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Pairing-Plus-Quadrupole Model in Two Oscillator Shells

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Abstract. An extended pairing-plus-quadrupole model is introduced. The pairing part of the Hamiltonian consists of pp-, nn- and pn-pairing terms and terms describing the pair-scattering between two oscillator shells. Energy spectra and shape parameters for nuclei from different shells are calculated and the role of the two parts in the interaction depending on the parameter strengths is discussed in detail. This investigation is the initial step towards creating an extended two-shell version of the microscopic (pseudo-)SU(3) shell model.

1 Introduction

In order to build a complete shell-model theory, intruder opposite-parity levels from the upper shell need to be included in the model space, especially if experimentally observed high-spin or negative-parity states are to be described. Intruder levels are present in heavy deformed nuclei where the strong spin-orbit interaction destroys the underlying harmonic oscillator symmetry of the nuclear mean-field potential. The role they play for the overall dynamics of the system has been the topic of many questions and debates [1–4]. Until now, the problem has been either approached within the framework of a truncation-free toy model [1] or by just considering the role of the single intruder level detached from its like-parity partners [2, 4]. It was argued in [1] that particles in these levels contribute in a complementary way to building the collectivity in nuclei. However, some mean-field theories suggest that these particles play the dominant role in inducing deformation [3].

Up to now, algebraic methodologies based on the existing good SU(3) [5] or pseudo-SU(3) symmetry [6] have not been applied to nuclei with mass numbers $A = 56$ to $A = 100$, which is an intermediate region where conventional wisdom suggests the break down of the assumptions that underpin their use in the other domains. In particular, the $g_{9/2}$ intruder level that penetrates down from the shell above due to the strong spin-orbit splitting appears to be as spectroscopically relevant to the overall dynamics as the normal-parity $f_{5/2}, p_{3/2}, p_{1/2}$ levels. Specifically, in this region the effect of the intruder level cannot be ignored or mimicked through a "renormalization" of the normal-parity dynamics which is how it has been handled to date. Moreover, both protons and neutrons occupy the same oscillator shells which suggests strong proton-neutron correlations. These facts should be addressed with the appropriate choice of a Hamiltonian.

The purpose of the present work is to introduce a pairing-plus-quadrupole model in two oscillator shells which, for the first time, explicitly includes particles from the complete unique-parity sector. After we shortly introduce the basics of the model, we present some results for the ground-state properties of two nuclear systems. Finally, we give suggestions for the use of a more realistic Hamiltonian, which will provide an opportunity for comparison with experimental results.

2 Pairing-Plus-Quadrupole Model in Two Shells

The pairing-plus-quadrupole model, first introduced by Bohr and Mottelson and Pines [7], and Belyaev [8], has been widely used to reproduce both few-particle non-collective and many-particle collective features of nuclei [9–11]. It incorporates those features that are most important in nuclear mean-field theories: the interaction between particles can be summed up, in a first approximation, to an average spherical single-particle potential; and long range particle-hole correlations and short-range particle-particle correlations can be taken into account by a deformation of the field and a pairing potential, respectively [12]. The model has never been applied for full-space calculations in more than one of the low-lying oscillator shells or for restricted number of particles in higher-lying shells.

SU(3) realization of the pairing-plus-quadrupole model in up to two (proton and neutron) spaces has been developed [13,14] and the effects of the quadrupole-quadrupole, identical-particle pairing, and even single-particle interactions have been studied. The basis states that were used in just one space were labeled as $|N[f]\alpha(\lambda,\mu)\kappa L, S; JM_J\rangle$ for one type of particles and as

$$|N[f]\alpha(\lambda,\mu)\kappa L,[f^c][PP'P'']\beta(ST);JM_JM_T\rangle \tag{1}$$

-for proton-neutron systems. In the above expression, N denotes the number of particles in the corresponding space, $[f]$ and $[f^c]$ are the spatial and its conjugate spin-isospin symmetry labels, and (λ,μ) - the SU(3) irrep label. Multiplicity indices α and β count different occurrences of (λ,μ) in $[f]$ and of (ST) in $[PP'P'']$ which will be explained later. These basis states are by construction directly linked to the shell-model Lie algebra $U(4\Omega)$, which contains the $SU(3)$ quantum numbers (λ_τ,μ_τ) for the proton and neutron systems. These labels correspond to the following chain of groups typical for the SU(3) model:

$$\begin{aligned} U(4\Omega) &\supset [U(\Omega)\times U(4)] \supset [SU(3)\times SU(4)] \\ &\supset [[SO(3)\times SU_S(2)]\times SU_T(2)] \\ &\supset [SU_J(2)\times SU_T(2)] \supset U_J(1)\times U_T(1). \end{aligned} \tag{2}$$

If we want to address the problem in more than one spaces (for example, four), we have to use SU(3) basis states of the type

$$|\{a_\pi; a_\nu\}\rho(\lambda,\mu)\kappa L, \{S_\pi, S_\nu\}S; JM_J\rangle \tag{3}$$

which are built as $SU(3)$ proton (π) and neutron (ν) strongly-coupled configurations with well-defined particle number and good total angular momentum J. Here, the proton and neutron quantum numbers are indicated by $a_\sigma = \{a_{\sigma N}, a_{\sigma U}\}\rho_\sigma(\lambda_\sigma, \mu_\sigma)$, where the $a_{\sigma\tau} = N_{\sigma\tau}[f_{\sigma\tau}]\alpha_{\sigma\tau}(\lambda_{\sigma\tau}, \mu_{\sigma\tau})$ are the basis-state labels for the four spaces in the model (σ stands for π or ν, and τ stands for normal (N) or unique (U) parity levels). First, the particles from the normal and the unique spaces are coupled for both protons and neutrons. Then, the resulting proton and neutron irreps are coupled to a total final set of irreps. The total angular momentum J results from the coupling of the total orbital angular momentum L with the total spin S. The ρ and κ are, respectively, the multiplicity indices for the different occurrences of (λ,μ) in $\{(\lambda_\pi,\mu_\pi)\times(\lambda_\nu,\mu_\nu)\}$ and L in (λ,μ).

The SU(3) classification of many-body states has the advantage of allowing for a geometrical analysis of the eigenstates of a nuclear system via the relations between the microscopic parameters (λ,μ) and the collective parameters (β,γ) of the collective model [15] and hence it gives an insight into phenomena associated with nuclear deformation. The deformation parameter β and the triaxiality parameter γ are given by the following formulae [16]:

$$k\beta = \frac{2}{3}\sqrt{C_2+3} \tag{4}$$

$$\cos(3\gamma) = \frac{C_3}{2\sqrt{(C_2+3)^3}}, \tag{5}$$

Here the constant $k = \sqrt{\frac{5}{9\pi}A\langle r^2\rangle}$, where A is the total number of nucleons, $\langle r^2\rangle$ – the nuclear mean square radius of the system, and C_2 and C_3 are the invariant Casimir operators of second and third order, respectively. This relation can be visualized on a plot shown in Figure 1 where β is the radius vector and γ the azimuthal angle. The (β,γ) vary continuously, while λ and μ take on positive integer values only.

Next we explain the other labels $[PP'P'']\,\beta(ST)$ in the basis (1) coming from the proton-neutron pairing that we consider in the present approach. Hence we account simultaneously for the isovector $(S=0, T=1)$ and the isoscalar $(S=1, T=0)$ pairing which have the SO(8) dynamical symmetry. The problem of the classification of the states for the case of the total pairing is solved in [17] for particles in one shell. It is also based on chain of the shell-model Lie algebra $U(4\Omega) \supset [U(\Omega) \supset SO(\Omega)] \otimes [SU_{ST}(4) \supset SU_S(2) \otimes SU_T(2)]$ and its complementarity to the chains of subalgebras typical for the SO(8) model. For example, the chain $SO(8) \supset SO_{ST}(6) \supset SO_S(3) \otimes SO_T(3)$ has been identified as complementary to the latter and used in the classification of the states for a given number of nucleons. The labels $[PP'P'']\,\beta(ST)$ are related to this last

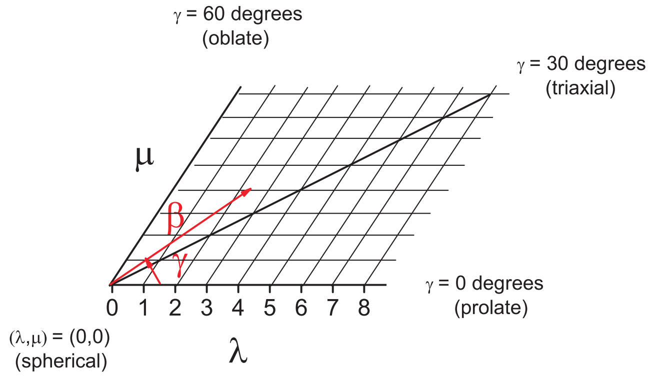

Figure 1. Traditional (β, γ) plot which demonstrates the relationship between the collective shape variables (β, γ) and the SU(3) irrep labels (λ, μ).

chain of groups, more specifically to the part $SO_{ST}(6) \supset SO_S(3) \otimes SO_T(3)$. For a certain nucleus (good T_z value), these pairing eigenstates can also be expressed as linear combinations of the SU(3) basis states (1). Solution to the scattering of pairs of particles between two orbitals has also been provided in the special case of seniority $\nu = 0$ and 1 [18].

We use a Hamiltonian that is more general by considering additionally identical-particle pair-scattering terms $(\tau \neq \tau')$, proton-neutron terms $(\pi\nu)$ and isoscalar terms which is obviously a more sophisticated form of the well-known pairing-plus-quadrupole model. It has the form

$$H_{P+Q.Q} = -\frac{\chi}{2} Q.Q - G \sum_{(JT)} \Big\{ \sum_{\sigma,\tau} (S^+)^{JT}_{\sigma\tau} (S^-)^{JT}_{\sigma\tau} + \sum_{\sigma,\tau\neq\tau'} (S^+)^{JT}_{\sigma\tau} (S^-)^{JT}_{\sigma\tau'} + \sum_{\tau,\tau'} (S^+)^{JT}_{\pi\nu,\tau} (S^-)^{JT}_{\pi\nu,\tau'} \Big\} \quad (6)$$

where, for simplicity, all pairing terms are taken with the same strength. The standard pair-creation and annihilation operators are given by

$$(S^+)^{JT}_{\sigma\tau} = \frac{1}{2} \sum_{\eta l j m_j} (-)^{l+j-m_j} [(a^\dagger_{\eta l j m_j})_{\sigma\tau} (a^\dagger_{\eta l j -m_j})_{\sigma\tau}]^{JT}$$

and

$$(S^-)^{\sigma\tau} = ((S^+)^{\sigma\tau})^\dagger.$$

The sum over (JT) describes the isovector $(J = 0, T = 1)$ and the isoscalar $(J = 1, T = 0)$ both at $L = 0$ identical-particle pairing (first term) and pair-scattering (second term), and proton-neutron pairing and pair-scattering (third term in the braces).

3 Results

We present the results from calculations performed in two shells for two $N = Z$ nuclear systems which we, although not entirely accurate, dub ^{20}Ne and ^{60}Zn. There are no imposed restrictions on the model space and we do not compare with experimental data because the interaction (6) we use could not pretend to be complete and realistic.

3.1 Pure Pairing Interaction in Two Shells

In Figures 2 and 3 (4 and 5), we can see the energy spectrum of the isovector- and the total-pairing interaction for the systems ^{20}Ne (^{60}Zn). By definition, the isovector pairing spectrum does not depend on the spin S and the isoscalar part does not depend on the isospin T which makes the difference between the Figures 2 and 3 (correspondingly 4 and 5) almost non observable. The states are clustered in groups for the different values of the total seniority quantum number which can be $\nu = 0, 2, 4$ for $J = 0$ but only $\nu = 2, 4$ for the remaining J values. While the first excited state in the isovector results is due to developing rotational features in isospace, the one in the total pairing results just reflects the pairing gap in the lower-lying (namely ds) shell of the model space.

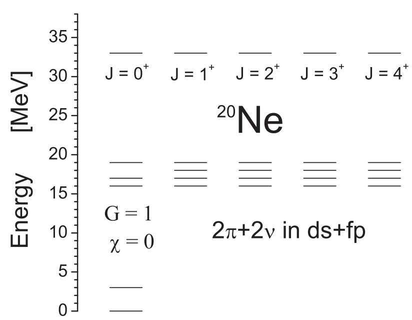

Figure 2. Isovector pairing for ^{20}Ne in the dsfp shell

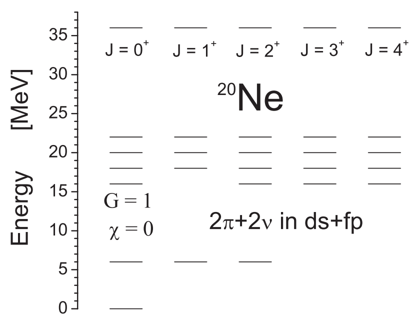

Figure 3. Total pairing for ^{20}Ne in the dsfp shell.

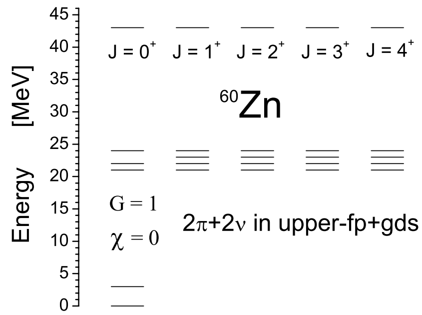

Figure 4. Isovector pairing for ^{60}Zn in the upper-fp-gds shell.

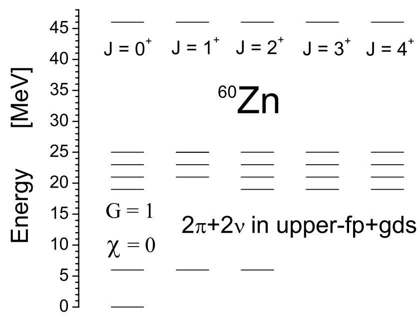

Figure 5. Total pairing for ^{60}Zn in the upper-fp-gds shell.

3.2 The Ground State: Energy and Shape

Results from calculations with the Hamiltoinan (6) of the ground-state energy for the nuclei are presented in Figure 6. The quadrupole parameter χ takes different values while the values $G = 0.2$ MeV and $G = 0.1$ MeV for the pairing interaction are fixed as the ones for a typical ds-shell (fp-shell) nucleus. For low values of the quadrupole parameter χ the isovector part has prominent contribution which becomes comparable with the effect from the isoscalar part for values of χ bigger than 0.04 MeV.

Finally, we calculate the expectation values of the two shape parameters - the deformation parameter β and the triaxiality parameter γ, given by equations (4) and (5). The results shown in Figures 7 and 8 for the ground states of the

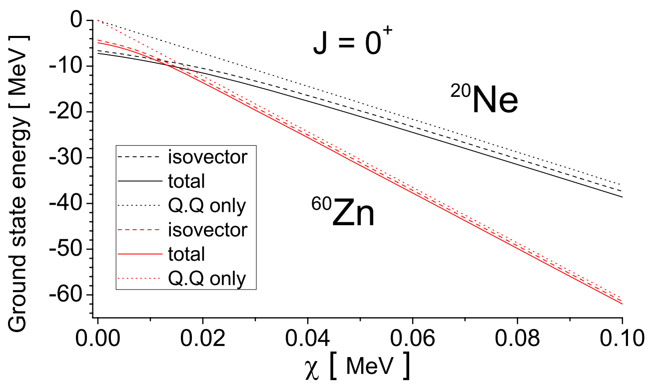

Figure 6. Ground-state energy for ^{20}Ne and ^{60}Zn in the pairing-plus-quadrupole model in two shells.

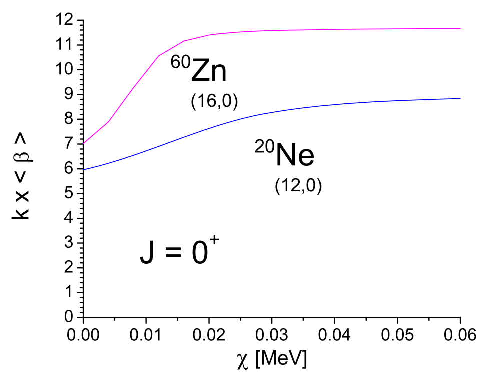

Figure 7. Deformation parameter β for ^{20}Ne and ^{60}Zn in the pairing-plus-quadrupole model in two shells.

two nuclear systems, reveal a steady rise (fall) in β (γ) for both nuclei with the increase of the parameter χ. Beyond the point marked with an arrow in Figure 8, the values of χ lead to eigenfunctions, composed primarily (at least 50 percent) of the leading irreducible representation, which can also be found on both figures under the symbol for the corresponding isotope.

4 The Extended (Pseudo-) SU(3) Model - Transition to More Elaborated Hamiltonians

The above analysis is useful for further and more realistic investigation of the role that intruder levels play in the dynamics of the system by using an extended

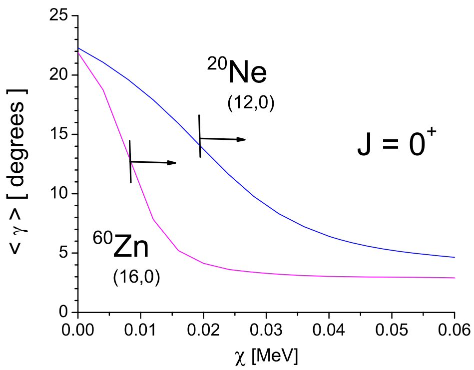

Figure 8. Triaxiality parameter γ for ^{20}Ne and ^{60}Zn in the pairing-plus-quadrupole model in two shells.

version of the (pseudo-)SU(3) model. It is a microscopic theory in the sense that both SU(3) generators - the angular momentum and the quadrupole operators - are given in terms of individual nucleon coordinate and momentum variables. Results for the quality of the pseudo-SU(3) symmetry for the nuclei ^{64}Ge, ^{68}Se [19] with realistic interactions suggest that one can perform symmetry-adapted calculations thus reducing significantly the size of the model space.

Until recently, SU(3) shell-model calculations - real SU(3) [5] for light nuclei and pseudo-SU(3) [6] for heavy nuclei - have been performed in either only one space (protons and neutrons filling the same shell, e.g. the ds shell) or two spaces (protons and neutrons filling different shells, e.g. for rare-earth and actinide nuclei). Number of interesting and important results for low-energy features like energy spectra, shape description and electromagnetic transition strengths, have been published over the years [20–22].

The Hamiltonian that one can use for extended-SU(3)-model calculations will be of the form

$$H = H_{s.p.} + H_{P+Q.Q} + aJ^2 + bK_J^2 + c_2C_2 + c_3C_3. \tag{7}$$

In addition to the Hamiltoian (6) it includes single-particle interaction $H_{s.p.}$ as well as four rotor-like terms. These terms - the square of the total angular momentum J^2, its projection on the intrinsic body-fixed axis K_J^2 and the Casimir operators of SU(3) of second and third order C_2 and C_3 - are used to “fine tune” the energy spectra. The first one (J^2) adjusts the moment of inertia of the ground band, the second (K_J^2) - the position of the gamma $K = 2^+$ bandhead, the third (C_2) distinguishes representations with both λ and μ even from the others, having zero strength in the first case and a positive value in the second, and the fourth (C_3) drives representations with $\mu \gg \lambda$ lower in energy than those with $\mu \ll \lambda$.

5 Conclusion

The study presented here for the pairing-plus-quadrupole Hamiltonian in two oscillator shells should serve as a base for more elaborated investigations of these and other interesting nuclear systems. More sophisticated interactions should be developed and tested in order to reveal the role of the particles from the unique-parity sector as well as the effectiveness of various truncations on the model space.

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