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$E0$ Decay of 0_2^+ States in the Rare-Earth Region: The Case of ^{156}Dy and ^{160}Er

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Abstract. The branching between the $E0$ $0_2^+ \rightarrow 0_1^+$ and the $E2$ $0_2^+ \rightarrow 2_1^+$ transitions in ^{156}Dy and ^{160}Er were measured following the ε decay of ^{156}Ho and ^{160}Tm, respectively. The experimental results are compared to theoretical calculations, using a generalized potential in the 4^{th} order of the deformation parameter β, which describes the violation of spherical symmetry in nuclei. This potential was chosen to cover the whole transitional path from from a spherical harmonic vibrator to an axially deformed rotor, since a dynamic symmetry transition, denoted $X(5)$, is suggested to occur for the $N = 90$ rare-earth nuclei. The lifetimes of these states provide independent information for the $E2$ strength and are used for the extraction the $E0$ strength.

Introduction

In the geometrical description of collective nuclear motion there are three limiting cases, corresponding to the harmonic vibrator, the symmetrically deformed rotor and the triaxial rotor. Each of them is associated with a particular nuclear shape, spherical, axial-ellipsoidal, and triaxial. The transition from a spherical harmonic vibrator to an axially deformed rotor was described analytically by Iachello, introducing a dynamic symmetry, denoted X(5) [1]. It is interesting in this context to investigate the $E0$ transition strength, in the decay of the first excited 0^+ state, because this quantity is known to have particularly large values in the transitional region between spherical and deformed nuclei [2,3]. This fact can be explained by strong mixing of states with different deformation [4], or within the framework of the Interacting Boson Model (IBM) [5], by mixing of states with different number of bosons [3]. Electric monopole, $E0$, transitions are forbidden by the γ-decay selection rules and can occur only via the emission of atomic electrons. Electron emission is the only process by which a 0^+ state can decay to another 0^+ state. If the nuclear spin is non-zero, electric quadrupole $E2$, and $E0$ transitions compete with each other. The $E0$ transition probability is factorized into electron and nuclear terms [6],

$$W = \Omega \rho^2(E0)\,, \tag{1}$$

where Ω represents all the "non-nuclear" contributions and can be calculated from several atomic models [7–9]. The nuclear structure information is contained in $\rho^2(E0)$ and can be directly related to different nuclear models.

Considering a simple collective geometrical model, Rasmussen estimated the dimensionless ratio of the reduced $E0$ and $E2$ transition probabilities and demonstrated that it is proportional to the deformation, β [10]

$$X(E0/E2; 0_2^+ \to 0_1^+) = e^2 R_0^4 \cdot \frac{\rho^2(E0; 0_2^+ \to 0_1^+)}{B(E2; 0_2^+ \to 2_1^+)} = 4\beta^2\,, \tag{2}$$

where R_0 is the nuclear radius. This ratio can be compared with the experiment as [11]

$$X(E0/E2; 0_2^+ \to 0_1^+) = 2.54 \cdot 10^9 A^{4/3} E_\gamma^5 q^2 \alpha_K(E_\gamma)/\Omega\,, \tag{3}$$

where E_γ is the energy of the $E2$ transition in MeV and $\alpha_K(E2)$ is the K-conversion coefficient for the $E2$ transition. The quantity q^2 is measured in the experiment as

$$q^2 = \frac{A_{e,K}(E0)}{\epsilon_{e,K}(E0)} \cdot \frac{\epsilon_{e,K}(E2)}{A_{e,K}(E2)}\,, \tag{4}$$

where $A_{e,K}(E\lambda)$ is the intensity of the K line for the corresponding transition and $\epsilon_{e,K}(E\lambda)$ is the efficiency of the β spectrometer.

The ratio of the reduced transtion probabilities $X(E0/E2)$ in ^{156}Dy and ^{160}Er was studied recently [12]. The $N = 90$ nucleus ^{156}Dy is considered as a good candidate for $X(5)$ symmetry [13], while ^{160}Er, which has a structural parameter $R_{4/2} = E(4_1^+)/E(2_1^+) = 3.1$, lies on the transition path between the critical point of the vibrator-rotor shape phase transition and the rigid rotor limit.

Experiment and Results

The experiments were performed at the INFN Laboratori Nazionali del Sud (LNS) in Catania. Partial level schemes of ^{156}Dy and ^{160}Er are presented in Figure 1.

Levels in ^{156}Dy were populated by the $^{156}\text{Er} \to {}^{156}\text{Ho} \to {}^{156}\text{Dy}$ ϵ decay chain. Excited states in ^{156}Er were populated in the ^{148}Sm(^{12}C,$4n$) reaction at 73 MeV. The ^{12}C beam was provided by the LNS tandem accelerator and the target was a thin self-supporting isotope-enriched foil having a thickness of $\approx$ 0.8 mg/cm^2. Since the $^{156}\text{Er} \to {}^{156}\text{Ho}$ and the $^{156}\text{Ho} \to {}^{156}\text{Dy}$ decays have half lives of 19.5 min and 56 min, respectively, the experiment was performed by repeating cycles in which the ^{148}Sm target was irradiated for one hour and, after a 5 min delay, the decay of ^{156}Ho was measured for one hour by detecting off-beam γ rays and conversion electrons. In-beam spectra, produced by the prompt de-excitation of ^{156,157}Er levels, were also collected during the irradiation periods for calibration purposes.

Excited states in ^{160}Er were populated through the ^{160}Tm $\rightarrow$ ^{160}Er ϵ decay chain. ^{160}Tm was produced by the ^{150}Sm(^{14}N,$4n$) reaction at 72 MeV. The target was a thin self-supporting isotope-enriched foil with a thickness of 0.612 mg/cm^2. The half live of ^{160}Tm is 9.4 min, therefore, a ten-minute on-off beam cycle was used. The data was collected in the beam-off intervals.

In both cases the β decay of the parent nuclei populates low-spin states in the daughter nuclei. Schematically the technique is illustrated in Figure 2. It has the advantage that the β decay passes through low-spin states. As a result,

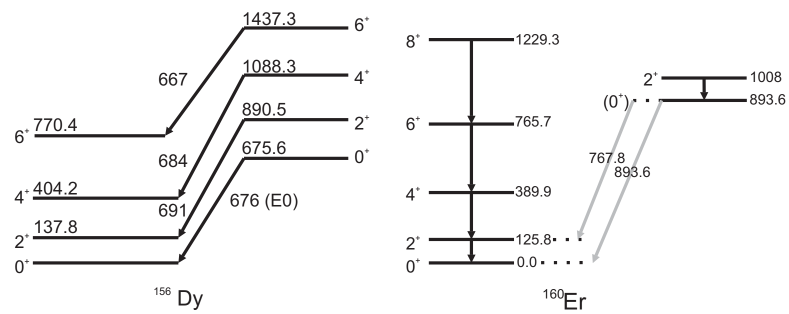

Figure 1. Partial level schemes of ^{156}Dy (left) and ^{169}Er (right), revealing the decay of the bands which are built on the 0_2^+ states.

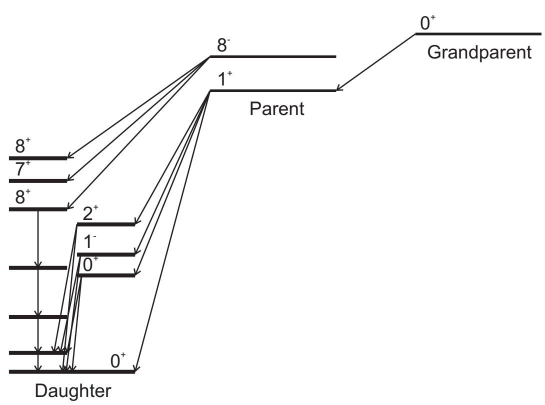

Figure 2. Schematic presentation of the population of non-yrast states in off-beam β-decay experiments.

the population of non-yrast states in the nucleus of interest is enhanced and relatively clean spectra are obtained. Another advantage of the technique is that the measurement is done off-beam, which reduces the background of the electron spectra.

Excited states in ^{156}Dy are populated in the β decay of ^{156}Ho. There are three β-decaying states in ^{156}Ho – the 4^- ground state ($T_{1/2} = 56(1)$ min) and the 1^- ($T_{1/2} = 9.5(15)$ s) and 9^+ ($T_{1/2} = 7.8(3)$ min) isomers. In the reaction the grandparent even-A nucleus ^{156}Er is produced, which β-decays to low-spin states in ^{156}Ho. In this way the β-decay of the 9^+ isomer is suppressed, because this state is weekly populated and the decay goes through the 4^- ground state and the 1^- isomer.

Non-yrast states in ^{160}Er are populated by the β decay of ^{160}Tm. In this case the β decay goes through the 1^- ground state ($T_{1/2} = 9.4(3)$ min) and the $I = 5$ isomer ($T_{1/2} = 74.5(15)$ s). With the selected beam-on/beam-off cycle the isomer β decay contributes little to the recorded spectra.

The measured γ-ray and conversion electron spectra, revealing the decay of the 0_2^+ states in ^{156}Dy and ^{160}Er are presented in Figure 3. The γ rays were measured with a coaxial HPGe detector which was positioned at $90°$ degrees with respect to the beam. Conversion electrons were measured with a mini-orange spectrometer, a magnetic lens made by permanent Sm-Co magnets, shaped like orange slices, and a 3 mm-thick Si(Li) detector cooled at liquid-nitrogen temperature. The mini-orange spectrometer was positioned at $135°$ with respect to the beam, in backward direction.

Results and Discussion

The 676-keV $E0$ transition in ^{156}Dy is rather weak (see the lower panel in the right part of Figure 3). Therefore, possible sources of contamination were analysed carefully to avoid systematic errors. Using the γ-ray intensities from the experiment and the conversion coefficients of Ref. [14], $q^2 = 1.97(70)$ was obtained. Estimating the electronic factor with the method of Kantele [9] as $\Omega = 4.05 \cdot 10^{10}$ s^{-1}, the ratio of the reduced transition probabilities is $X(E0/E2; 0_2^+ \to 0_1^+) = 0.045(17)$. The γ-ray intensities, which were measured by Caprio et al. [13], can also be used in the analysis, since excited states in ^{156}Dy were populated in exactly the same way in both experiments. They yield $q^2 = 2.17(74)$, in perfect agreement with the value above.

These values can be compared with results from a recent compilation of the $E0$ strength [11], where the authors re-evaluated the existing data. For ^{156}Dy the results are: $q^2 = 3(2)$ and $X(E0/E2; 0_2^+ \to 0_1^+) = 0.08(5)$. The results are in agreement with each other, but with the present experiment, the uncertainty was reduced considerably.

In Table 1 the $X(E0/E2)$ ratios of reduced transition probabilities for several other transitions, which de-excite the band, which is built on the 0_2^+ state, are presented [15]. These, combined with measurements of lifetimes [16] and

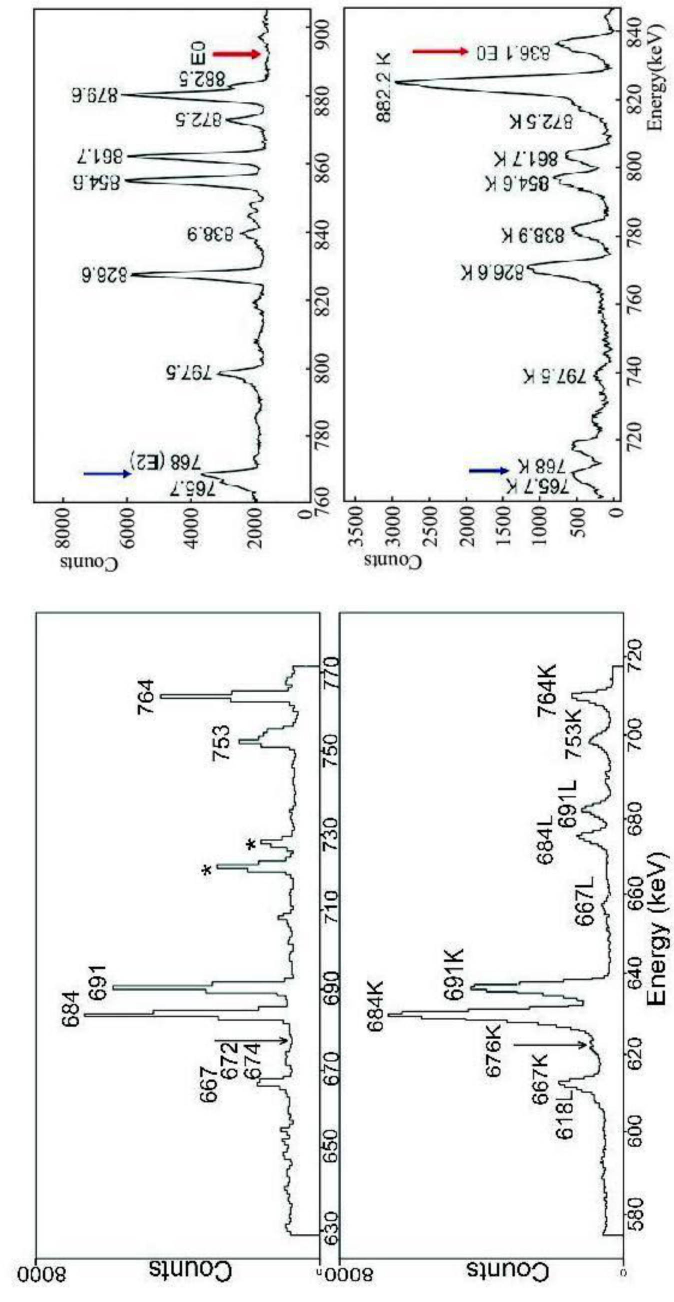

Figure 3. (left) Partial off-beam γ-ray spectrum (upper panel) and conversion electron spectrum (lower panel), obtained in the ϵ decay of ^{156}Ho; (right) Partial off-beam γ-ray spectrum (upper panel) and conversion electron spectrum (lower panel), obtained in the ϵ decay of ^{160}Tm

Table 1. Ratios of reduced transition probabilities, lifetimes and $E0$ strength of transitions connecting the β band and the ground-state band in ^{156}Dy.

E_γ, keV	$I^\pi_\beta \rightarrow I^\pi_{gs}$	q^2	$X(E0/E2)$	τ, ps	$\rho^2(E0)$
666.9	$6^+_\beta \rightarrow 6^+_{gs}$	3.8(8)	0.163(34)	5.14(34)	58(26)
675.6	$0^+_\beta \rightarrow 0^+_{gs}$	1.9(7)	0.045(17)		
684.1	$4^+_\beta \rightarrow 4^+_{gs}$	3.4(7)	0.158(32)	6.5(17)	61(5)
690.9	$2^+_\beta \rightarrow 2^+_{gs}$	2.7(6)	0.127(26)		

conversion coefficients [15] of the levels of interest, allow the $E0$ strength of these transitions to be extracted.

The $X(E0/E2)$ ratios of the $0^+_2 \rightarrow 0^+_1$ transitions in the $N = 90$ nuclei are presented in Table 2, together with values of the deformation parameter β, which was calculated within the approximation of Rasmussen [10]. The results indicate that the $N = 90$ nuclei have moderate deformation.

Table 2. $E0/E2$ branching ratio of $0^+_2 \rightarrow 0^+_1$ transitions in $N = 90$ nuclei and deformation of transitional nuclei.

Isotope	$X(E0/E2; I^\pi_\beta \rightarrow I^\pi_{gs})$	β
^{152}Sm	0.074(6)	0.14(2)
^{154}Gd	0.048(20)	0.11(7)
^{156}Dy	0.045(17)	0.11(6)
^{158}Er	0.039(7)	0.10(2)

In the case of ^{160}Er a more intense $0^+_2 \rightarrow 0^+_1$ transition was observed (see the lower panel in the right part of Figure 3). With the mini-orange efficiency calibration and the γ-ray intensities of the experiment and the conversion coefficients of Ref. [14], $q^2 = 4.3(7)$ was obtained [17]. Estimating the electronic factor with the method of Kantele [9] as $\Omega = 7.04 \cdot 10^10$ s^{-1}, the ratio of the reduced transition probabilities is $X(E0/E2; 0^+_2 \rightarrow 0^+_1) = 0.18(4)$. The deformation parameter β in this case takes a somewhat larger value, $\beta = 0.22(13)$ compared to the deformation parameters of the $N = 90$ nuclei from Table 2.

Theoretical calculations

The $X(5)$ critical point symmetry [1] is a solution of the Bohr Hamiltonian with a special choice of the potential: $v(\beta, \gamma) = u(\beta) + v(\gamma)$. This potential allows an approximate separation of variables and then the potential in β is chosen as an infinite square well. This choice comes from the fact that, using the coherent state formalisms in the IBM, one can obtain the potential that (only) at the critical point goes as $\sim \beta^4$ and therefore it can be approximated with an infinite square well [18]. In a similar way, for the $U(5) - SU(3)$ first order shape phase transition, a more general potential in β can be introduced, the "Lo Bianco potential”, that is parametrized as follows:

$$u(\beta) = V_0(\zeta\beta^4 - 2\zeta\beta_0\beta^3 + (1-\zeta)\beta_0^2\beta^2)\,, \qquad (5)$$

with $0 < \zeta < 1$. When $\zeta = 0$ there is a spherical minimum, at the critical point with $\zeta = 1/2$ there are two coexisting minima (with a very small bump in between), one in $\beta = 0$ and the other in $\beta = \beta_0$, and at $\zeta = 1$ there is only a unique deformed minimum in $3/2\beta_0$. This potential has the virtue of covering the whole transitional path of the shape phase transition at the price of having three parameters instead of the parameter-free predictions of the $X(5)$ model. The parameters, especially V_0, can be adjusted according to phenomenology to reproduce a subset of low-lying energy levels. Results for the excitation spectra of ^{160}Er are displayed in Figure 4.

The calculations indicate that ^{156}Dy lies in the spherical region, $\zeta \approx 0.1$, while ^{160}Er is rather close to the critical point, $\zeta \approx 0.53$, but the deformed minimum is already winning. Matrix elements of the $E0$ operator between 0^+ states in ^{156}Dy and ^{160}Er were obtained, which allows a straightforward calculation of the $E0$ strength [12]. However, the comparison with the experimental data is not very good because the model underestimates the experimental results by a factor of four. One could wonder whether this is a shortcoming of the collective model, rather than attributing it to details of the potential.

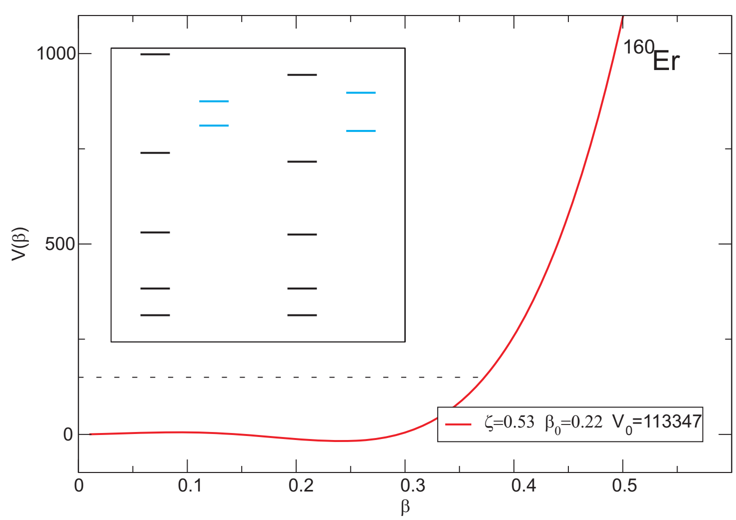

Figure 4. Potential energy (large panel) to fit with the experimental levels for ^{160}Er. The ground state is indicated by a dotted line. The inner panel contains the experimental (left) and calculated (right) lowest energy levels, 0^+, 2^+, 4^+, 6^+ and 8^+ of the *gs* band, and 0^+ and 2^+ of the β band in reduced energy units.

Conclusions

The ratios of the reduced transition probabilities $X(E0/E2)$ in ^{156}Dy and ^{160}Er have been measured and compared with calculations using a potential, which is a combination of quadratic, cubic and quartic powers of β. The results indicate that ^{156}Dy is rather in the spherical region, while ^{160}Er is located in the deformed region, but quite close to the critical point for the $U(5)-SU(3)$ first order shape phase transition.

Acknowledgements

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