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Nuclear Symmetry Energy and Surface Properties of Exotic Nuclei

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Abstract. We study the correlation between the thickness of the neutron skin in finite nuclei and the nuclear symmetry energy for isotopic chains of even-even Ni, Sn, and Pb nuclei in the framework of the deformed self-consistent mean-field Skyrme HF+BCS method. The symmetry energy, the neutron pressure and the asymmetric compressibility in finite nuclei are calculated within the coherent density fluctuation model using the symmetry energy as a function of density within the Brueckner energy-density functional. The mass dependence of the nuclear symmetry energy and the neutron skin thickness are also studied together with the role of the neutron-proton asymmetry. A correlation between the parameters of the equation of state (symmetry energy and its density slope) and the neutron skin is suggested in the isotopic chains of Ni, Sn, and Pb nuclei.

1 Introduction

Recently, the interest in the symmetry energy has been stirred up by novel astrophysical observations and by the availability of exotic beams in accelerators that provide additional information to the standard nuclear asymmetry studies based on stable nuclei. Particularly important in the different areas, and similarly uncertain, is the density dependence of the symmetry energy in uniform matter. The neutron skin thickness, generally defined as the difference between neutron and proton rms radii in the atomic nucleus, is closely correlated with this dependence. Moreover, it has been shown that the neutron skin thickness in heavy nuclei, like ^{208}Pb, calculated in mean-field models with either nonrelativistic or relativistic effective nuclear interactions, displays a linear correlation with the slope of the neutron equation of state (EOS) obtained with the same interactions at a neutron density $\rho \approx 0.10$ fm^{-3} $[1, 2]$.

The symmetry energy of finite nuclei at saturation density is often extracted by fitting ground state masses with various versions of the liquid-drop mass formula within liquid-drop models [3–5]. It has been also studied in the random phase approximation based on the Hartree-Fock (HF) approach [6] or effective relativistic Lagrangians with density-dependent meson-nucleon vertex functions [7], energy density functionals of Skyrme force [8, 9] as well as relativistic nucleon-nucleon interaction [10, 11]. In the present work the symmetry energy will be studied in a wide range of finite nuclei on the basis of the Brueckner energy-density functional for nuclear matter [12, 13] and using the coherent density fluctuation model (CDFM) (e.g., Refs. [14, 15]). The CDFM has been successfully applied for different tasks: to calculate nuclear properties of the ground and first monopole states, in scaling analyses etc.

In the present work (see also [16]) we investigate the relation between the neutron skin thickness and some nuclear matter properties in finite nuclei, such as the symmetry energy, symmetry pressure (proportional to the slope of the bulk symmetry energy), and asymmetric compressibility, considering nuclei in given isotopic chains and within a certain theoretical approach. In addition to various linear relations between several quantities in bulk matter and for a given nucleus that have been observed and tested within different theoretical methods (e.g. nonrelativistic calculations with different Skyrme parameter sets and relativistic models), we are looking forward to establish a possible correlation between the skin thickness and these quantities and to clarify to what extent this correlation is appropriate for a given isotopic chain. As in our previous papers [17, 18], we choose some medium and heavy Ni and Sn isotopes, because, first, they are primary candidates to be measured at the upcoming experimental facilities, and second, for them several predictions have been made concerning the nuclear skin emergence. In addition, we present some results for a chain of Pb isotopes being inspired by the significant interest (in both experiment [19] and theory [20]) to study, in particular, the neutron distribution of ^{208}Pb and its rms radius. The densities of these nuclei were calculated within a deformed HF+BCS approach with Skyrme-type density-dependent effective interactions [21, 22]. As already mentioned, in the present work we use the CDFM as a link between the description of nuclear matter quantities (on the example of the Brueckner *et al.* [12, 13] energy-density functional) and the calculations of the corresponding intrinsic properties of realistic finite nuclear systems.

2 Theoretical Framework

2.1 The Key EOS Parameters in Nuclear Matter

The symmetry energy $S(\rho)$ is defined by a Taylor series expansion of the energy per particle for nuclear matter (NM) in terms of the isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$

$$E(\rho,\delta) = E(\rho,0) + S(\rho)\delta^2 + O(\delta^4) + \cdots \ , \tag{1}$$

where $\rho = \rho_n + \rho_p$ is the baryon density with ρ_n and ρ_p denoting the neutron and proton densities, respectively (see, e.g. [23, 24]). Odd powers of δ are forbidden by the isospin symmetry and the terms proportional to δ^4 and higher orders are found to be negligible.

Near the saturation density ρ_0 the energy of isospin-symmetric matter, $E(\rho,0)$, and the symmetry energy, $S(\rho)$, can be expanded as

$$E(\rho,0) = E_0 + \frac{K}{18\rho_0^2}(\rho-\rho_0)^2 + \cdots \ , \tag{2}$$

and

$$\begin{aligned} S(\rho) &= \frac{1}{2}\left.\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\right|_{\delta=0} \\ &= a_4 + \frac{p_0}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K}{18\rho_0^2}(\rho-\rho_0)^2 + \cdots \ . \end{aligned} \tag{3}$$

The parameter a_4 is the symmetry energy at equilibrium ($\rho=\rho_0$). The pressure p_0^{NM}

$$p_0^{NM} = \rho_0^2 \left.\frac{\partial S}{\partial \rho}\right|_{\rho=\rho_0} \tag{4}$$

and the curvature ΔK^{NM}

$$\Delta K^{NM} = 9\rho_0^2 \left.\frac{\partial^2 S}{\partial \rho^2}\right|_{\rho=\rho_0} \tag{5}$$

of the nuclear symmetry energy at ρ_0 govern its density dependence and thus provide important information on the properties of the nuclear symmetry energy at both high and low densities. The widely used "slope" parameter L^{NM} is related to the pressure p_0^{NM} [Eq. (4)] by

$$L^{NM} = \frac{3p_0^{NM}}{\rho_0}. \tag{6}$$

2.2 Symmetry Energy Parameters of Finite Nuclei in CDFM

The CDFM was suggested and developed in Refs. [14,15]. The model is related to the delta-function limit of the generator coordinate method [15,25]. It is shown in the model that the one-body density matrix of the nucleus $\rho(\mathbf{r},\mathbf{r}')$ can be written as a coherent superposition of the one-body density matrices for spherical "pieces" of nuclear matter (so-called "fluctons") with densities

$$\rho_x(\mathbf{r}) = \rho_0(x)\Theta(x-|\mathbf{r}|), \tag{7}$$

where

$$\rho_0(x) = \frac{3A}{4\pi x^3}. \tag{8}$$

The generator coordinate x is the radius of a sphere containing Fermi gas of all A nucleons uniformly distributed in it.

Considering the pieces of nuclear matter with density $\rho_0(x)$ (8) one can use for the matrix element $V(x)$ of the nuclear Hamiltonian the corresponding nuclear matter energy from the method of Brueckner *et al.* [12, 13]. In this energy-density method the expression for $V(x)$ reads

$$V(x) = AV_0(x) + V_C - V_{CO}, \tag{9}$$

where

$$\begin{aligned} V_0(x) = {} & 37.53[(1+\delta)^{5/3} + (1-\delta)^{5/3}]\rho_0^{2/3}(x) \\ & + b_1\rho_0(x) + b_2\rho_0^{4/3}(x) + b_3\rho_0^{5/3}(x) \\ & + \delta^2[b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x)] \end{aligned} \tag{10}$$

with $b_1 = -741.28$, $b_2 = 1179.89$, $b_3 = -467.54$, $b_4 = 148.26$, $b_5 = 372.84$, and $b_6 = -769.57$. $V_0(x)$ in Eq. (9) corresponds to the energy per nucleon in nuclear matter (in MeV) with the account for the neutron-proton asymmetry. V_C is the Coulomb energy of protons in a "flucton" and V_{CO} is the Coulomb exchange energy. Thus, in the Brueckner EOS [Eq. (10)], the potential symmetry energy turns out to be proportional to δ^2. Only in the kinetic energy the dependence on δ is more complicated. Substituting $V_0(x)$ in Eq. (3) and taking the second derivative, the symmetry energy $S^{NM}(x)$ of the nuclear matter with density $\rho_0(x)$ (the coefficient a_4 in Eq. (3)) can be obtained:

$$S^{NM}(x) = 41.7\rho_0^{2/3}(x) + b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x). \tag{11}$$

The corresponding analytical expressions for the pressure $p_0^{NM}(x)$ and asymmetric compressibility $\Delta K^{NM}(x)$ of such a system in the Brueckner theory have the form:

$$p_0^{NM}(x) = 27.8\rho_0^{5/3}(x) + b_4\rho_0^2(x) + \frac{4}{3}b_5\rho_0^{7/3}(x) + \frac{5}{3}b_6\rho_0^{8/3}(x) \tag{12}$$

and

$$\Delta K^{NM}(x) = -83.4\rho_0^{2/3}(x) + 4b_5\rho_0^{4/3}(x) + 10b_6\rho_0^{5/3}(x). \tag{13}$$

Our basic assumption within the CDFM is that the symmetry energy, the slope and the curvature for finite nuclei can be defined weighting these quantities for nuclear matter (with a given density $\rho_0(x)$ (8)) by means of the weight function $|\mathcal{F}(x)|^2$ given below. Thus, in the CDFM they will be an infinite superposition of the corresponding nuclear matter quantities. Following the CDFM scheme, the symmetry energy, the slope and the curvature have the following forms:

$$s = \int_0^\infty dx|\mathcal{F}(x)|^2 S^{NM}(x), \tag{14}$$

$$p_0 = \int_0^\infty dx|\mathcal{F}(x)|^2 p_0^{NM}(x), \tag{15}$$

$$\Delta K = \int_0^{\infty} dx |\mathcal{F}(x)|^2 \Delta K^{NM}(x). \tag{16}$$

If one takes the delta-function approximation to the Hill-Wheeler integral equation in the generator coordinate method one gets a differential equation for the weight function $\mathcal{F}(x)$ [15, 25]. Instead of solving this differential equation we adopt a convenient approach to the weight function $|\mathcal{F}(x)|^2$ proposed in Refs. [14, 15]. In the case of monotonically decreasing local densities (*i.e.* for $d\rho(r)/dr \leq 0$), the latter can be obtained by means of a known density distribution $\rho(r)$ for a given nucleus:

$$|\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)} \left. \frac{d\rho(r)}{dr} \right|_{r=x}. \tag{17}$$

The normalization of the weight function is:

$$\int_0^{\infty} dx |\mathcal{F}(x)|^2 = 1. \tag{18}$$

3 Results of Calculations and Discussion

As the main emphasis of the present study is to inspect the correlation of the neutron skin thickness ΔR of nuclei in a given isotopic chain with the s, p_0 and ΔK parameters extracted from the density dependence of the symmetry energy around saturation density, we show first in Figure 1 the results for Ni isotopes. The symmetry energy, the pressure and the asymmetric compressibility are calculated within the CDFM according to Eqs. (14)-(16) by using the weight functions (17) calculated from the self-consistent HF+BCS densities. The differences between the neutron and proton rms radii of these isotopes are obtained from HF+BCS calculations using four different Skyrme forces, SLy4, SG2, Sk3

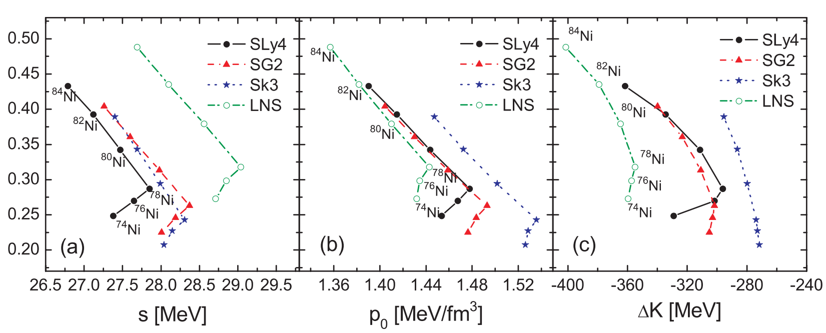

Figure 1. HF+BCS neutron skin thicknesses ΔR for Ni isotopes as a function of the symmetry energy s (a), pressure p_0 (b), and asymmetric compressibility ΔK (c) calculated with SLy4, SG2, Sk3, and LNS forces.

and LNS. It is seen from Figure 1(a) that there exists an approximate linear correlation between ΔR and s for the even-even Ni isotopes with $A = 74 - 84$. We observe a smooth growth of the symmetry energy till the double-magic nucleus ^{78}Ni ($N = 50$) and then a linear decrease of s while the neutron skin thickness of the isotopes increases. This behavior is valid for all Skyrme parametrizations used in the calculations, in particular, the average slope of ΔR for various forces is almost the same. The LNS force yields larger values of s comparing to the other three Skyrme interactions. In this case the small deviation can be attributed to the fact that the LNS force has not been fitted to finite nuclei and therefore, one cannot expect a good quantitative description at the same level as purely phenomenological Skyrme forces. As a consequence, the neutron skin thickness calculated with LNS force has larger size with respect to the other three forces whose results for ΔR are comparable with each other.

The analysis of the correlation between the neutron skin thickness and some macroscopic nuclear matter properties in finite nuclei is continued by showing the results for a chain of Sn isotopes. This is done in Figure 2, where the results obtained with SLy4, SG2, Sk3, and LNS Skyrme forces are presented for isotopes with $A = 124 - 152$. Similarly to the case of Ni isotopes with transition at specific shell closure, we observe a smooth growth of the symmetry energy till the double-magic nucleus ^{132}Sn ($N = 82$) and then an almost linear decrease of s while the neutron skin thickness of the isotopes increases. In Ref. [17] we have studied a formation of a neutron skin in tin isotopes with smaller A where very poor experimental information is available. For instance, a large uncertainty is shown to exist experimentally in the neutron skin thickness of ^{124}Sn, *i.e.*, its value varies from 0.1 to 0.3 fm depending on the experimental method. Our theoretical prediction $\Delta R = 0.17$ fm for this nucleus is found to be within the above experimental band. A similar approximate linear correlation between ΔR and p_0 for Sn isotopes is also shown in Figure 2(b). The asymmetric compressibility ΔK given in Figure 2(c) is less correlated than p_0 with ΔR within the Sn isotopic chains.

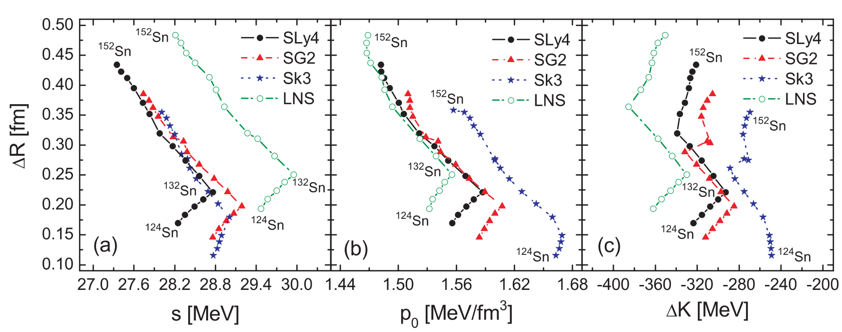

Figure 2. Same as in Figure 1, but for Sn isotopes.

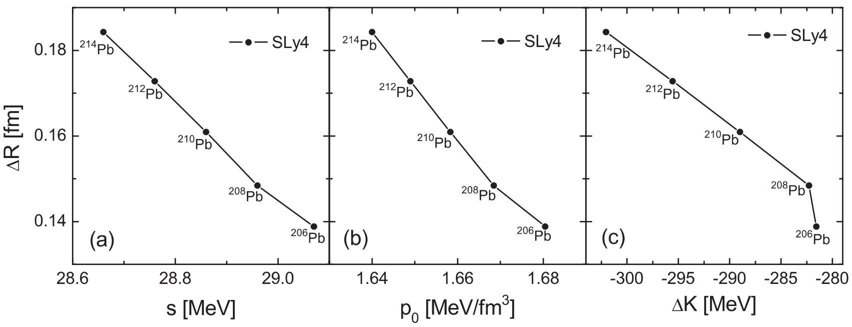

Figure 3. HF+BCS neutron skin thicknesses ΔR for Pb isotopes as a function of the symmetry energy s (a), pressure p_0 (b), and asymmetric compressibility ΔK (c) calculated with SLy4 force.

In Figure 3 we display the theoretical neutron skin thickness ΔR of nuclei from a Pb isotopic chain against the parameters of interest s, p_0 and ΔK. Similarly to the three panels for Ni and Sn isotopes presented in Figures 1 and 2, respectively, the predicted correlations manifest the same linear dependence. However, in this case the kink observed at the double magic ^{208}Pb nucleus is much less pronounced that it can be seen at ^{78}Ni and ^{132}Sn isotopes and it does not change its direction. The value of ΔR for ^{208}Pb deduced from present HF+BCS calculations with SLy4 force is lower than the predicted thickness by the Skyrme Hartree-Fock model [8] and by the extended relativistic mean-field model [11], but fits well the values calculated with self-consistent densities of several nuclear mean-field models (see Table I in Ref. [26]). The p_0 and ΔK values for ^{208}Pb are in a good agreement with those from Ref. [8].

As it has been shown by Brown and Typel (see [1,2]), and confirmed later by others [8, 23], the neutron skin thickness calculated in mean-field models with either nonrelativistic or relativistic effective interactions is very sensitive to the density dependence of the nuclear symmetry energy and, in particular, to the slope parameter L (respectively to the pressure p_0 [Eq. (6)]) at the normal nuclear saturation density. We note that while the basic idea behind studies of this type of correlations, namely between the neutron skin thickness and the symmetry energy, is to constrain the symmetry energy in bulk matter from experimental results in finite nuclei, the main aim of our work is to extract quantitative information about s, p_0 and ΔK values for finite nuclei in the CDFM. Moreover, so far in our work we present another type of correlation, namely how these quantities are related to ΔR within a given isotopic chain apart from the same one for a specific nucleus using different theoretical models. Along this line we analyze the latter correlation and the results for several Sn isotopes (^{132}Sn, ^{134}Sn, ^{156}Sn) and ^{78}Ni nucleus are shown in Figure 4. Using the four sets of Skyrme interaction parameters, a linear fit to the correlation between ΔR and p_0 is performed to illustrate qualitatively the above correlation. It is seen from Figures 4(a) and

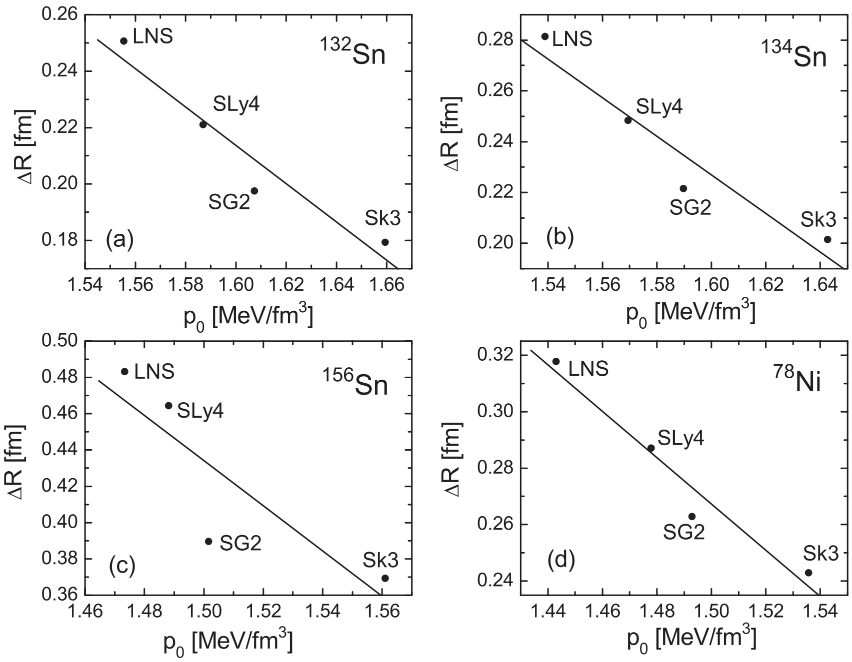

Figure 4. HF+BCS neutron skin thicknesses ΔR for ^{132}Sn (a), ^{134}Sn (b), ^{156}Sn (c), and ^{78}Ni (d) as a function of the pressure p_0 for four sets of Skyrme interaction parameters: SLy4, SG2, Sk3, and LNS. The lines in all panels represent linear fits.

4(d) that in the cases of double-magic ^{132}Sn and ^{78}Ni nuclei ΔR almost linearly correlates with p_0. If one goes from ^{132}Sn nucleus further to unstable nuclei within the same Sn isotopic chain the correlation becomes weaker, as it can be seen from Figures 4(b) and 4(c), being almost the same in the case of neighbouring ^{134}Sn nucleus and poorly expressed in the case of far away ^{156}Sn isotope. Due to the limited number of Skyrme parametrizations used in our theoretical approach, the results presented in Figure 4 cannot provide a definite constraint of the slope (p_0 value) of the symmetry energy. Obviously, more complete confirmation of the existing correlation between the neutron skin thickness and the pressure for a given nucleus can be drawn when large set of nuclear models may be considered.

4 Conclusions

A theoretical approach to the nuclear many-body problem combining the deformed HF+BCS method with Skyrme-type density-dependent effective interactions and the CDFM has been used to study nuclear properties of finite nuclei. For this purpose, we examined three chains of neutron-rich Ni, Sn, and

Pb isotopes, most of them being far from the stability line and representing an interest for future measurements with radioactive exotic beams. Four Skyrme parametrizations were involved in the calculations: SG2, Sk3, SLy4, and LNS.

For the first time, we have demonstrated the capability of CDFM to be applied as an alternative way to make a transition from the properties of nuclear matter to the properties of finite nuclei investigating the nuclear symmetry energy s, the neutron pressure p_0 and the asymmetric compressibility ΔK in finite nuclei. This has been carried out on the base of the Brueckner energy-density functional for infinite nuclear matter. One of the advantages of the CDFM is the possibility to obtain transparent relations for the intrinsic EOS quantities analytically by means of a convenient approach to the weight function.

We have found that there exists an approximate linear correlation between the neutron skin thickness of even-even nuclei from the Ni ($A = 74 - 84$), Sn ($A = 124 - 152$), and Pb ($A = 206 - 214$) isotopic chains and their nuclear symmetry energies. Our HF+BCS calculations lead to a symmetry energy in the range of 27-30 MeV, which is in agreement with the empirical value of 30 ± 4 MeV [27]. In the cases of Ni and Sn isotopes the symmetry energy is found to increase almost linearly with the mass number A till the double-magic nuclei (^{78}Ni and ^{132}Sn) and then to decrease, being larger when LNS force is used. A similar linear correlation between ΔR and p_0 is also found to exist, while the relation between ΔR and ΔK is less pronounced. The same behavior containing an inflexion point transition at specific shell closure is observed for these correlations. The calculated values of $p_0 = 1.36 - 1.68$ MeV/fm^3 obtained in our calculations lead to values of the slope parameter $L = 26 - 32$ MeV that are in agreement with other theoretical predictions (see, for example [8]). Hence, the study of other two EOS parameters (whose determination is more uncertain than the symmetry energy), namely the slope and the curvature of s, performed in our work may provide more constraints on both the density dependence of the nuclear symmetry energy and the thickness of the neutron skin in neutron-rich medium and heavy nuclei.

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