On the Extraction of the Probability of Two- and Three-Nucleon Short Range Correlations in Nuclei

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Abstract. Recent experimental data on inclusive and exclusive lepton and hadron scattering off nuclei have renewed the interest in theoretical and experimental studies of Short Range Correlations (SRC), due to the relevant impact they may have not only on the structure of ordinary nuclei but on the structure of hadronic matter at high densities as well. One of the ultimate aim of these studies is the determination of the probability of two- and three-nucleon correlations in nuclei. To this end, we have studied the possibility to extract these probabilities from a novel analysis of inclusive $A(e, e')X$ processes in terms of relativistic scaling variables which incorporate effects from two- and three-nucleon SRC, with a resulting scaling function strictly related to longitudinal momentum distributions; such an approach led to a satisfactory explanation of the cross section ratios recently found at JLab and interpreted as strong evidence of SRC in nuclei.

1 Introduction

New data on inclusive quasi elastic (q.e.) electron scattering off nuclei, $A(e, e')X$, at high momentum transfer ($2.5 \lesssim Q^2 \lesssim 7.4$ GeV^2) are under analysis at the Thomas Jefferson National Accelerator Facility (JLab) [1]. Nowadays one of the aims of the investigation of q.e. scattering off nuclei is to obtain information on Nucleon-Nucleon (NN) short range correlations (SRC); to this end various approaches are being pursued, such as the investigation of the scaling behavior of the ratio of the inclusive cross section σ_2^A of heavy nuclei to that of ^{2}H and ^{3}He plotted versus the Bjorken scaling variable x_{Bj} [2, 3], or the analysis of cross sections in terms of Y-scaling [4]. The aim of this paper is to critically review these analyses and propose a novel approach to $A(e, e')X$ processes particularly suited to treat the effects of SRC. In order to illustrate the basic ideas of our approach [5], some general concepts of Y-scaling have to be recalled.

2 Inclusive Lepton Scattering and Y-scaling

Within the Plane Wave Impulse Approximation (PWIA), the inclusive q.e. cross section can be written as follows [6]

$$\sigma_2^A(q,\nu) \equiv \frac{d^2\sigma(q,\nu)}{d\Omega_2\,d\nu} = F^A(q,\nu)\,K(q,\nu)\,[Z\sigma_{ep} + N\sigma_{en}] \tag{1}$$

where

$$F^A(q,\nu) = 2\pi \int_{E_{\min}}^{E_{\max}(q,\nu)} dE \int_{k_{\min}(q,\nu,E)}^{k_{\max}(q,\nu,E)} k\,dk\,P^A(k,E) \tag{2}$$

is the nuclear structure function, $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ and $\nu = \epsilon_1 - \epsilon_2$ are the three-momentum and energy transfers ($Q^2 = q^2 - \nu^2 = 4\epsilon_1\epsilon_2 \sin^2\frac{\theta}{2}$, with $q \equiv |\mathbf{q}|$), σ_{eN} is the elastic electron cross section off a moving off-shell nucleon with momentum $k \equiv |\mathbf{k}|$ and removal energy E, $K(q,\nu)$ is a kinematical factor, and, eventually, $P^A(k,E)$ is the spectral function of nucleon N. As is well known, $P^A(k,E) = P_0^A(k,E) + P_1^A(k,E)$, where $P_0^A(k,E)$ is the (trivial) shell-model part and $P_1^A(k,E)$ is the (interesting) component generated by NN correlations [7]. Considering, for ease of presentation, high values of the momentum transfer such that $E_{\max}(q,\nu)$ and $k_{\max}(q,\nu,E)$ become very large, the replacement $E_{\max} = k_{\max} = +\infty$ is justified by the rapid falloff of $P^A(k,E)$ with k and E. Without any loss of generality, we can substitute the energy transfer ν with a generic scaling variable $Y = Y(q,\nu)$; in this case, the scaling function (2) can be cast as follows [6]

$$F^A(q,Y) = f^A(Y) - B^A(q,Y) \tag{3}$$

where $f^A(Y) = 2\pi \int_{|Y|}^\infty k\,dk\,n^A(k)$ represents the longitudinal momentum distribution, and

$$B^A(q,Y) = 2\pi \int_{E_{\min}}^\infty dE \int_{|Y|}^{k_{\min}(q,Y,E)} k\,dk\,P_1^A(k,E) \tag{4}$$

is the so called binding correction. The longitudinal momentum distribution depends only upon the nucleon momentum distributions $n^A(k) = \int P^A(k,E)\,dE$, which, as is well known [8] and illustrated in Figure 1, at high values of the momentum k approximately scale with A according to $n^A(k) \simeq C^A\,n^D(k)$, where $n^D(k)$ is the momentum distribution of the Deuteron; the binding correction $B^A(q,Y)$, on the contrary, depends upon the correlated part of the spectral function $P_1^A(k,E)$. In the Deuteron case, one has $E = E_{\min} = 2.22$ MeV, $k_{\min}(q,Y,E_{\min}) = |Y|$, $B^D(q,Y) = 0$ and $F^D(q,Y) = f^D(Y)$, from which the nucleon momentum distributions can be obtained by the relation $n^A(k) = -[df^A(Y)/dY]/[2\pi Y]$; in general, however, $B^A(q,Y) \neq 0$ and $F^A(q,Y) \neq f^A(Y)$ and the momentum distributions cannot be obtained. The central idea
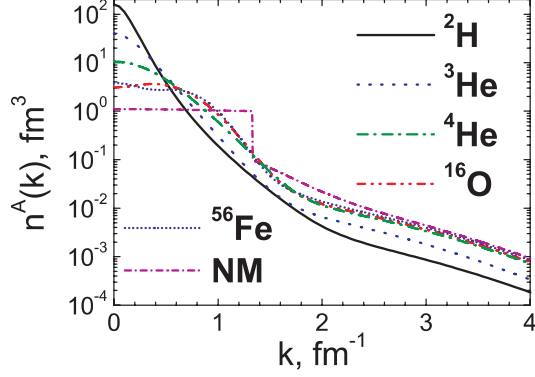
Figure 1. The nucleon momentum distributions $n^A(k)$ for nuclei ranging from ^{2}H to NM. It can be seen that, at high values of the momentum k, $n^A(k)$ can be approximately considered as a rescaled version of the momentum distributions of ^{2}H. After Ref. [9].

of our approach [5], is that the contribution arising from the binding correction could be minimized by a proper choice of the scaling variable Y, such that $k_{\min}(q, Y, E) \simeq |Y|$, with the resulting cross section (1) depending only upon the nucleon momentum distributions; by this way, a direct access to high momentum components generated by SRC could be obtained. It is clear that the outlined picture can in principle be modified by the effects of the final state interactions (FSI); this important point will be discussed later on.

2.1 Traditional Approach to Y-Scaling: the Mean Field Scaling Variable y

The traditional scaling variable $Y \equiv y$ is obtained by placing $k = |y|$, $\cos \alpha = (\mathbf{k} \cdot \mathbf{q}/kq) = 1$ and $E^*_{A-1} = 0$ in the energy conservation law given by

$$\nu + M_A = \sqrt{(M_{A-1} + E^*_{A-1})^2 + \mathbf{k}^2} + \sqrt{m_N^2 + (\mathbf{k} + \mathbf{q})^2} \qquad (5)$$

where E^*_{A-1} is the intrinsic excitation energy of the $(A - 1)$-nucleon system and the other notations are self explained. In such an approach, y represents the minimum longitudinal momentum of a nucleon having the minimum value of the removal energy $E = E_{\min} + E^*_{A-1} = E_{\min} = m_N + M_{A-1} - M_A$. In the asymptotic limit $(q \to \infty)$, one has $k^\infty_{\min}(q, y) = |y - (E - E_{\min})|$ [6], so that, when $E = E_{\min}$, one gets $k^\infty_{\min}(q, y) = |y|$ and $B^A(q, y) = 0$; this occurs only when $A = 2$, whereas in the general case, $A > 2$, the excitation energy E^*_{A-1} of the residual system is different from zero, leading to $B^A(q, y) > 0$. The binding correction plays indeed a relevant role in the traditional approach to Y-scaling. To illustrate this, the ratio $B^A(q, y)/F^A(q, y)$ is shown in Figure 2; it can be seen that at high (negative) values of y, the effects from binding are very large. Moreover, the experimental scaling function
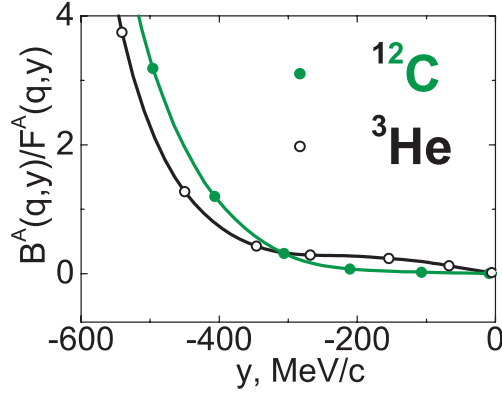
Figure 2. The ratio of the binding correction $B^A(q, y)$ given by (4) to the scaling function $F^A(q, y)$ given by (2) for ^{3}He (open dots) and ^{12}C (full dots), calculated using the scaling variable y. After Ref. [5].

$F_{\exp}^A(q, y) = \sigma_{\exp}/[K(q, y)\ (Z\sigma_{ep} + N\sigma_{en})]$ plotted versus the scaling variable y confirms, as shown in Figure 3, that the scaling function strongly differs from the longitudinal momentum distribution, and therefore does not exhibits any proportionality to the Deuteron scaling function $f^D(y)$.
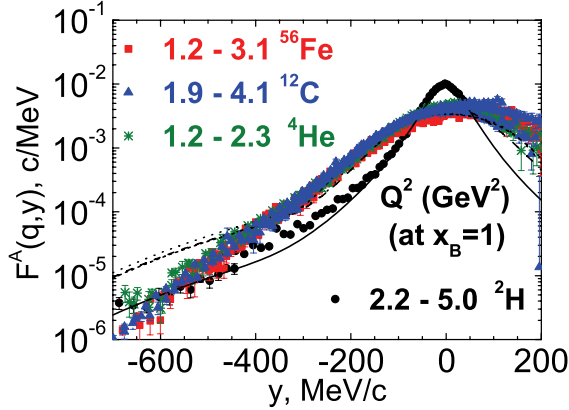

Figure 3. The experimental scaling function $F_{\exp}^A(q, y)$ of ^{4}He, ^{12}C, and ^{56}Fe obtained from the experimental data of Refs. [10,11]. The longitudinal momentum distributions of ^{2}H (full line), ^{4}He (long-dashed), ^{12}C (dashed) and ^{56}Fe (dotted) are also shown. After Ref. [5].

2.2 A Novel Approach to Y-Scaling: the Scaling Variable Embedding Two-Nucleon Correlations (2NC)

2NC are defined as those nucleon configurations where momentum conservation in the ground state of the target nucleus ($\sum_1^A \mathbf{k}_i = 0$) is almost entirely exhausted by two correlated nucleons with high and opposite momenta, with the spectator $(A - 2)$-nucleon system being almost at rest. Since high excitation states of the final $(A - 1)$-nucleon system are generated by SRC in the ground state of the target nucleus, the traditional (mean field) scaling variable y does not incorporate, by definition, SRC effects, for it is obtained by placing $E^*_{A-1} = 0$ in the energy conservation law (5). Motivated by this observation, in Ref. [4], a new scaling variable $Y \equiv y_{CW} \equiv y_2$ has been introduced by setting in (5) $k = |y_2|$, $\cos \alpha = (\mathbf{k} \cdot \mathbf{q}/kq) = 1$ and $E^*_{A-1} = \langle E^*_{A-1}(k) \rangle_{2NC}$, which represents the momentum dependent average excitation energy of $(A - 1)$ generated by 2NC. Let us stress that this quantity is not a kind of parameter, but is a quantity that can realistically be calculated in terms of the nucleon Spectral function [9]. By this way, y_2 properly includes the momentum dependence of the average excitation energy of the $(A - 1)$-nucleon system generated by SRC. The approach of Ref. [4] has been further improved in Ref. [5], obtaining a scaling variable y_2 which, through the k-dependence of $\langle E^*_{A-1}(k) \rangle_{2NC}$, interpolates between the correlations and the mean field regions of the q.e. cross section. The relevant feature of y_2 is that it leads to $k_{\min}(q, y_2, E) \simeq |y_2|$ and therefore to a minor role of the binding correction; this is indeed demonstrated in Figure 4, which clearly shows that $B^A(q, y_2)$ vanishes in the whole region of y_2 considered. One can therefore conclude that, using the new scaling variable y_2, one obtains $F^A(q, y_2) \sim f^A(y_2) \sim C^A f^D(y_2)$.

The experimental scaling function $F^A(q, y_2)$ of ^4He, ^{12}C and ^{56}Fe is plotted in Figure 5 versus the scaling variable y_2; it can be seen that at high values
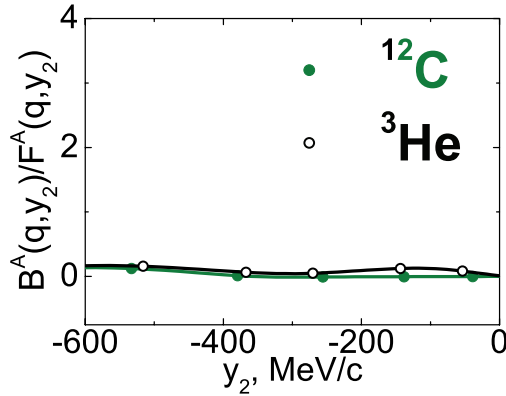

Figure 4. The same as in Figure 2, obtained using in (2) and (4) the scaling variable $y_2 \equiv y_{CW}$. After Ref. [5].
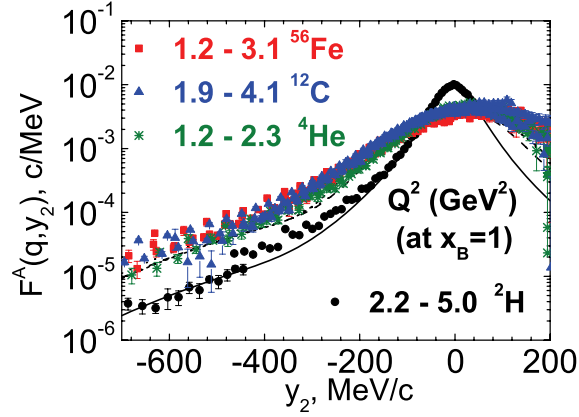
Figure 5. The same as in Figure 3 *vs.* the scaling variable $y_2 \equiv y_{CW}$. After Ref. [5].
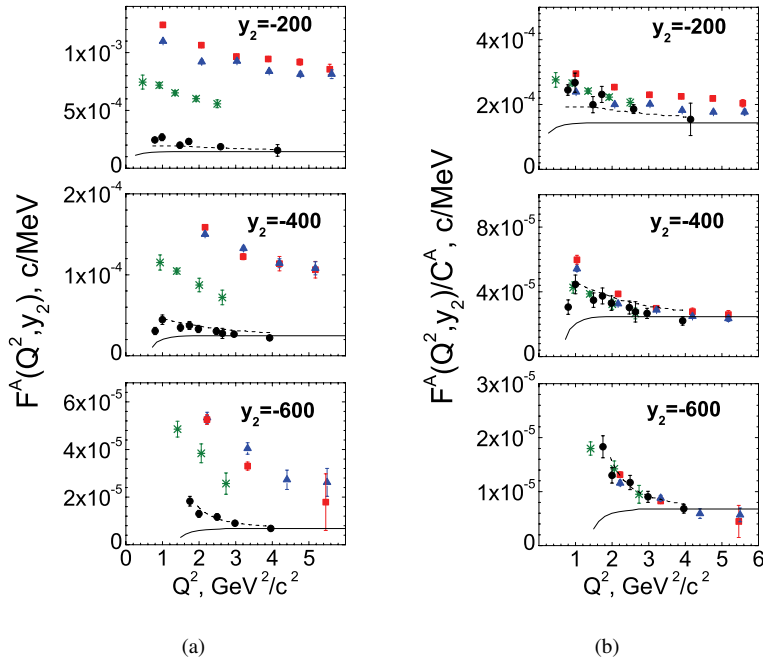

(a)   (b)

Figure 6. (a) The scaling function $F^A(Q^2, y_2)$ *vs.* Q^2, at fixed values of $y_2 \equiv y_{CW}$; (b) the same data divided by the constants $C^4 = 2.7$, $C^{12} = 4.0$ and $C^{56} = 4.6$, respectively for ^{4}He, ^{12}C and ^{56}Fe. The theoretical curves represent the longitudinal momentum of the Deuteron, calculated (AV18 interaction) in PWIA (full line) and including FSI (dashed line) effects. After Ref. [5].

of $|y_2|$, the relation $F^A(q, y_2) \sim f^A(y_2) \sim C^A \, f^D(y_2)$ is indeed experimentally confirmed. In order to analyze more quantitatively the scaling behavior of $F^A(q, y_2)$, the latter has been plotted versus Q^2, at fixed values of y_2.

The result is shown in Figure 6, together with the theoretical scaling function for $A = 2$, calculated in PWIA (solid line), and taking FSI into account (dashed line) [6]. It can be seen in Figure 6(a) that, due to FSI effects, scaling is violated and approached from the top, and not from the bottom, as predicted by the PWIA. However, the violation of scaling seems to exhibit a Q^2-dependence which is very similar in Deuteron and in complex nuclei. This is illustrated in more details in Figure 6(b), which shows $F^A(Q^2, y_2)$ divided by a constant C^A, chosen so as to obtain the Deuteron scaling function $F^D(Q^2, y_2)$. It clearly appears that the scaling function of heavy and light nuclei scales to the Deuteron scaling function; it is also important to stress that, although FSI are very relevant, they appear to be similar in Deuteron and in a nucleus A, which is evidence that, in the SRC region, FSI are mainly restricted to the correlated pair.

2.3 A Novel Approach to Y-Scaling: the Scaling Variable Embedding Three-Nucleon Correlations (3NC)

3NC correspond to those nucleon configurations when the high momentum $\mathbf{k}_1 \equiv \mathbf{k}$ of nucleon "1" is almost entirely balanced by the momenta $\mathbf{k}_2$ and $\mathbf{k}_3$ of nucleons "2" and "3". Let us investigate the presence and relevance of 3NC configurations in the spectral function of the 3-nucleon system for which the Schroedinger equation has been solved exactly. In Figure 7, the realistic spectral function of ^{3}He obtained [12] using realistic wave functions [13] corresponding to the AV18 interaction [14] (full squares), is compared with the predictions of the 2NC model (solid line) [9]. It can be observed that 2NC reproduce the exact
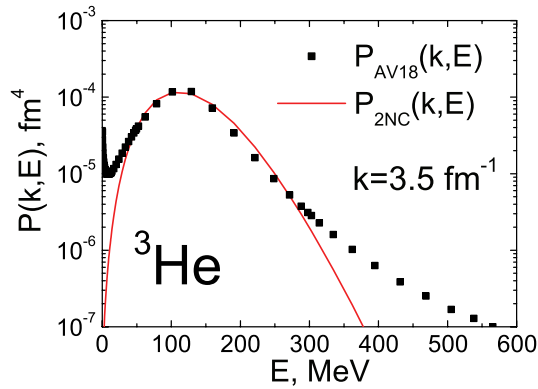

Figure 7. The spectral function of ^{3}He *vs.* the removal energy E, at $k = 3.5$ fm^{-1} [12], corresponding to realistic wave functions (squares) [13] and to the 2NC model of Ref. [9] (full line) [15].
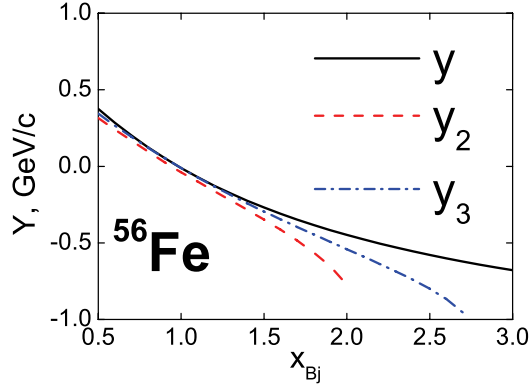
Figure 8. The scaling variables y, y_2 and y_3 vs. x_{Bj} for $A = 56$, calculated at $\langle Q^2 \rangle = 2.8\ (\text{GeV/c})^2$

spectral function in a wide range of removal energies ($50 \lesssim E \lesssim 200$ MeV), but fail at very low and very high values of E, where the effects from 3NC are expected to provide an appreciable contribution. Let us investigate how 3NC can show up in available experimental data. The scaling variables y and y_2 have been obtained by placing different values of E^*_{A-1} in (5), namely $E^*_{A-1} = 0$ and $E^*_{A-1} = \langle E^*_{A-1}(k)\rangle_{2NC}$, respectively. We have derived the scaling variable embedding 3NC, $Y \equiv y_3$, by placing in (5) $E^*_{A-1} = \langle E^*_{A-1}(k)\rangle_{3NC}$. The explicit expression of $\langle E^*_{A-1}(k)\rangle_{3NC}$ and y_3 will be given elsewhere [15]. Here we show in Figure 8, in the case of ^{56}Fe, the values of y, y_2 and y_3 plotted versus x_{Bj}. It can be seen that, because of the different values of E^*_{A-1} used in (5), different limits of existence of the three scaling variables are obtained: y describes the mean field configuration and is defined in the whole range of $x_{Bj} \leq A$; y_2 represents 2NC in heavy nuclei resembling the ones acting in Deuteron and is defined only for $x_{Bj} \leq 2$; y_3, eventually, describes 3NC as in ^{3}He, and is defined only for values of x_{Bj} up to 3.

3 Cross Section Ratio: Preliminary Results

As mentioned in previous sections, our novel approach to inclusive lepton scattering off nuclei is based upon the introduction of proper scaling variables that effectively include the energy E^*_{A-1} of the residual system and allow one to describe the $A(e, e')X$ cross section only in terms of nucleon momentum distributions generated by 2N and 3N SRC, *i.e.*

$$
\frac{d^2\sigma}{d\Omega_2\, d\nu} \propto \int_{E_{\min}}^{E_{\max}(q,\nu,E)} dE \int_{k_{\min}(q,\nu,E)}^{k_{\max}(q,\nu,E)} kdk\; P^A(k, E)
$$

$$
\simeq \int_{|y|}^{\infty} n_0^A(k)\, kdk + \int_{|y_2|}^{\infty} n_2^A(k)\, kdk + \int_{|y_3|}^{\infty} n_3^A(k)\, kdk \quad (6)
$$

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where $n_0^A(k)$ is the component of the nucleon momentum distribution generated by the mean field,

$$n_2^A(k) = \int dk_{CM}\, n_{\mathrm{rel}}(\mathbf{k} + \mathbf{k}_{CM})\, n_{CM}^{\mathrm{soft}}(\mathbf{k}_{CM}) \qquad (7)$$

is the one due to 2NC and, eventually,

$$n_3^A(k) = \int dk_{CM}\, n_{\mathrm{rel}}(\mathbf{k} + \mathbf{k}_{CM})\, n_{CM}^{\mathrm{hard}}(\mathbf{k}_{CM}) \qquad (8)$$

is the one due to 3NC; here, $n_{CM}^{\mathrm{soft}}(\mathbf{k}_{CM})$ and $n_{CM}^{\mathrm{hard}}(\mathbf{k}_{CM})$ include only "soft" and "hard" momentum components, respectively. Within such an approach, the cross section ratio $r(A/A')$ reduces to the scaling function ratio of nuclei A and A'.
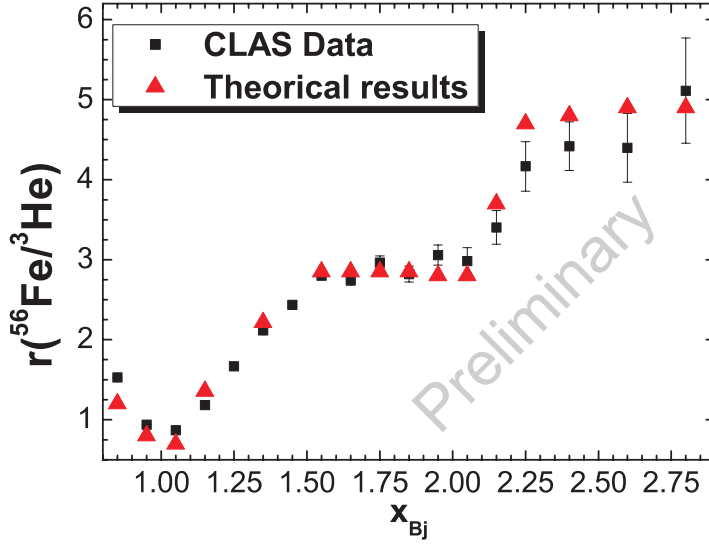

Figure 9. The experimental cross section ratio from CLAS data [3] compared with our preliminary theoretical results.

This picture is modified by the effects of the final state interaction, which can be implemented by replacing the momentum distributions with the distorted momentum distributions. Our preliminary results of the calculations of the ratio $r(^{56}\mathrm{Fe}/^3\mathrm{He}) = (2/56)\sigma_2^{56}/\sigma_D$ are shown in Figure 9, and a qualitative agreement with CLAS data can be observed.

4 Conclusions

The main findings of our analysis can be summerized as follows: (i) the experimental scaling function in the 2NC region scales to the Deuteron scaling func-

tion and exhibits A-independent FSI effects, mostly due to the FSI in the correlated pair; (ii) proper scaling variables have been introduced which effectively include the excitation energy $\langle E^*_{A-1}(k) \rangle$ of the residual system generated by 2NC and 3NC, and allow one to describe the $A(e, e')X$ cross section in terms of the corresponding momentum distributions generated by 2NC and 3NC; (iii) the experimental ratio $r(^{56}\text{Fe}/^3\text{He})$ in the 2NC and 3NC region qualitatively agrees with our preliminary results. Calculations for other nuclei are in progress [15].

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