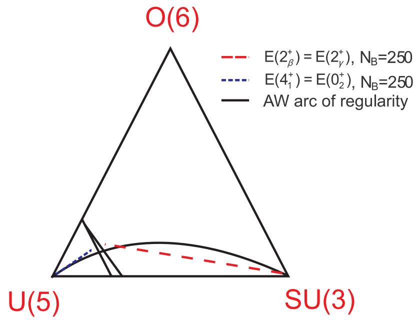

Figure 1. IBA symmetry triangle in the parametrization of Eq. (1) with the three dynamical symmetries and the Alhassid–Whelan arc of regularity. The shape coexistence region [13] is shown in slanted lines. The loci of the degeneracies $E(2^+_\beta) = E(2^+_\gamma)$ (dashed line on the right, corresponding to the QDS discused in [10] and $E(4^+_1) = E(0^+_2)$ (doted line on the left) are shown for $N_B = 250$.

In what follows we use the IBM Hamiltonian [7–9]

$$H(\eta, \chi) = c\left[\eta \hat{n}_d + \frac{\eta - 1}{N_B}\hat{Q}_\chi \hat{Q}_\chi\right], \tag{1}$$

where $\hat{n}_d = d^\dagger \tilde{d}$ is the d boson number operator, $\hat{Q}_\chi = (s^\dagger \tilde{d} + d^\dagger s) + \chi(d^\dagger \tilde{d})^{(2)}$ is the quadrupole operator, and N_B is the number of valence bosons. The parameters (η, χ) are the coordinates of the triangle and serve for symmetry breaking. η ranges from 0 to 1, and χ ranges from 0 to $\frac{-\sqrt{7}}{2} = -1.32$. Numerical calculations of energy levels have been performed using the code IBAR [14] which can handle bosons up to N=250.

3 Quantal Signatures of Chaos

In order to analyze the statistical fluctuations of the spectrum one first has to follow the procedure of the purification of the spectrum, which means remove structural details and keep just the statistical properties. To do so, one constructs a staircase function of the spectrum, N(E), and fits a 6th order polynomial to the staircase function. Then, the unfolded spectrum is obtained using the mapping $\tilde{E}_i = N(E_i)$ and the nearest spacing neighbour level spacings are obtained from $S_i = \tilde{E}_{i+1} - \tilde{E}_i$. These normalised spacings are assigned to bins and the histogram which is constructed represents the spacing distribution P(S) [9].

The distribution P(S) can then be fitted to the Brody distribution [15] parametrized by ω,

$$P(S) = (1+\omega)RS^{\omega}\exp(-RS^{1+\omega}), \tag{2}$$

where $R = \Gamma\left[\frac{2+\omega}{1+\omega}\right]^{1+\omega}$. The Brody distribution interpolates between Poisson statistics ($\omega = 0$), which characterize a regular system, and the Wigner distribution ($\omega = 1$), which corresponds to a chaotic system.

4 Numerical Results

Calculations were performed at four different points, as is demonstrated in Figure 2, namely on the SU(3) vertex, a point on the arc having parameters $(\eta, \chi) = (0.632, -0.882)$ and at two random points, at $\chi = -0.7$ and $\chi = -1.1$. In this work the boson number is chosen to be $N_B = 175$. This gives 2640 states for L=0 and 5192 states for L=2. The results obtained for P(S) (with the fitted parameter ω) are illustrated in Figures 3–5 and in Table 1.

Some comments are now in place.

The nearest neighbour level spacing distribution, P(S), for the four different cases, for L=0, is shown in Figure 3.

At the SU(3) vertex the distribution is pushed to the left, indicating a very high degree of regularity. The fit with the Brody distribution results in a negative number ($\omega = -0.52$), an indicator of strong presence of degeneracies [9]. At the point on the arc, the distribution is again pushed to the left, while the Brody

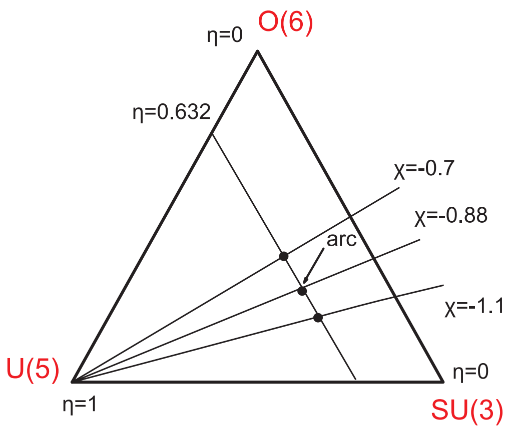

Figure 2. The four different points where calculations were performed.

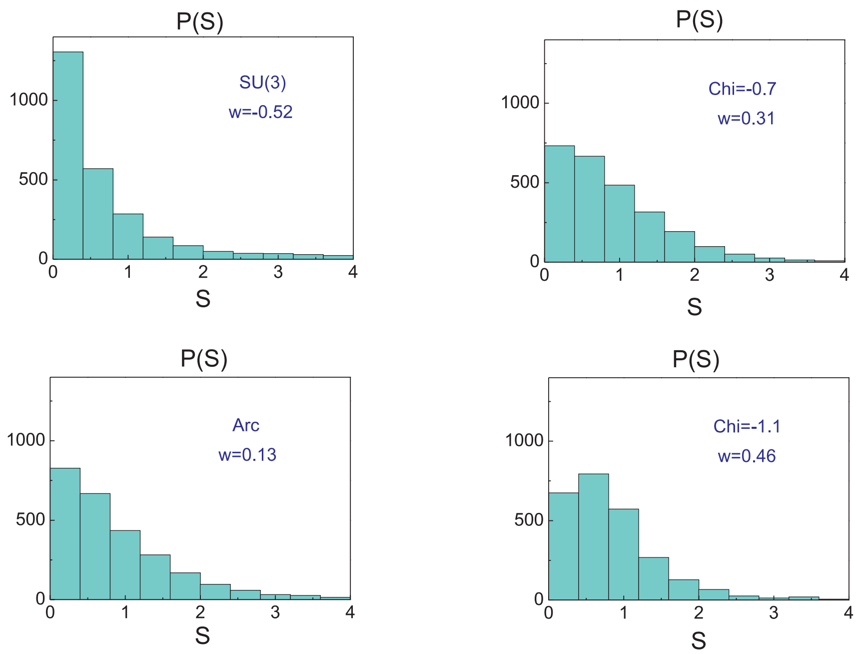

Figure 3. The histograms for the whole spectrum, for the 4 different points described in the text, for L=0 and $N_B = 175$ bosons. The ω values are also shown for each particular point.

fit provides a rather low number ($\omega = 0.13$), indicating a high degree of regularity. At the points off the arc, with $\chi = -0.7$ and $\chi = -1.1$ respectively, we remark that the distribution starts moving to the right, especially in the second case, indicating an increasing degree of chaoticity. The Brody fits give $\omega = 0.31$

Table 1. Numerical values of ω, for the 8 parts of each spectrum. The states 1-330 are labelled as interval 1, the states 2311-2640 are labelled as interval 8.

Interval	SU(3) ω	Arc ω	$\chi = -0.7$ ω	$\chi = -1.1$ ω
1	-0.16	-0.02	0.07	-0.12
2	0.33	1.15	0.67	0.75
3	0.16	0.41	0.66	1.13
4	0.14	0.32	0.40	0.93
5	0.10	0.01	0.20	0.56
6	-0.05	0.07	0.22	0.36
7	0.13	0.08	0.23	0.19
8	0.16	-0.12	0.05	0.01

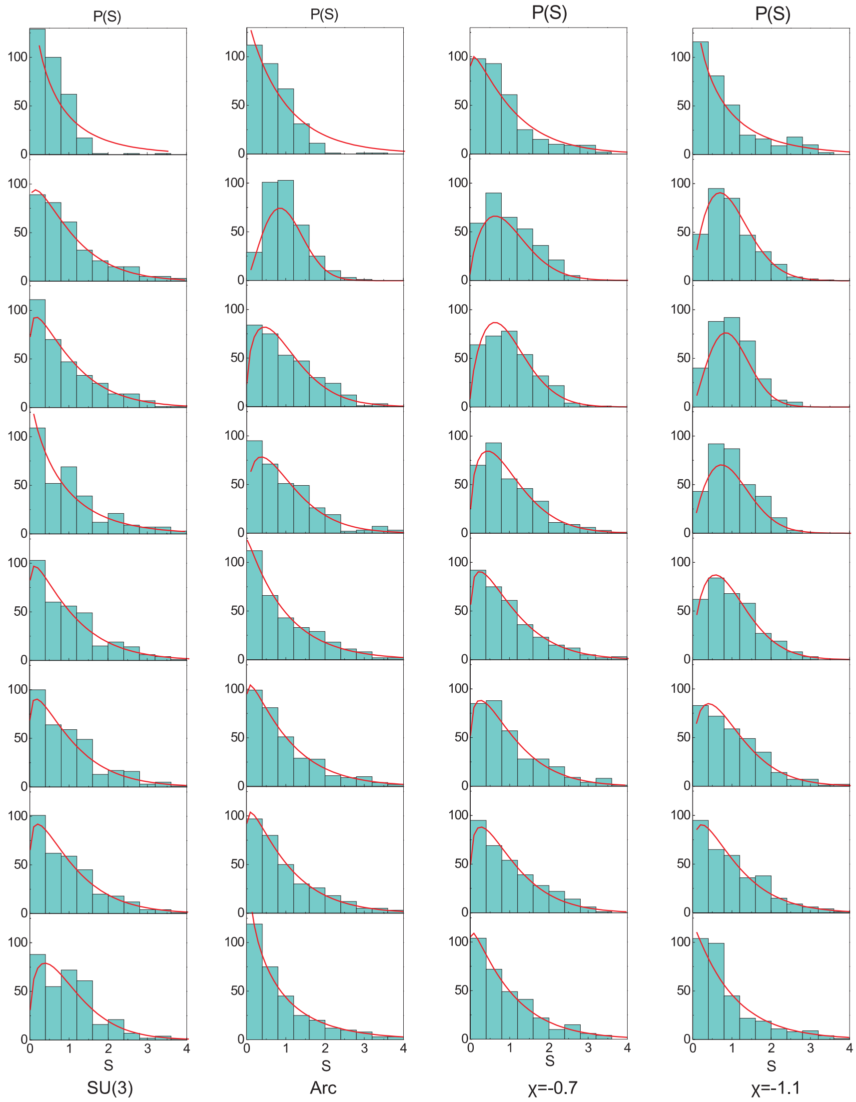

Figure 4. The histograms obtained when the set of 0^+ states in each of the cases depicted in Figure 3 is divided into 8 sets of 330 states each. The states 1-330 (labelled as interval 1 in Table 1) appear on the top, the states 2311-2640 (labelled as interval 8 in Table 1) appear at the bottom.

and $\omega = 0.46$ respectively, in agreement with the qualitative expectation coming from the shape of the distribution. The same comments can be applied to Figure 5 (with the SU(3) case missing).

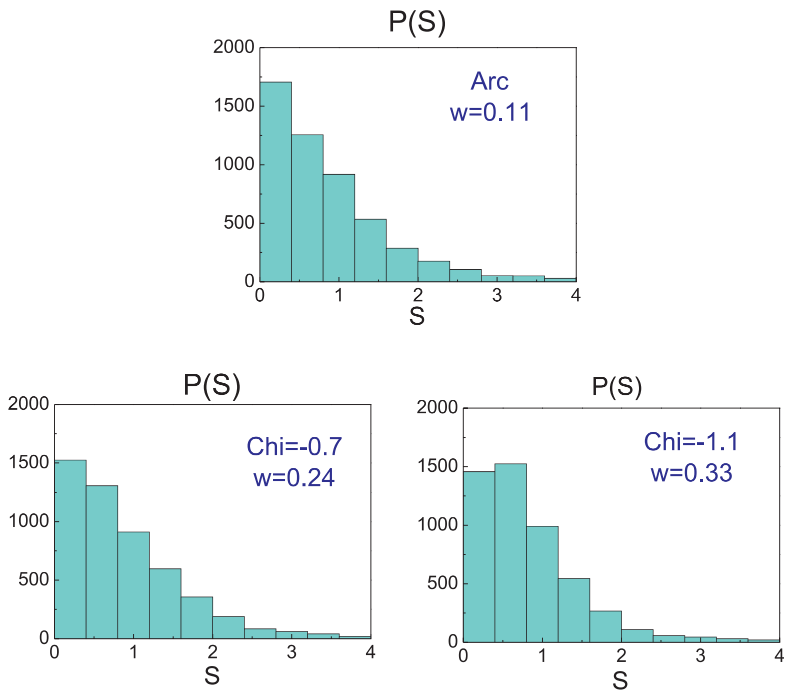

Figure 5. The histograms for the whole spectrum, for the 3 different points described in the text for L=2 and $N_B = 175$ bosons. The ω values are also shown for each particular point.

It is instructive to divide the set of 0^+ states, in each of the cases mentioned above, into 8 sets of 330 number of states each. Then, as seen in Figure 4, irregular behavior is spread all over the spectrum in the $\chi = -0.7$ and $\chi = -1.1$ cases (except the lowest and the highest parts). However, irregularity is confined to a smaller number of states as one approaches the arc or the SU(3) vertex.

5 Conclusion

We have shown the presence of a QDS based on SU(3) symmetry in the interior of the triangle using the nearest neighbour level spacing distribution P(S). Further calculations with different number of bosons, different L values and other statistical measures of chaos promise to give intriguing results.

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